Method for assessing discharge in meandering compound channels

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Laboratory experiments were conducted in a large-scale meandering compound channel to investigate the flow patterns at the apex and cross-over sections. An equation in curvilinear co-ordinates was derived from the momentum equation and the flow continuity equation. In this equation, two shear stress terms are ignored due to their negligible contributions such that simple expressions are obtained. Several groups of experimental data from the published literature were used to verify this model. Comparison of the experimental and predictive results indicates that the proposed method is capable of accurately forecasting the stage discharge in meandering compound channels. Finally, the simple analytical solution of this model and the velocity parameter are further discussed.

Notation

B	total channel width
B_m	meander belt width
b	meandering main channel width
C_p	contribution percentage
D_r	relative flow depth, defined as $D_r = (H-h)/H$
F_c	centrifugal force
f	Darcy–Weisbach friction factor
g	local gravitational acceleration
H	flow depth
h	bankfull depth
k	dimensionless coefficient reflecting the relation between secondary flows and F_c
n	Manning's coefficient
Q	total discharge
Q_{fp}	floodplain discharge
Q_{mc}	main channel discharge
q	local unit discharge, defined as $q = \int_0^H U dz$
R	outer radius
R_h	hydraulic radius
R_m	medium radius
r	inner radius
S	bed slope in Equation 1
S_{fp}	valley slope
S_{mc}	meandering main channel slope, defined as $S_{mc} = S_{fp}/s$

s	sinuosity of a meandering main channel
U, V, W	velocity components corresponding to the x, y, z directions, respectively
U_*	shear velocity
U_d	depth-averaged velocity
V_o	original secondary current cell
V_{fp}	component of floodplain flow
x, y, z	streamwise, lateral and vertical directions, respectively
ε	depth-averaged eddy viscosity
ε_r	absolute relative error
Γ	velocity parameter
λ	dimensionless eddy viscosity
ρ	flow density
$\tau_{xx}, \tau_{yx}, \tau_{zx}$	Reynolds shear stresses
$\overline{\tau_{yx}}$	depth-averaged Reynolds shear stress
ϕ	dimensionless coefficient

Subscripts

i	'mc' or 'fp', indicating the value in the main channel or on floodplains
j	term number in Equation 3

1. Introduction

A meandering channel is often observed in natural alluvial rivers, and the curved main channel balances sediment transport

and bed erosion. When a flood occurs, the floodplains aid the meandering main channel to convey the extra water downstream such that a meandering compound channel is observed. The flow characteristics in the meandering compound channel are different from those in the straight channel. In a straight channel, a velocity difference is observed between the main channel flow and the floodplain flow, which causes strong lateral shear stresses in the mixing region. In a meandering channel, the flow under the bankfull level prefers to progress downstream in the meandering main channel, but the floodplain flow travels along the valley direction (Shiono and Muto, 1998), which causes strong vertical shear stresses near the bankfull level in the cross-over sections. Owing to the effects of centrifugal force and upstream flood plain flow, the secondary flow in the meandering main channel is strengthened, which may affect lateral sediment transport and longitudinal bed morphology (Lyness *et al.*, 2001; Shiono *et al.*, 2008, 2009; Spooner, 2001).

The stage–discharge curve is important in hydraulic engineering projects. With consideration of the complicated flow interaction between the meandering main channel flow and the floodplain flow, many methods have been presented for predicting the stage–discharge curve (Greenhill and Sellin, 1993; Patra and Kar, 2000; Patra *et al.*, 2004; Shiono *et al.*, 1999) based on the divided panel method and the classical Manning’s equation. However, the predictive capabilities of these methods were not satisfactory compared with the performance of these methods in straight channels. In addition, several two-dimensional (2D) and three-dimensional (3D) numerical models offer high-accuracy modelling performance in the curved channel (Guo *et al.*, 2008; Jing *et al.*, 2009; Morvan *et al.*, 2002; Shao *et al.*, 2003; Zarrati *et al.*, 2005). Also, certain 2D analytical methods that model the lateral distribution of depth-averaged velocity at the apex section have been presented (Ervine *et al.*, 2000; Huai *et al.*, 2009; McGahey and Samuels, 2003) and, recently, a 2D analytical method capable of predicting lateral velocity profiles along a meander was proposed (Liu *et al.*, 2014a). These models indicate that modelling work in a meandering channel is difficult because additional factors must be considered (Ervine *et al.*, 1993). In addition, curvilinear co-ordinates are more suitable for the meandering channel than vertical co-ordinates (Schlichting, 1968). Therefore, for engineering purposes, a simple method that is able to accurately predict the stage–discharge curve is worth investigating.

In this research, three groups of experiments were conducted in a large-scale meandering compound channel (Figure 1). The detailed 3D velocities and Reynolds shear stresses were recorded along a meander. Based on these experimental data, the appropriate cross-section for predicting discharges is determined after the flow patterns are analysed in the meandering main channel. A governing equation in curvilinear co-ordinates is derived from the momentum equation and the flow continuity equation. In this equation, two shear stress terms are reasonably ignored due to their negligible

Figure 1. Photograph of the large-scale meandering compound channel at SKLH (looking downstream). This channel is a 35 m-long, 4 m-wide and 1 m-high experimental flume

contributions. Thus, simple analytical solutions are obtained in the main channel and on the floodplain. Finally, the proposed model is verified using several groups of experimental data from the published literature.

2. Experimental data used

2.1 New experiments

The present experiments were conducted in a large-scale meandering compound channel constructed in a 35 m-long, 4 m-wide and 1 m-high flume at the State Key Laboratory of Hydraulics and Mountain River Engineering (SKLH) at Sichuan University in China (Figure 1). In this meandering channel, the stage–discharge curve was measured using an automatic ultrasonic measurement system produced by the Sinfotek Corporation. The 3D velocities, turbulences and Reynolds shear stresses were recorded by a three-component Sontek acoustic Doppler velocimeter for three cases (i.e. MN1, MN2 and MN3) with relative flow depths of $D_r \approx 0.25$, 0.35 and 0.45, respectively, where $D_r = (H - h)/H$, H is the flow depth and h is the bankfull height (Figure 2). The original point of the curvilinear co-ordinate system was set at the right-side corner of the main channel at CS1. The shape of the main channel was rectangular

Figure 2. Cross-section of the meandering main channel with 13 vertical measurement lines at CS1. In the experiments, the channel width (b) and the bankfull depth (h) are 0.7 and 0.14 m, respectively. The lateral interval between vertical measurement lines is 0.05 m

with a sinuosity $s=1.381$. The valley slope (S_{fp}) was 0.001. In the meandering main channel, the lateral interval of 13 vertical measurement lines was 0.05 m. The measurement intervals between two vertical points were arranged as 0.015 m.

Four and a half meanders were constructed in this large flume. The top view of the test meander is shown in Figure 3. The width of the main channel (b) was designed as 0.7 m, giving an aspect ratio (b/h) of 5.0. The inner radius (r) and the outer radius (R) were 0.9 and 1.6 m, respectively, such that the medium radius (R_m) was 1.25 m. The details of the geometry parameters are shown in Table 1. All experiments were conducted under quasi-uniform flow conditions by ensuring that

the water surface slope remained parallel to the bed slope at each meander. This condition was achieved by manually adjusting the tailgate at the end of the flume. When the deviation between the water surface and the bed slope reached less than 5%, it was determined that quasi-uniform flow had been attained, and then measurements commenced.

2.2 Data from the published literature

Data were selected from the published literature (Lambert and Sellin, 1996; Shiono *et al.*, 1999; Spooner, 2001), and details of the experimental parameters are shown in Table 2. Different scales and geometry parameters might have different effects on the predictive accuracy, and, therefore, these data were selected from large and small meandering channels with different sinuosities. Brief introductions for these experiments are provided for convenience.

2.2.1 Source 1

The experimental flume consisted of a rectangular cross-section that was 13 m long, 2.4 m wide and 0.3 m deep with a valley slope $S_{fp}=0.002$ located at Loughborough University. This flume was constructed from grass and acrylic. The width of the main channel was 0.4 m, with a bankfull level of 0.04 m, giving an aspect ratio of $h/b=10$. The data acquisition system and experimental set-up are described in detail by Spooner (2001).

Figure 3. Top view of the test meander with the curvilinear co-ordinate system

Total channel width, B : m	Meander belt width, B_m : m	Wavelength: m	Inner radius, r : m	Outer radius, R : m	Cross-over length, L_0 : m	Side slope: degrees	Valley slope, S_{fp}	Sinuosity, s
4.0	2.99	5.53	0.9	1.6	1.2	90	0.001	1.381
Series name	Discharge, Q : m ³ /s	Flow depth, H : m	Bankfull depth, h : m	Relative flow depth, D_r	Floodplain condition			
MN1	0.189	0.255	0.14	0.451	Smooth			
MN2	0.113	0.216	0.14	0.352	Smooth			
MN3	0.085	0.189	0.14	0.259	Smooth			

Table 1. Geometry parameters and experimental conditions

Source	Valley slope, S_{fp}	Main channel, S_{mc}	Sinuosity, s	Main channel width, b : m	Channel width, B : m	Bankfull depth, h : m	Medium radius, R_m : m	Cross-over length, L_0 : m
Spooner (2001)	0.002	0.001445	1.384	0.4	2.4	0.04	0.765	0.75
Case 2, Shiono <i>et al.</i> (1999)	0.001–0.002	0.00073–0.00146	1.372	0.152	1.2	0.052	0.425	0.376
Case 4, Shiono <i>et al.</i> (1999)	0.0005–0.002	0.000458–0.00183	1.092	0.15	1.2	0.052	0.425	0.377
Case 5, Shiono <i>et al.</i> (1999)	0.0005–0.002	0.000318–0.00127	1.571	0.15	1.2	0.052	0.425	0
Lambert and Sellin (1996)	0.001	0.000847	1.18	0.85	4.9	0.15	1.365	0

Table 2. Experimental parameters for the cases from the published literature

2.2.2 Source 2

A series of experiments was conducted in a 9.2 m-long, 1.2 m-wide and 0.35 m-high flume in the hydraulics laboratory at the University of Bradford (Shiono *et al.*, 1999). The entire flume was made of acrylic, with different meandering channel configurations. Five cases (1–5) were investigated in different meandering channels. The data measured in the channel with small ($s=1.092$, case 4), medium ($s=1.372$, case 2) and large sinuosities ($s=1.571$, case 5) were selected. Different valley slopes ($S_{fp}=0.0005–0.002$) were also considered. In these three cases, the detailed measurements of the velocities were collected by way of laser Doppler anemometry under two relative flow depths of $D_r=0.15$ and 0.5 (Shiono and Muto, 1998). These velocity data were provided by Professor Koji Shiono at Loughborough University and Professor Muto Yasunori at the University of Tokushima. Hence, further discussion of the velocity parameter is possible.

2.2.3 Source 3

A distorted 1:5 scale model was constructed in the 56 m-long Flood Channel Facility at the laboratory of HR Wallingford in

the UK. This model was moulded in cement mortar with a valley slope $S_{fp}=0.001$. The main channel was 0.85 m wide and 0.15 m deep, giving an aspect ratio (b/h) of 5.67, and the sinuosity was 1.18. Once the stage–discharge relationship was obtained, detailed velocity measurements were conducted for three typical flow depths. Hence, sub-area discharges were obtained. Additional details can be found in Lambert and Sellin (1996).

3. Flow patterns at the apex and the middle cross-section

Before introducing the theoretical background, the flow patterns at different cross-sections in the meandering compound channel must be understood. Based on the experimental data, the streamwise velocity distribution is presented at two typical cross-sections (the apex section CS1 and the middle cross-section CS4), as shown in Figure 4. From this figure, it can be observed that the magnitude of the velocity at CS1 appears larger than that at CS4. At the apex, dense contour lines are noted at the upper region next to the left-hand floodplain; however, dense contour lines are observed at the bankfull level near the right-side floodplain (black dashed circles in Figure 4)

Figure 4. Streamwise velocity (U) distributions in the meandering main channel at two typical sections (the black dashed circle indicates the high vertical gradient of the U zone): (a) apex section (CS1), (b) middle cross-section (CS4)

in CS4 because the upstream flood plain flow plunges into the meandering main channel and collides with the main channel flow. This process converts additional kinetic energy into heat by producing much stronger vertical shear stresses (Figure 5) and local super elevation. Therefore, the conveyance capability at the middle cross-section is weaker than that at the apex. In addition, Liu *et al.* (2014a) and Shiono and Muto (1998) have observed that the flow directions in the main channel and floodplain are the same at the apex section (CS1) and different at the middle cross-section (CS4). In summary, the flow pattern is uniform at the apex section, where the flow directions in the main channel and on the floodplain are the same. Therefore, it is suggested that the apex section is the best cross-section in the meandering compound channel for prediction of the stage–discharge curve.

4. Theoretical background

It is more suitable to use the momentum equation and the flow continuity equation in curvilinear co-ordinates in the meandering channel. Spooner and Shiono (2003) used 2D curvilinear equations, and developed a method for modelling the depth-averaged velocity in meandering compound channels.

Under the curvilinear co-ordinate system, the x axis is parallel to the meandering main channel, and the y axis is normal to the meandering main channel (see Figure 3). For the condition of quasi-uniform flow in the meandering channel, the streamwise momentum equation and the flow continuity equation in curvilinear co-ordinates are combined, and the result is

$$1. \quad \frac{r}{r+y} \frac{\partial \rho U^2}{\partial x} + \frac{\partial \rho UV}{\partial y} + \frac{\partial \rho UW}{\partial z} + \frac{2\rho UV}{r+y} = \frac{r}{r+y} \rho g S + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \frac{2\tau_{yx}}{r+y}$$

where x , y and z are the streamwise, lateral and vertical directions, respectively; U , V and W are the velocity components corresponding to the x , y and z directions, respectively; ρ is the flow density; g is the local gravitational acceleration; S is the bed slope; and τ_{xx} , τ_{yx} and τ_{zx} are the Reynolds shear stresses.

In Equation 1, $\partial \rho UV / \partial y$ and $2\rho UV / (r+y)$ represent the same dimension, and thus it is assumed that $2\rho UV / (r+y) = \phi (\partial \rho UV / \partial y)$, where ϕ is a dimensionless coefficient. $\phi = 0$ when $r \rightarrow \infty$.

Figure 5. Vertical shear stress (τ_{zx}) distributions in the meandering main channel at two typical sections (the black dashed circle indicates the strong τ_{zx} zone): (a) apex section (CS1); (b) middle cross-section (CS4)

According to the generation mechanism of secondary flows in the meandering channel (Liu *et al.*, 2014a), in the main channel the secondary currents consist of the original secondary current cell (V_o) enhanced by the centrifugal force (F_c) and the component of floodplain flow (V_{fp}). Liu *et al.* (2014a) suggested $F_c = k[\partial(\rho UV_o)/\partial y]$, where k is a dimensionless coefficient reflecting the relationship between secondary flows and F_c . Hence, $V = (1+k)V_o + V_{fp}$. Thus, Equation 1 may be rewritten as

$$\begin{aligned} & \frac{\partial \rho U^2}{\partial x} + \frac{r+y}{r}(1+\phi) \left[(1+k) \frac{\partial \rho UV_o}{\partial y} + \frac{\partial \rho UV_{fp}}{\partial y} \right] \\ & + \frac{r+y}{r} \frac{\partial \rho UW}{\partial z} \\ & = \rho g S + \frac{r+y}{r} \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) + \frac{2\tau_{yx}}{r} \end{aligned} \quad (2)$$

In this research, the focus is on the sub-area discharge and the total discharge. Darby and Thorne (1996) and Liu *et al.* (2014b) proposed simplified versions of the flow momentum and continuity equations for the lateral distribution of the local unit discharge ($q = \int_0^H U dz$) in a vegetated straight compound channel. As such, Equation 2 is integrated over the flow depth, and Equation 3 is obtained as

$$\begin{aligned} \underbrace{\Gamma}_1 &= \underbrace{\rho g HS}_2 + \underbrace{\frac{\partial}{\partial y} \left(\frac{1}{2} \rho \frac{r+y}{r} \lambda \sqrt{\frac{f}{8}} \frac{\partial q^2}{\partial y} \right)}_3 \\ & - \underbrace{\frac{\rho}{H^2} \frac{r+y}{r} \frac{f}{8} q^2}_4 + \underbrace{\frac{2}{r} \left(\frac{1}{2} \rho \lambda \sqrt{\frac{f}{8}} \frac{\partial q^2}{\partial y} \right)}_5 \end{aligned} \quad (3)$$

$$\begin{aligned} \overline{\tau}_{yx} &= \frac{\rho \varepsilon}{H} \frac{\partial q}{\partial y} \quad \tau_b = \frac{\rho}{H^2} \frac{f}{8} q^2 \\ \varepsilon &= \lambda H U_* \quad U_* = \frac{1}{H} \sqrt{\frac{f}{8}} q \end{aligned} \quad (4)$$

where Γ is the velocity parameter that represents the longitudinal velocity variation and lateral secondary flow variation; λ is the dimensionless eddy viscosity; f is the Darcy–Weisbach friction factor; $\overline{\tau}_{yx}$ is the depth-averaged Reynolds shear stress; ε is the depth-averaged eddy viscosity; and U_* is the shear velocity. In Equation 3 ‘term 1’ through ‘term 5’ represent the velocity variation term, the gravity term, the gradient of shear stress term, the bed shear stress term and the shear stress term, respectively.

At the apex section, the component of floodplain flow (V_{fp}) can be ignored because the angle between the streamwise

direction and the flow direction is nearly 0° (Liu *et al.*, 2014a). Therefore, the velocity parameter is

$$\Gamma = \frac{r}{r+y} \frac{\partial \rho H U_d^2}{\partial x} + (1+\phi) \left[(1+k) \frac{\partial (\rho H U V_o)_d}{\partial y} \right] \quad (5)$$

where U_d is the depth-averaged velocity.

Noting that the velocity term in Equation 5 is expressed in velocity components (U and V), it is easy to understand its physical significance associated with the longitudinal velocity variation and the lateral secondary flow variation. Using the present study’s experimental data (MN1), longitudinal profiles of U_d^2 at seven vertical measurement lines (lines 1, 3, 5, 7, 9, 11 and 13) are shown in Figure 6. From this figure, it is noted that for these seven vertical lines the maximal U_d^2 value always occurs at the apex sections. In addition, the mirror longitudinal profiles of U_d^2 at lines 1 and 13 are observed because of the periodical flow pattern in the meandering channel. Therefore, $\partial U_d^2 / \partial x = 0$ at the apexes due to the extreme-value theory. Thus, Equation 5 can be further simplified as

$$\Gamma = (1+\phi)(1+k) \frac{\partial (\rho H U V_o)_d}{\partial y} = (1+\phi) \frac{\partial (\rho H U V)_d}{\partial y} \quad (6)$$

Equation 6 advances the understanding that, at the apex section, the secondary flow is enhanced only by the centrifugal force, and it demonstrates that good prediction at the apex section can still be obtained by ignoring the longitudinal velocity variation, similar to the work of Ervine *et al.* (2000), Huai *et al.* (2009) and McGahey and Samuels (2003).

Figure 6. Longitudinal distributions of U_d^2 at measurement lines 1, 3, 5, 7, 9, 11, and 13 (see Figure 3 for lateral locations) in the meandering main channel based on the experimental data (MN1)

It is practically impossible to solve Equation 3 analytically because it takes the form of an inhomogeneous differential equation when the parameters are given. This difficulty arises because the bed shear stress term (term 4) simultaneously contains the independent variable (y) and the dependent (q). Therefore, one or more terms must be ignored to obtain the analytical solution of Equation 3. To address this difficulty, the orders of magnitude of the five terms are compared. Several cases are selected from different sources with different channel scales, flow depths and geometry conditions (from the data from the experiments reported here and those of Shiono and Muto (1998) and Spooner (2001)). Most importantly, the sub-area discharges are reported in these cases. To conveniently conduct comparisons, the contribution percentage (C_p) is defined as

$$7. \quad C_p = |\text{term } j| / \sum_{j=1}^5 |\text{term } j| \times 100\%$$

where $j=1-5$ in Equation 3. For different cases, the C_p values of five terms are examined, as shown in Figure 7. From this figure, the gravity term shows the highest proportion of contribution (approximately 50%). The second largest contribution (35.42%) is from the velocity term, and the third comes from the bed shear stress term (14.21%). However, the shear stress term and its gradient term account for only 0.31% and 0.89%,

Figure 7. Comparison of the contribution percentage (C_p) for five terms in Equation 3 based on the data from the experiments reported here and those of Shiono and Muto (1998) and Spooner (2001)

respectively, which indicates that the two shear stress terms can be ignored due to their negligible contributions. Therefore, the simple relationship

$$8. \quad \Gamma = \rho g H S - \frac{\rho}{H^2} \frac{r + y f}{r} q^2$$

is obtained. In Equation 8, the error induced by ignoring two shear stress terms is noted. In Equations 3 and 8, the gravity term and the bed shear stress term are the same, and hence, the velocity parameter (Γ) reflects the prediction error. From this analysis, the C_p values of term 3 and term 5 in Equation 3 are only 0.89% and 0.31%, respectively. Therefore, the difference between the velocity parameters in Equations 3 and 8 is only 1.2%. The impact of this difference on the prediction will be addressed in the discussion.

Equation 8 is similar to the simplified equation proposed by Spooner and Shiono (2003), but the largest difference between the two approaches is the definition of the velocity variation term (i.e. the left-hand side of Equation 8). Spooner and Shiono (2003) used a term that included a parameter K and U_a^2 to replace the velocity variation term. However, in this paper, the velocity variation term is assumed to be a constant across the channel. To verify this assumption, the velocity parameter is back-calculated using the experimental data (MN1, MN2 and MN3), as shown in Figure 8. From this figure, the lateral profiles of Γ indicate that this assumption is reasonable but is only demonstrated at $D_r \geq 0.25$. Spooner and Shiono (2003) observed large lateral variations of the velocity variation term in low-discharge cases ($D_r = 0.0$ and 0.2). This point will be discussed further with respect to applications.

Figure 8. Lateral profiles of the velocity parameter Γ_j in the meandering main channel based on the experimental data (MN1, MN2 and MN3)

Next, the analytical solution of the discharge per unit width in the meandering main channel is presented as

$$9. \quad q = \sqrt{\frac{H^2}{\rho} \frac{r}{r+y} \frac{8}{f}} (\rho g H S - \Gamma)$$

Laterally integrating Equation 9, the main channel discharge (Q_{mc}) is obtained as

$$10. \quad Q_{mc} = 2(\sqrt{r(r+b)} - r) \sqrt{\frac{H^2}{\rho} \frac{8}{f_{mc}}} (\rho g H S_{mc} - \Gamma_{mc})$$

where S_{mc} is the slope of meandering main channel ($= S_{fp}/s$).

On the floodplain, $r \rightarrow \infty$. Therefore, the floodplain discharge (Q_{fp}) is

$$11. \quad Q_{fp} = (B-b) \sqrt{\frac{(H-h)^2}{\rho} \frac{8}{f_{fp}}} (\rho g (H-h) S_{fp} - \Gamma_{fp})$$

where B is the width of the entire channel.

Finally, the total discharge ($Q = Q_{mc} + Q_{fp}$) is obtained. It should be noted that Equations 10 and 11 contain only two undetermined parameters (i.e. the Darcy–Weisbach friction factor f_i and the velocity parameter Γ_i) when the geometry parameters are given in a meandering channel, where i is ‘mc’ or ‘fp’, indicating the value in the main channel or on the floodplain, respectively. Two simple relationships, Equations 10 and 11, are thus helpful in predicting the sub-area discharge and the stage–discharge.

5. Determinations of parameters

5.1 Darcy–Weisbach friction factor

The Darcy–Weisbach friction factor (f_i) is usually back-calculated by the depth-averaged velocities and boundary shear stresses (Rameshwaran and Shiono, 2007; Shiono and Knight, 1991). However, instruments were not available for measuring the boundary shear stresses in the experiments reported here, and the data for bed shear stress were not reported by Lambert and Sellin (1996), Shiono *et al.* (1999) or Spooner (2001). Moreover, bed shear stresses are not often collected, particularly in natural rivers. Therefore, Equation 12, proposed by Huai *et al.* (2008) and Knight *et al.* (2007), is used to predict f_i in the main channel and on the floodplain.

$$12. \quad f_i = \frac{8gn^2}{R_h^{1/3}}$$

where R_h is the hydraulic radius, and n is Manning’s coefficient.

From Equation 12, the friction factor is only associated with Manning’s coefficient (n) and the hydraulic radius (R_h). Therefore, f_i is determined when the channel bed condition and the flow depth are given. Liu *et al.* (2014a) used Equation 12 to predict f_i in the meandering compound channel, and obtained accurate lateral distributions of depth-averaged velocity along a meander, thus demonstrating the predictive validity of Equation 12 for f_i .

5.2 Velocity parameter

The only undetermined parameter is the velocity parameter (Γ_i). Although the expression for Γ_i (see Equation 6) at the apex sections was obtained, its determination is still not straightforward. Shiono and Knight (1991) noted that the secondary flow term was not easily obtained by direct measurement, and back-calculation might be the best way to obtain its value. To predict discharges (Q) at other flow depths, the sub-area discharges (Q_{mc} and Q_{fp}) are first needed, to back-calculate the values of Γ_i in the main channel (Equation 10) and on the floodplain (Equation 11), respectively. Next, the three-order function between Γ_i and D_r can be built. To obtain good predictions, at least three groups of Γ_i and D_r are required. Once this information is at hand, the proposed model can predict the stage–discharge relationship for that same geometry. Therefore, two methods for obtaining Q_{mc} and Q_{fp} are shown below.

Method 1. Q_{mc} and Q_{fp} are obtained from the detailed experimental data, which are often available in indoor laboratory experiments, such as those reported here and Spooner’s experiments (Spooner, 2001). Next, the velocity parameters are obtained by back-calculation.

However, in most cases, only the total discharges and flow depths are provided, particularly in natural rivers. Therefore, another method is presented to obtain the sub-area discharges.

Method 2. A classical equation was proposed by Knight and Demetriou (1983) to predict the main channel discharge, as follows.

$$13. \quad \frac{Q_{mc}}{Q} \% = 100 \left(\frac{A_{mc}}{A} + A_{\text{difference}} \right)$$

where $Q_{mc}/Q\%$ is the percentage of the flow carried by the main channel and A_{mc}/A is the ratio of the main channel flow area to the total flow area, which can be expressed as $1/[(\alpha-1)D_r+1]$ with $\alpha=B/b$, and $A_{\text{difference}}$ is a difference factor. Patra and Kar (2000) investigated $A_{\text{difference}}$, and proposed the empirical expression

$$14. \quad A_{\text{difference}} = 108 \frac{1+36D_r \ln(s)}{\delta} \left(\frac{\alpha-1}{\alpha} \right)^{1/4} \times (3.3D_r)^{4/a} e^{-9.9D_r}$$

where δ is the ratio between the main channel width and its depth (b/h). This equation reduces to the form of Knight and Demetriou (1983) in the straight channel when the sinuosity (s) is 1.0. After the main channel discharge (Q_{mc}) is obtained, Q_{fp} ($=Q-Q_{mc}$) is accordingly obtained. Finally, the velocity parameters are back-calculated in the main channel and on the floodplain.

6. Application and discussion

6.1 Application in different cases

Different sources of experimental data are used to verify the proposed model. Based on the detailed sub-area discharges from the experiments reported here and Spooner's experiments (Spooner, 2001), method 1 determines the velocity parameters, and Figure 9 shows that the predictive stage-discharge curves closely follow the experimental data. However, the data for the sub-area discharges are not sufficient (less than three groups) in the experiments of Lambert and Sellin (1996) and Shiono *et al.* (1999). Therefore, the velocity parameters are determined by method 2 in those cases. Figure 10 indicates that predictive discharges are approximately equal to the experimental data, although the sub-area discharges are initially unknown. However, it should be noted that, in case 2, large discrepancies were observed at low discharges, which may be attributed to the assumption of the velocity parameter, which is considered to be a constant across the apex section. Spooner and Shiono (2003) reported that the velocity variation term varied significantly in the lateral direction in low-discharge cases ($D_r=0.0$ and 0.2). Therefore, the assumption in this paper requires further investigation when the discharge is low. Nevertheless, in most cases, Figures 9 and 10 demonstrate that this model is capable of predicting the stage-discharge curve in meandering compound channels, whether method 1 or 2 is used to determine the velocity parameter.

6.2 Discussion

As previously mentioned in the foregoing section, the difference between the velocity parameters in Equations 3 and 8 is only 1.2%. To show the impact of this difference on the predictive discharge, the absolute relative error ε_r is proposed to avoid the offsetting of positive and negative values.

$$15. \quad \varepsilon_r = \frac{1}{N} \sum_{i=1}^N \left| \frac{Q_{\text{predicted}} - Q_{\text{measured}}}{Q_{\text{measured}}} \right| \times 100\%$$

where $Q_{\text{predicted}}$ and Q_{measured} are the predicted and measured discharges, respectively, and N is the number of Q_{measured} . In this formulation, Q is the total discharge ($=Q_{mc}+Q_{fp}$).

The Γ_i value is calculated using method 1 or 2, and Equation 3 and used to predict the discharges. The values of ε_r for all cases are shown in Table 3. From this table, the ε_r values from method 1 or 2 are within 5%, indicating that the predictive capability of this model is good. The small difference

Figure 9. Comparison of the experimental data and the predictive stage-discharge curve with Γ_i calculated by method 1 (see Section 5.2): (a) data from the experiments reported here; (b) data from Spooner (2001)

(0.13–2.98%) between the values of ε_r^a (Γ_i determined by method 1 or 2) and ε_r^b (Γ_i determined by Equation 3) is also noted. The average difference of $\varepsilon_r^b - \varepsilon_r^a$ is only 1.55%, thus demonstrating that it is reasonable to ignore the two shear stress terms in Equation 3. In addition, such a small error is acceptable for engineering purposes.

The sub-area discharges were reported in selected cases such that the vertical profiles of Γ_i calculated by method 1 are shown in Figure 11. From this figure, Γ_i is apparently a vertical variable that varies significantly in different meandering channels. In general, the Γ_{mc} values are larger than the Γ_{fp} values in the same meandering channel, which may be attributed to the centrifugal force in the main channel that enhances the secondary flow. The effect of the centrifugal force on the floodplain can be ignored such that $k=0$ in Equation 6. In addition, vertical co-ordinates may be more suitable on the floodplains,

Figure 10. Comparison of the experimental data and the predictive stage–discharge curve with Γ_i calculated by method 2 (see Section 5.2): (a) case 2, (b) case 4 and (c) case 5 from Shiono *et al.* (1999); (d) data from Lambert and Sellin (1996)

	Experimental data	Spooner (2001)	Shiono <i>et al.</i> (1999)			Lambert and Sellin (1996)
			Case 2	Case 4	Case 5	
ε_r^a (%)	3.45	3.05	4.24	3.11	3.25	1.15
ε_r^b (%)	4.41	5.26	4.36	4.47	6.23	2.83
$\varepsilon_r^b - \varepsilon_r^a$ (%)	0.96	2.21	0.12	1.37	2.98	1.68

^a Γ_i calculated by method 1 or 2 in Section 5.2.
^b Γ_i calculated by Equation 3.

Table 3. Absolute relative error (ε_r) for predictions using different velocity parameters

and, thus, $r \rightarrow \infty$. Accordingly, $\phi = 0$. Therefore, Equation 6 is simplified as $\Gamma = \partial(\rho H U V_o)_d / \partial y$, which is the expression of secondary flow on the floodplains.

An interesting phenomenon is shown in Figure 11. In the three channels with similar geometrical parameters (Shiono and Muto, 1998), the Γ_{fp} values are nearly identical, whereas the Γ_{mc} values vary with the sinuosities. These vertical distributions of Γ_i indicate that the velocity parameter is determined by several factors (i.e. the channel scale, geometry and flow depth). For these reasons, an initial relationship for Γ_i and D_r must be calibrated, following the steps introduced in Section 5.2 in a new meandering channel.

Indeed, Equation 9 is a modification of Manning's equation. If the secondary flow in the meandering channel is ignored (i.e. $\Gamma_i = 0$) and the Darcy–Weisbach friction factor (f_i) is substituted by Equation 12, Equation 9 may be rewritten as

$$16. \quad q = \frac{H}{n} \sqrt{\frac{r}{r+y}} R^{1/3} H S$$

If the flow is located in a shallow and wide channel ($R \approx H$), Equation 16 is changed into

$$17. \quad \frac{q}{H} = \sqrt{\frac{r}{r+y}} \frac{1}{n} H^{2/3} S^{1/2}$$

Figure 11. Vertical distributions of the velocity parameter in different cases based on the data from the experiments reported here and those of Shiono and Muto (1998) and Spooner (2001)

which is Manning's equation in curvilinear co-ordinates. However, the classical Manning's equation is only a 1D method that cannot reflect the complicated 3D flow in the meandering channel. Therefore, poor results are obtained if Manning's equation is applied to a curved channel (Greenhill and Sellin, 1993). Although complex 3D flow occurs in the meandering channel, it is considered to be a 2D flow because it is reasonable to ignore the longitudinal velocity variation (see Section 4). This analysis explains why such good predictions are obtained in different meandering channels and also indicates that good predictions in the meandering channel must contain the effect of secondary flow.

The limitations of the proposed model are worth noting. In the process of derivation, the flow condition in the meandering channel is considered to be quasi-uniform flow. Therefore, further verification is needed under other flow conditions. In addition, initial calibration of the velocity parameter is important before the proposed model is used in a new channel. It is not easy to record the velocities and boundary shear stresses in a large range of discharges in rivers, but these data can be obtained in only three typical discharges. Next, the velocity parameter can be determined by method 1 or 2, and accordingly the function between Γ_i and D_r is obtained. Once these values are determined, the proposed model can predict the stage discharges in the same channel.

7. Conclusion

- Three groups of experiments were conducted in a large-scale meandering compound channel, and the detailed experimental data along a meander were recorded. Based on these data, the distributions of streamwise velocity and vertical shear stress at the apex and the middle cross-section were compared and analysed. The best position for predicting the stage–discharge curve is the apex section.
- The method for assessing stage discharge in curvilinear co-ordinates is derived from the momentum equation and the flow continuity equation. Two shear stress terms in the control equation (Equation 3) are ignored due to their negligible contributions. The impact of this simplification on the prediction is further discussed, and the corresponding error is found to be acceptable. In addition, simple analytical solutions are presented with the effect of secondary flows.
- Several cases from the published literature are selected to verify this model. The results demonstrate that the proposed model is capable of predicting the stage discharges in the meandering channel, even if the sub-area discharge is unknown.
- Finally, the velocity parameter is further discussed and associated with many factors such that its initial calibration is important in a new channel. Once this calibration is

completed, this method can predict discharges at other flow depths for that same geometry.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (Grant Nos. 51479128, 51279117), the Program for New Century Excellent Talents in University of China (NCET-13-0393) and the National Science and Technology Ministry (No. 2012BAB05B02).

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