Discussion: Hydraulic jump in arbitrary prismatic channel

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1.1 Introduction

The contributors would like to thank the authors for proposing an approximation of the sequent depth ratio of the hydraulic jump formed in an arbitrary gradually expanding channel (Zhang *et al.*, 2012) and would like to draw attention to a few points.

A prismatic channel has an unvarying cross-section with position along the channel (Jeppson, 2011). A gradually expanding channel is not a prismatic one from a hydraulic standpoint and thus using the term ‘expanding prismatic channel’ in the paper by Zhang *et al.* (2012) is inappropriate.

1.2 Assumption of a quarter-elliptical longitudinal water surface profile

The authors assumed that the projected area of the water surface onto the y - z plane is elliptic, as shown in Figure 8. In this case, for a large expansion angle ($b_2 - b_1 \geq 2mh_1$), the side-wall pressure force in the x -direction, F_{sx} , can be calculated from

Figure 8. Projected area onto y - z plane for an expanding trapezoidal channel assuming a quarter-elliptical longitudinal water surface profile

$$\begin{aligned} \frac{F_{sx}}{\rho g} = & \int_0^{mh_1} (y/m)^2 dy + \int_{mh_1}^{(b_2-b_1)/2} \left(h_1 + (h_2 - h_1) \right. \\ & \times \left[1 - \left(\frac{y - (b_2 - b_1)/2 - mh_2}{(b_2 - b_1)/2 + m(h_2 - h_1)} \right)^2 \right]^{1/2} \Bigg)^2 dy \\ & + \int_{(b_2-b_1)/2}^{mh_2+(b_2-b_1)/2} \left(h_1 + (h_2 - h_1) \right. \\ & \times \left[1 - \left(\frac{y - (b_2 - b_1)/2 - mh_2}{(b_2 - b_1)/2 + m(h_2 - h_1)} \right)^2 \right]^{1/2} \\ & \left. - \frac{[y - (b_2 - b_1)/2]}{m} \right)^2 dy \end{aligned} \quad 14.$$

It is worth noting that in a prismatic trapezoidal channel ($b_2 = b_1$), F_{sx} should be zero. But, according to Figure 9, for a prismatic trapezoidal channel ($b_2 - b_1 = 0$ and $m \neq 0$), A_2 and F_{sx} are not zero. In this case, Equation 14 takes the form

Figure 9. Projected area onto y - z plane for a prismatic trapezoidal channel assuming a quarter-elliptical longitudinal water surface profile as suggested by Zhang *et al.* (2012)

$$\begin{aligned} \frac{F_{sx}}{\rho g} &= \int_{mh_1}^{mh_2} \left(h_1 + (h_2 - h_1) \left[1 - \left(\frac{y - mh_2}{m(h_2 - h_1)} \right)^2 \right]^{1/2} - y/m \right)^2 dy \\ &= \frac{10 - 3\pi}{6} m(h_2 - h_1)^3 \end{aligned}$$

15.

This result is different from that obtained by substituting $b_2 = b_1$ into Equation 5 of Zhang *et al.* (2012); that is

$$\frac{1}{6} \times m(h_2 - h_1)^2 [4(h_2 - h_1) + 3h_1\pi]$$

As noted, $F_{sx} \neq 0$, but, in a prismatic trapezoidal channel, the side-wall pressure force should be zero. Figure 9 indicates that the projected area onto the y - z plane for a prismatic trapezoidal channel is not acceptable from a hydraulic viewpoint. This may be attributed to an incorrect projecting procedure or an unrealistic assumption of the quarter-elliptical longitudinal water surface profile. This inconsistency may be removed by assuming a linear longitudinal water surface profile as shown in Figure 10.

1.3 Using a linear longitudinal water surface profile

For computation of the side-wall pressure force in the x -direction, two cases should be considered – a large expansion angle ($b_2 - b_1 \geq 2mh_1$) and a small expansion angle ($b_2 - b_1 < 2mh_1$).

For a large expansion angle in a trapezoidal expanding channel

Figure 10. Projected area onto y - z plane for an expanding trapezoidal channel assuming a linear longitudinal water surface profile: (a) large expansion angle; (b) small expansion angle

$$\begin{aligned} \frac{F_{sx}}{\rho g} &= \int_0^{mh_1} (y/m)^2 dy + \int_{mh_1}^{(b_2 - b_1)/2} \left(h_1 + \frac{h_2 - h_1}{(b_2 - b_1)/2 + m(h_2 - h_1)} (y - mh_1) \right)^2 dy \\ &\quad + \int_{(b_2 - b_1)/2}^{mh_2 + (b_2 - b_1)/2} \left(h_1 + \frac{h_2 - h_1}{(b_2 - b_1)/2 + m(h_2 - h_1)} (y - mh_1) - \frac{[y - (b_2 - b_1)/2]}{m} \right)^2 dy \\ &= \frac{(b_2 - b_1) [(b_2 - b_1)(h_1^2 + h_1h_2 + h_2^2) - 2mh_1^3]}{6(b_2 - b_1) + 2m(h_2 - h_1)} \end{aligned}$$

16.

In a rectangular expanding channel $m = 0$, and thus the governing equation takes the form

$$\begin{aligned} \frac{F_{sx}}{\rho g} &= \int_0^{(b_2-b_1)/2} \left(h_1 + \frac{h_2-h_1}{(b_2-b_1)/2} y \right)^2 dy \\ 17. \quad &= \frac{(b_2-b_1)(h_1^2 + h_1 h_2 + h_2^2)}{6} \end{aligned}$$

Similarly, for a small expansion angle in a non-prismatic trapezoidal channel ($m \neq 0$)

$$\begin{aligned} \frac{F_{sx}}{\rho g} &= \int_0^{(b_2-b_1)/2} (y/m)^2 dy + \int_{mh_1}^{mh_2+(b_2-b_1)/2} \\ &\times \left(\frac{y}{m} - \frac{[y-(b_2-b_1)/2]}{m} \right)^2 dy + \int_{mh_1}^{mh_2+(b_2-b_1)/2} \\ &\times \left(h_1 + \frac{h_2-h_1}{(b_2-b_1)/2 + m(h_2-h_1)} (y-mh_1) \right. \\ &\left. - \frac{[y-(b_2-b_1)/2]}{m} \right)^2 dy \\ 18. \quad &= \frac{(b_2-b_1)^2 [mh_2 + 2mh_1 - (b_2-b_1)/2]}{12 m^2} \end{aligned}$$

1.4 Conclusion

This discussion has dealt with the side-wall pressure force in the x -direction for a hydraulic jump in a horizontal, gradually expanding trapezoidal channel. A distinction is made between assuming a quarter-elliptical longitudinal water surface profile and a linear one. From a hydraulic standpoint, using the elliptic water surface profile yields a non-zero side-wall pressure force in the x -direction for prismatic channels. Thus, a linear longitudinal water surface profile is proposed, which seems more realistic. Moreover, it has been shown that the side-wall pressure force formulation is different for small and large expansion angles. The proposed linear equation for the water surface profile needs to be verified using experimental data.

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Authors' reply

First, we would like to thank the contributors for their attention and contribution to the original paper (Zhang *et al.*, 2012).

The assumption of an elliptic curve for the water surface profile and its projection in the x -direction is used in the original paper

and is based on experiments by Hager (1985) and Omid *et al.* (2007). The contributors proposed a linear longitudinal water surface profile for computing the side-wall pressure force in the x -direction. It is the opinion of the authors that the linear distribution is also a kind of simple approximation for determining the distribution of the side-wall pressure force.

Furthermore, computing the side-wall pressure force with the water surface profile and then projecting onto the x -direction is also an approximate method. In a hydraulic jump, due to the effect of entrainment, the side-wall pressure force and the water surface profile distribution are not consistent. The side-wall pressure force is neither elliptical nor linear in distribution. However, we agree with the linear approximation proposed here.

The purpose of this discussion is to unify the hydraulic jump formula for an expanding trapezoidal channel and a prismatic trapezoidal channel. In order to achieve this, one problem that needs to be addressed is that, for projection of the side-wall pressure force in the x -direction, the projected area is

$$A_{sx} = A_s \times \sin \left(\frac{b_2 - b_1}{2L_j} \right)$$

when $b_2 - b_1 = 0$, $A_s = 0$.

In Equation 14, when $b_2 - b_1 = 0$, $F_{sx} \neq 0$; the main reason for this is the use of the water surface profile of the hydraulic jump to represent the side-wall pressure force. Because of the good agreement between the experimental results and prediction by the proposed formula, which is based on the assumption of an elliptic curve for the water surface profile in the original paper, the applicability of the formula for a prismatic trapezoidal channel is ignored in the original paper. In this discussion, the contributors propose a linear distribution for the projection of the side-wall pressure force to the x -direction. Furthermore, they proposed a unified formula that can be used for a hydraulic jump in both an expanding trapezoidal channel and a prismatic trapezoidal channel. This is an improvement on work presented in the original paper. We therefore agree with the points raised regarding the side-wall pressure force and appreciate this contribution.

In fact, no part of the distribution of the side-wall pressure force along the flow direction and its projection in the x -direction is linear or elliptic. We suggest that it is more realistic to compute the component of the side-wall pressure force in the x -direction, F_{sx} , based on the actual side-wall pressure force and including the effect of aeration. However, so far, using a linear distribution still is the most simple and effective.

Once again, we thank the discussers for their attention to the original paper and for this contribution.

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