Shell Structures in Civil and Mechanical Engineering

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Theory and analysis

Second edition

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To
Ratidzo, Tafadzwa and Simbarashe

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Preface

Shell structures find application in many fields of engineering, notably civil, mechanical, marine and aeronautical disciplines. Since about the beginning of the twentieth century, considerable effort has been expended on the development of rigorous theories – both general and specialist – to describe the behaviour of shells in the elastic range as realistically as possible. The problem with such rigorous formulations has been that, in almost all cases, they have led to sets of differential equations for which exact mathematical solutions cannot be found, or for which the mathematical solutions that are obtained are not very practical from an engineering point of view. Not surprisingly, the development of numerical formulations over the past 60 years, notably the finite-element method, has seen the gradual decline of attempts to find analytical solutions to rigorous mathematical formulations of shell behaviour.

The finite-element method is a general numerical procedure that can be traced back to the early 1960s. It can be used to tackle any shell problem to any desired degree of accuracy; one needs only to model the behaviour of the basic individual elements making up a structure (rather than the behaviour of the whole structure), and then assemble the relevant structural properties by enforcing compatibility conditions between adjacent elements, conditions of equilibrium of forces and moments, as well as boundary conditions at the edges or supports of the structure. In this way, the overall behaviour of the complete structure is approximated. The finite-difference method is another popular numerical approach, where the exact differential equations of the problem are approximated with equivalent finite-difference equations, which can then be solved more conveniently as linear algebraic equations. Numerical modelling is particularly useful in those instances where the mathematical formulation results in sets of differential equations that are difficult or impossible to solve analytically, particularly where non-linearities in material and/or geometric behaviour have to be accounted for.

However, for situations where the mathematical formulation results in sets of differential equations that can be solved analytically, the numerical solution approach, while still effective as a tool for obtaining actual values of stresses and displacements in a structure, is not necessarily the most efficient for the purposes of studying the effect of varying a single parameter (such as the thickness of a shell) during the course of design, since each time such a change is made, the entire analysis procedure has to be carried out, albeit by a computer. Numerous runs of the computer program may be necessary in order to obtain a comprehensive understanding of the effect (on the stresses and deformations in the shell) of varying a parameter of interest through the full range of possibilities. Thus the process of preliminary design can easily become very expensive if several structural parameters need to be studied.

It is in situations such as these that analytical formulations can be extremely useful. In fact, we may justify the pursuit of exact mathematical solutions to the differential equations of the shell, in an era where numerical solution approaches have become very popular (owing to their ease of programming), on the grounds that analytical solutions: (a) provide valuable insights on the key parameters that govern the behaviour of the shell; (b) are more amenable to systematic parametric study than numerical formulations; and (c) provide a means (sometimes the only means) for checking the correctness of numerical solution procedures.

In many situations, particularly where the aim is simply to calculate the stresses and deformations in the shell under normal service conditions, the assumption of linear-elastic material and geometric behaviour is usually made right from the outset. This certainly simplifies the problem from a mathematical point of view. Unfortunately for many types of shells, linear-elastic assumptions still result in sets of differential equations that are very difficult to solve exactly. To be able to obtain analytical solutions for such problems, we may either (a) make further simplifying assumptions at the beginning of our formulation, or (b) introduce simplifying approximations in the rigorously obtained sets of governing differential equations. Either way we end up with differential equations that, while not exact, have the merit of having known mathematical solutions. Provided the simplifying assumptions are good, this approach can afford reasonably accurate closed-form analytical solutions to a wide range of practical shell problems. It is an effective strategy that we will adopt throughout this book.

Closed-form analytical solutions are highly suitable for practical calculations, enabling quantities that are of interest to a designer (such as stresses and deformations) to be computed with ease. General expressions (in terms of the parameters defining the geometry of the shell and the nature of the loading) can easily be programmed on portable scientific calculators or small computers, permitting the designer to rapidly evaluate stresses in a shell structure of a given set of parameters, or to obtain important design insights by simply examining the form of the mathematical expressions, or to obtain a deeper understanding of the effect of varying a particular parameter by simply plotting the results with respect to that parameter.

This book is intended to be both a textbook for senior undergraduate and postgraduate students of civil and mechanical engineering, and a reference book for practising engineers in these disciplines. It is also intended to serve as a useful resource for researchers involved with the development of more effective methods of shell analysis, as well as software developers. As with the first edition, the distinguishing features of the book are as follows:

- (a) emphasis is placed on the presentation of simplified shell theory for which closed-form analytical solutions can be obtained;
- (b) for a wide range of shell problems, closed-form results for stresses and deformations are derived and presented in a way that allows convenient evaluation of these quantities by the designer, and that permits any desired parametric studies to be conducted. For many problems, the results of such parametric studies, drawn from the author's previous work, are presented and design implications pointed out, enhancing the reference value of the book.

The main differences between the first edition and the present edition are (i) the inclusion of new sections on non-conventional shell forms for liquid-containment applications in Chapters 3, 7 and 8, based on recent work of the author, and (ii) the addition of three new chapters covering buckling of thin shells, finite-element analysis and design considerations for shells. Although the main emphasis of this book remains the development of simplified mathematical theory and presentation of closed-form analytical solutions for a wide variety of shell problems, the last three chapters have been included for the sake of completeness; it is

acknowledged that no treatment of shells would be regarded as complete without some consideration of the important topics of buckling, finite-element modelling and design.

In shells where there is a significant amount of compression or shear, and the shell happens to be very thin, there is always the possibility of failure by buckling rather than material crushing or rupture. It is therefore important to be able to predict the critical loading that will trigger failure by buckling, and be able to design against this instability phenomenon. Also, where a shell problem cannot be solved analytically owing to the complexity of the differential equations or boundary conditions, resort is usually made to finite-element analysis, so there is merit in understanding how the finite-element method works, and how it can be implemented in the context of shell analysis. Ultimately, all analysis serves the purpose of informing design, and the connection between the two is important. Thus an overview of design considerations for shell structures seems an appropriate way of concluding a book of this type.

Chapter 1 explains the essential properties of shell structures and the manner in which shells resist loads. It traces the historical development of shell theory and analytical solution procedures, and discusses the assumptions and limitations of the membrane hypothesis in shell analysis. Chapters 2 to 4 develop the membrane theory of shells of revolution, and present membrane solutions for pressure vessels, liquid-containment vessels, domes, roofs and cooling towers. Chapters 5 and 6 present the theories of the axisymmetric bending of cylindrical shells and general shells of revolution. Chapter 7 explains the flexibility formulation for shell structures, and applies this to shell-ring problems and multi-shell assemblies. Chapter 8 features a series of parametric studies of spherical tanks and domes, as well as feasibility studies of certain non-conventional shapes for sludge digesters. The use of shell theory as a tool for exploring alternative new shell forms is also illustrated.

Chapter 9 presents the membrane theory of general cylinders and its application to liquid-containment vessels and barrel shell roofs of circular, elliptic and parabolic cross-sections. The membrane theory of shells of arbitrary shape is developed in Chapter 10, and applied to the analysis of elliptic and hyperbolic paraboloidal shell roofs. An approximate theory for the linear elastic buckling of cylindrical shells is presented in Chapter 11, and results for critical buckling loads derived for the cases of axial loading, uniform lateral pressure and torsion. Results for other common shell geometries (spherical, conical, ellipsoidal and toroidal shells) are also presented. The finite-element formulation for shells is outlined in Chapter 12, with considerations covering static, dynamic and stability behaviour. Various types of shell elements are discussed, and the process of assembly and solution of system equations explained.

The last chapter (Chapter 13) is devoted to design considerations for concrete and steel shells. For steel vessels, Eurocode design criteria and guidelines of the American Petroleum Institute are reviewed, while for concrete shells and thin shell roofs in particular, the key recommendations of the American Concrete Institute and of the International Association for Shell and Spatial Structures are summarised.

While every effort has been made to ensure that all mathematical expressions are correct, it is possible a few errors have escaped detection, given the large number of equations and mathematical results that are presented. The author welcomes all feedback from readers in this regard. It is hoped the book will succeed in serving its intended purpose.

Alphose Zingoni
Cape Town, 2017

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