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A new technique for the measurement of the electrical resistivity of concrete

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Contribution by K. Snyder

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The paper proposed a new method to calculate the electrical resistivity of concrete. Since electrical resistivity is directly related to diffusivity, measurement of electrical resistivity offers a simple means to assess the durability of concrete. Currently, no standard procedure exists for electrical resistivity measurements and the authors have proposed a new and simple technique.

To measure resistivity the authors have embedded a 5 mm diameter steel rod in a cylinder of concrete (32–110 mm diameter), both of which are oriented vertically. The midsections of both the steel and outside cylinder surface are exposed by masking off the end portions of both, resulting in purely radial current flow. Using a fixed-frequency alternating current technique, the authors attempted to calculate the resistivity of the concrete from the resistance measured between the embedded steel and the outside surface of the concrete cylinder.

The authors' equation for resistivity ρ , assuming that the thickness of cover is much less than the radius of the steel rod, as a function of measured resistance R , was given in equation (3) of their text

$$\rho = R2\pi h \quad (1)$$

where h is the height of the exposed surfaces. I believe that the text contains a misprint. This equation, based upon the authors' previous mathematical development, is correct assuming the thickness of the concrete is far greater than the radius of the steel rod, which is the more

logical limiting case in this experiment. However, the fact that the above equation is independent of the concrete thickness is a little disconcerting. How could doubling the thickness of the concrete cover have no effect on the measured resistance?

This apparent inconsistency may be due to an error in the development of the equation relating resistivity and resistance. The authors began with $\rho = RA/L$, where A and L are the area and length of the specimen, respectively. However, this equation is valid only in cartesian coordinates with one-dimensional current flow. The authors' experiment has cylindrical symmetry; therefore, the relationship between resistivity and measured resistance has to be derived from an electric field in cylindrical coordinates.

The electric field due to a line charge can be found in any introductory electricity and magnetism text¹

$$E_r = \frac{V_0}{\ln(b/a)} \frac{1}{r} \quad (2)$$

The measured current I_0 , is calculated from the charge flux

$$I_0 = \int_{\theta,h} \mathbf{J} \cdot d\mathbf{a} = \int_{\theta,h} \sigma E_r \cdot d\mathbf{a} \quad (3)$$

$$= \sigma \frac{V_0}{\ln(b/a)} 2\pi h \quad (4)$$

where h is the height of the opening and σ is the

conductivity ($\sigma = 1/\rho$). Note that the radius at which the integral is evaluated factors out of the equation. Since $\nabla \cdot \mathbf{J} = 0$, the total current across any field radius in the interval $[a, b]$ is constant.

The measured resistance R_0 is calculated from Ohm's law

$$R_0 = \frac{V_0}{I_0} = \frac{\rho}{2\pi h} \ln(b/a) \quad (5)$$

$$= \frac{\rho}{2\pi h} \ln \left[\frac{a + L}{a} \right] \quad (6)$$

where L is the concrete cover thickness. For large values of cover thickness L with respect to rod diameter a , the resistance can be approximated by

$$R_0 \approx \frac{\rho}{2\pi h} \ln(L/a) \quad (7)$$

which increases as $\ln(L)$. However, for the smaller sample radii used in the authors' experiment, equation (7) approximates equation (6) with a 9% error. This suggests that the large cover approximation may not be accurate for this experiment.

With this, readers may convince themselves that the resistances given in Table 1 of the authors' paper increase with increasing cover thickness. However, the lack of replications, or associated statistics, makes validating any equation difficult. Repeated measurements could have been used to perform an analysis of variance to demonstrate whether the data were from populations with the same means, as the authors' equation suggests, or whether the mean values followed a relationship more like equation (6) above.

Even with an equation that corrects for the geometry of the problem, questions remain concerning the validity of the authors' technique. Consider that the only difference between each measurement in Table 1 is the $\ln(b/a)$ term from equation (6) above. Therefore, all else being constant, the measured resistance should have ranged from 3.09 to 1.86 times some arbitrary constant; this is a factor of 1.66. The fact that the data do not vary by this magnitude indicates that this method, as described, performed and reported by the authors, may not be suitable for the measurement of the conductivity of a porous material like concrete.

Table 1. Resistance and resistivity values for cubic concrete specimens of different cover thicknesses and constant height

Length l : mm	Resistance: Ω	Resistivity: Ωm
47	470	23.5
54	462	23.1
67	441	22.0
77	434	21.7

Mix proportion 1 : 1 : 2; w/c = 0.4.

Number of days of curing was 3.

Height of specimens = 50 mm.

A technique with more consistent results has been demonstrated by Christensen and Mason.² This technique uses a broadband approach and determines the bulk conductivity from a well-defined feature of the complex impedance spectrum.

A minor point of this discussion concerns the general use of the word 'permeability' to characterize a material's resistance to transport; either permeability as defined by Darcy's Law or diffusion. Although it is common to use the term permeability to describe either permeability or diffusivity, when the mechanism is specified, the correct term should be used. The authors state in their abstract: '... the rate of transport of aggressive ions like chlorides and sulphates ... is controlled by permeability.' The ingress of ions is due to diffusion; the controlling physical property is diffusivity. Permeability involves the motion of a fluid, either liquid or gas, under a hydrostatic head.

The authors also state that '... the electrical resistivity of concrete is related to its permeability.' Electrical conductivity is directly related to diffusivity through the Nernst-Einstein relation.³ Conductivity is related to permeability when suitable length scale and shape factor are chosen. Calculation of proper length scale and shape factor are still debated; length scales relating permeability and electrical conductivity have been proposed by Katz and Thompson⁴ and Johnson *et al.*,⁵ and have been investigated by Martys and Garboczi.⁶

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Reply by the Authors

Mr Snyder makes a perfectly valid point, that the relationship between the resistance and resistivity derived by us is not valid for specimens with cylindrical configuration. However, the failure of the corrected expression to fit the observed experimental results does not in any way invalidate the suitability of the proposed technique for measuring the resistivity of concrete. In fact the validity of this technique does not rest on any one single experimental result; it is based on a systematic study of innumerable specimens. For instance, it can be seen from Fig. 4 of the paper that the resistance decreases linearly with the height of the specimens, as required by the expression. Again the resistivity varies in an accurately predictable manner for any given mix proportion (Table 3). This fact, along with the observation that the resistivity variation with time is akin to the behaviour of compressive

strength, shows that the method is not to be blamed for any seemingly anomalous result.

What then is the reason for the failure of the expression? To investigate this we have carried out the resistivity measurement on tetragonal concrete specimens under similar conditions. All the specimens have the same height (50 mm) and a square base, and two parallel mild steel plates were used as electrodes on both ends. The specimens were cured in saturated calcium hydroxide, taken out of the curing medium and the potential step measurements were carried out between the two electrodes immediately without any external electrolytic medium. The results are presented in Table 1.

The results show that at constant height and at different cover thicknesses the resistance actually shows a slight decrease with increasing cover thickness. The resistivity of course shows a similar trend.

The question, then, is whether it is proper to apply any simple mathematical expression based on the assumption of uniform electric field which in any case is only valid for a homogeneous isotropic medium. Obviously, for a highly inhomogeneous medium like concrete, with different conduction paths, no simple mathematical expression can be used to check the experimental results. The problem of local field intensification or field distortion introduced by different materials of differing conductivity must be taken into account. A study based on the

dielectrophoretic effect — the behaviour of matter in a non-uniform electric field — would throw some light on this behaviour.

The method does not use any fixed-frequency alternating current, as suggested by Mr Snyder. On the contrary, the technique consists in applying a step input, for example a controlled current pulse of any short duration (say < 10 ms). It can be merely a single pulse or a repetitive pulse train. It is the step response which provides the required resistance information.

The comments regarding the usage of the term 'diffusion' and 'permeability' are quite appropriate and correct.

We thank Mr Snyder for his extremely helpful comments. We would further like to stress the fact that the proposed technique relies on simple instrumentation and trivial data analyses. The method itself is highly reproducible and is based on a sound premise; i.e. for a short duration current pulse the electrode polarization effects are negligible and the initial potential jump corresponds to the ohmic drop. We also emphasize the fact that the results obtained for cylindrical specimens, far from revealing any incongruity in the technique, actually prove that for a porous medium like concrete much more work needs to be done to understand what determines the overall bulk resistivity of an inhomogeneous medium.