Discussion on papers published in the

Magazine of Concrete Research

Volume 32, Number 112 : September 1980

Bond stress along lapped reinforcing bars*

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The analogy between the transfer of load in a rivetted or welded joint and that in the anchorage zone of bars embedded in concrete is of interest. The theory developed in the first part of the paper assumes that, in a reinforcement splice, the load is transferred from bar to bar through the medium of an intermediate layer of concrete. It is only towards the end of the paper, as a suggested simplification, that the load is assumed to be transferred directly from bar to bar, the role of the concrete being relegated to that of a quasi-adhesive. Since the bars in a splice are in contact, it could well be argued that it is more likely that the transfer of force will be mainly from bar to bar rather than by a circuitous flow through the surrounding concrete. However, if this is the real mechanism, the bond-slip relationship should be that measured in a bar-to-bar test rather than in a bar-to-concrete test. Can Dr Tepfers give any information on the difference, if any, between such tests?

When the bond-slip curve is applied to the analysis of stresses in a splice it is found that, except in the case of a linear relationship as used by Dr Tepfers, a formal analytical solution is not possible. However, graphical or numerical methods can be used to obtain solutions to individual problems. It is well known that a ductile rivet has a highly non-linear deformation

curve and that such rivets allow an almost uniform load distribution before failure in a joint of several rows, whereas a line of brittle elastic rivets will fail at the outermost rivets while the innermost ones carry little load. The analogy raises the question whether a similar principle will apply to the splicing of reinforcing bars.

Calculations have been made using the curved bond-slip relationship given in Figure 1 of the paper for $f_c = 23.5 \text{ N/mm}^2$. The results are shown in Figure I for two splices of 16 mm dia. bars, one with a lap of 520 mm as used by Dr Tepfers and the other with a smaller lap of 300 mm. The presentation shows the degree of uniformity of the bond stress (as measured by the ratio of the minimum to the maximum values) over the length of the splice as a function of the load on the bar. The results of analyses assuming linear bond-slip relationships are also shown and they indicate that the ratio is independent of the load on the bar in a given splice.

It is seen that the 520 mm splice reveals little difference between the linear and non-linear analyses with a non-uniform distribution of bond stress at all loads. The 300 mm splice calculation with the non-linear bond-slip curve indicates that the bond stress becomes progressively more uniform as the load increases. In both cases the load on the splice is limited by the strength of the bar, assumed to be 450 N/mm^2 . It would be of interest to know whether

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Figure 1: Uniformity of bond stress in splices of 16 mm dia. bars 520 mm and 300 mm long. Vertical lines show values predicted from linear bond-slip relationships. Curved lines show values predicted from the bond-slip curve of Figure 1 with $f_c = 23.5 \text{ N/mm}^2$.

Dr Tepfers has any information on splices shorter than those reported in his paper.

It is realized that the maximum slip in the short splices will be greater than in the long splices and that excessive slip may lead to splitting failures. However, it would be usual to provide containing links around the bars in splices, which might be expected to prevent or at least delay splitting. Has the author any information to show whether, by the use of confining reinforcement, it is possible to utilize a

greater proportion of the bond-slip curves of Figure 1 of the original paper than is involved in the 520 mm splices. These long splices utilize only the initial linear part of the curve and so the use of the formula based on an elastic bond-slip calculation is adequate. The shorter splices utilize much more of the bond-slip curve, thus testing the theory more stringently, and indicate considerable differences between the elastic and inelastic approach to bond-slip relationships.

Reply by the author

I wish to thank Professor Brock for his interest in my paper, his deep understanding of the problem and the very interesting questions he has put forward.

First regarding the flow of force from one lapped bar to other: certainly the force is transferred from one bar to the other by the shortest route. However, the force cannot just jump across without creating reaction forces and thus activating the surrounding concrete. The strength and geometry of this concrete determines the degree of confinement, which makes the transfer of force possible. A bigger concrete mass and stronger concrete make possible a greater intensity of force transfer. If we consider the slip deformations, it becomes obvious that the slip movement between the bars is the sum of that between each bar and the concrete. This greater movement between the bars gives rise to increased wedging forces which must be resisted by the surrounding concrete. By its resistance to splitting forces, the surrounding con-

crete determines the force transfer intensity between the bars without giving rise to any really circuitous flow of force from the lapped bars. The confinement of the concrete is represented in the equations by the effective concrete area and the modulus of displacement.

A bar-to-concrete pull-out test will give information about the shear resistance of the zone of adhesion or the splitting resistance of the surrounding concrete or the shear strength of concrete or the blocking effect of crushed compacted concrete supported against surrounding concrete. Which information we gain is dependent upon the bar surface deformations and the concrete confinement. In a bar-to-bar pull-out test the information achieved will be essentially the same, but the splitting resistance and concrete confinement will be of much more importance, because the added slip between the bars gives rise to increased splitting forces.

The analogy of brittle rivets applies to the splicing

Figure II: Bond stress distribution along spliced bars in longitudinally cracked and uncracked parts of the concrete cover. The modulus of displacement theory is valid only within the range l_1 .

of reinforcing bars. In the outermost parts of the splice, where the bond stresses are high the concrete cover will be cracked along the bars.

A linear relationship for the bond-slip curve can be used because (references 8 and 9 of the paper), in a normal lapped splice, the concrete cover cracks along the bars at a bond stress of $\approx 2-4 \text{ N/mm}^2$; up to this stress level the relationship can still be regarded as linear. When longitudinal cracks appear in the cover, the possibility of movement between the concrete and the reinforcing bar increases and the bond-slip relationship determined in the pull-out test is no longer valid. The modulus of displacement theory applies only in parts where the bars overlap, where such cracks do not exist as is obvious from Figure 7 of the paper at high loads. Where the cover has cracked, the modulus of displacement has a low value and therefore the bond stress there becomes evenly distributed (Figure II). The bond capacity in these parts of the splice length is determined by the minimum ultimate splitting failure crack pattern in the surrounding concrete. When the necessary deformations are able to develop, this ultimate bond stress usually exceeds the bond stress required to crack the cover along the bars.

The splitting resistance of a certain concrete cover at the appearance of a longitudinal crack is the same regardless of splice length and determines the possible peak bond stress in the theory. This means that the possible bar stress at the end of splice, when the cover starts to crack for the two splice lengths with conditions given in Figure I, is 113 N/mm^2 for $l = 520 \text{ mm}$ and 98 N/mm^2 for $l = 300 \text{ mm}$. This limited cover resistance, as becomes evident from Figure I, practically excludes the use of non-linear parts of the bond-slip curve in theory for normal conditions. If the bond length becomes very short, the bond stress variation along the length becomes very small in theory as well as in practice. This fact is used in pull-out tests with short bond lengths with large concrete splitting resistance for the determination of bond-slip relationships (Figure 1 of the paper).

Stirrups around lapped bars become effective in concrete cover when splitting bond forces are sufficient to cause longitudinal cracks in the cover. The transverse reinforcement contributes with a confining force which corresponds to the ultimate tensile strain of the concrete ($\approx 0.015\%$) and thus delays the cracking. After the longitudinal crack in the cover has opened, the bond stress becomes evenly distributed. After that, the bond stress can increase up to failure load. The bond stress level at failure is determined by the minimum ultimate splitting failure pattern in concrete (reference 8 of the paper). Stirrups become effective in the ultimate failure pattern at a stress which corresponds to the ultimate tensile strain of concrete and, if necessary, in the longitudinal cover crack up to $\approx 400 \text{ N/mm}^2$. The modulus of displacement theory is useful for estimating the bond stress distribution in parts of an overlap where cover has not cracked.

CALCULATION OF STRESSES

The following parameters are used:

$$l = 0.52 \text{ m and } 0.30 \text{ m}$$

$$f_{cc} = 23.5 \text{ N/mm}^2$$

$$f_{ct} = 2.15 \text{ N/mm}^2$$

$$\phi = 16 \text{ mm, type Ks40}$$

$$K = 2400f_{cc} = 0.564 \times 10^5 \text{ MN/m}^3$$

$$c = 20 \text{ mm (smallest cover)}$$

Bond stress which, according to references 8 and 9 of the paper, starts to crack the cover along the lapped bars:

$$\begin{aligned} \tau_{\max} &= (1.3c + 0.15\phi) \frac{f_{ct}}{2\phi} \\ &= (1.3 \times 0.02 + 0.15 \times 0.016) \frac{2.15}{2 \times 0.016} \\ &= 1.91 \text{ N/mm}^2 \end{aligned}$$

Bar stress at end of splice $x = \frac{l}{2}$, equation 22b:

$$\begin{aligned} \sigma_{s0} &= \frac{\tau_{\max} \times 2u}{A_s \times \kappa_2} \frac{\sinh \frac{\kappa_2 l}{2}}{\cosh \frac{\kappa_2 l}{2}} = \frac{8\tau_{\max}}{\phi \times \kappa_2} \frac{\sinh \frac{\kappa_2 l}{2}}{\cosh \frac{\kappa_2 l}{2}} \\ \kappa_2 &= \sqrt{\frac{uK}{A_s \times E_s}} = \sqrt{\frac{4K}{\phi E_s}} \\ &= \sqrt{\frac{4 \times 0.564 \times 10^5 \times 10^6}{0.016 \times 2.1 \times 10^5 \times 10^6}} = 8.194 \text{ m}^{-1} \end{aligned}$$

For $l = 0.52 \text{ m}$,

$$\sigma_{s0} = \frac{8 \times 1.91}{0.016 \times 8.194} \times \frac{\sinh 2.130}{\cosh 2.130} = 113 \text{ N/mm}^2$$

For $l = 0.30 \text{ m}$,

$$\sigma_{s0} = \frac{8 \times 1.91}{0.016 \times 8.194} \times \frac{\sinh 1.229}{\cosh 1.229} = 98 \text{ N/mm}^2$$