Rigid-plastic theory v. truss analogy method for calculating the shear strength of reinforced concrete beams*

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Professor Campbell, Dr Chitnuyanondh and Professor Batchelor have presented an interesting comparison between the two approaches to predicting shear strength. There are, however, a number of points which may be raised concerning that comparison.

The truss-analogy method presented in the paper requires several empirical parameters to be defined, whereas the plastic method needs only the effectiveness factor. Yet a comparison of the shear capacities (v'/f_c'), as predicted by the variable-angle truss model and the plastic method (Figure 3 in the paper), shows a relatively small difference between the two methods. Both approaches give the same results for $\lambda < \lambda_b$ (where λ is the web reinforcement index), and a small difference is shown for $\lambda > \lambda_b$, which decreases as the amount of shear reinforcement increases.

It is worth pointing out that, as presented in Figure 3, the curve predicts zero shear strength for no shear reinforcement, despite the conditions reported by the authors for the two equations 1a and 3. Such prediction is obviously incorrect, since the concrete shear strength is not zero when $\lambda = 0$, and equation 1a does not apply at low values of λ . As λ decreases, the yield line decreases in slope until it eventually runs from load to support. The equation for this solution is different from equation 1a and, as derived by Nielsen and Bræstrup, assuming that the longitudinal reinforcement does not yield but using the authors' symbols, is:

$$\frac{v'}{f_c'} = \frac{k_o}{2} \left[\sqrt{\left(\frac{a}{h}\right)^2 + 1} - \rho \left(\frac{a}{h}\right) \right]$$

which is valid for $0 \leq \lambda \leq \lambda_o$ where

$$\lambda_o = \frac{k_o}{2} \frac{\sqrt{\left(\frac{a}{h}\right)^2 + 1} - \left(\frac{a}{h}\right)}{\sqrt{\left(\frac{a}{h}\right)^2 + 1}}$$

$$\rho = 1 - \frac{2\lambda}{k_o}$$

and h = depth of beam. Could the authors explain whether they have an equivalent equation for low values of λ in their truss-analogy approach?

Furthermore, in their comparison, the authors use a relatively high concrete effectiveness factor ($k_o = 0.85$) which is reported to have been established from tests on beams with web reinforcement, in comparison with an effectiveness factor 0.47 suggested by Nielsen and Bræstrup from tests on beams without web reinforcement. A smaller effectiveness factor would reduce the difference between the two solutions. This significant difference between the two effectiveness factors (almost 2 to 1) is certainly excessive and further experimental work is needed to determine the correct value.

These observations suggest that the truss-analogy method, although more complicated, shows little difference from the rigid-plastic theory. Indeed the rigid-plastic theory, despite its minor shortcomings, offers a simple and consistent approach to the prediction of the ultimate shear strength of a wide variety of reinforced concrete beams and it is to be hoped that more research effort will be devoted to its development.

*Pages 39 to 44 of MCR 110.

Reply by the authors

We thank Mr Al-Safi and Professor Kemp for their remarks and welcome the opportunity to comment on them.

The paper presents a comparison of the rigid-plastic theory and the truss analogy and comments on the subtle differences between them. One of the implications of the variable-angle truss model is that the shear strength of a beam is zero if the beam has no shear reinforcement. The model is, therefore, not intended to predict shear strength of such members. In such cases semi-rational methods, or the rigid-plastic theory proposed by Nielsen and Bræstrup (reference 9 of the paper), would be appropriate. However, in the ranges of practical web reinforcement indices, the predictions of the variable-angle truss model have been confirmed experimentally (reference 7 of the paper).

It is true that equation 1a does not apply at low values of λ . For low values of λ , the inclination (α_u) of the diagonal compression at failure, as given by equation B5, will be small. Obviously α_u cannot be smaller than $\cot^{-1}(a/h_t)$, at which point arch action occurs in the beam and truss action breaks down. Resort must then be had to an arch-type analysis, such as that given by Nielsen and Bræstrup, and referred to by Mr Al-Safi and Professor Kemp, for values of λ giving α_u less than $\cot^{-1}(a/h_t)$ in the truss analogy. This added restriction on the angle of the

inclined compression must be recognized in the truss analogy.

Mr Al-Safi and Professor Kemp point out the relatively small difference in the shear capacities predicted by the two methods for an over-reinforced ($\lambda > \lambda_b$) section. Although this is true, nevertheless it is perhaps more important to realize that, for a section with $\lambda_b < \lambda < k_o/2$, the rigid-plastic theory predicts an under-reinforced type of behaviour, whilst truss analogy predicts an over-reinforced behaviour. In design it is desirable to use an under-reinforced section, with accompanying ductility, rather than an over-reinforced section which behaves in a brittle manner. Hence, in this range of λ , although the shear capacities as predicted by the two methods may not differ significantly, the type of behaviour predicted at failure will be distinctly different.

The effective concrete strength factor, k_o , is incorporated in both methods and the value of this factor is one area of uncertainty. As pointed out in the paper, it has been suggested (reference 10 of the paper) that k_o decreases with increasing deformation in the shear span. This is in agreement with the trend pointed out by Mr Al-Safi and Professor Kemp, since a shear span without web reinforcement will exhibit larger shear deformations, and hence lower k_o , than an equivalent shear span with web reinforcement. Further work is being carried out in this area.