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The strength of restrained reinforced concrete columns – a new approach*

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Readers cannot fail to be impressed by the almost spectacular results of Professor Butler's new approach to column design – covering as it does a wide range of cases, all subjected to the worst pattern of beam loading for inducing buckling. Designers should note that Figure 1 could also stand for cases where opposite beams (top and bottom) have become fully plastic under full load, with remaining beams, under dead load only staying elastic, with M_0 then equalling M_p – (fixed-end moment). It is gratifying to note that a combination of my own 'variable stiffness approach' (reference 6 of the paper) and Professor Roik's available-moment diagram (Figure 7) has produced such remarkable accuracy. It remains to devise a direct design procedure to avoid iteration.

It is when we attempt to do this, however, that we discover that the basis of Butler's procedure is not at all clear, and it is then important that any critic should attempt to answer the question 'But why, apparently, does it work?'. For example, whereas curve C, Figure 2a, shows a buckling load before M_{ult} is reached, nowhere in the procedure is such a buckling load ever found. Secondly, if such buckling occurs, the correct 'variable-stiffness' definition (reference 6) of the collapse load is $N = C'R'N_E$, where R' is the instantaneous (tangential) over-all stiffness of the column under the system of moments at collapse; and I had previously adopted the form $R' = R_m(1 - \bar{N}^2)$ to distinguish a deterioration of R_m due to moments from

$(1 - \bar{N}^2)$ which causes all stiffness to vanish at the squash load. If there are no applied moments, then $R_m = 1$ and $(1 - \bar{N}^2)$ is a combined out-of-straightness plus locked-up-stress plus tangent-modulus effect. It is fascinating and puzzling to note that Butler has proceeded without mention of R_m . Is there some hidden simplification?

It will be shown below that case C4 is just about at the point of simultaneous buckling and moment failure. For this case, we find $\bar{N} = 0.269$, $R = 1 - \bar{N}^2 = 0.928$, which is the only expression used by Butler. However, simultaneous buckling demands $C' = 2.91$, $R' = 0.189$, $R_m = 0.204$. To try to understand this discrepancy requires us to go right back to basic moment-curvature (M, κ) relationships at a given N . If, generally,

$$M = EI' \kappa = (R_\kappa) EI \kappa$$

where R_κ is the stiffness reduction for a cross-section only, the curve of M versus κ (Figure 1) shows that R_κ is a *secant* stiffness which will tend to control end moments, whereas $dM/d\kappa$ is the *tangent* stiffness which deteriorates more rapidly and concerns buckling. Note that, as M tends to zero, the secant and tangent coincide. The complete set of (secant) contours of R_κ are plotted for the first time in Figure II and are highly instructive for the case of Cranston's cross-section with 1% reinforcement (Figures 3 and 4 of the paper). For example, if $M = 0$, the R_0 is given alongside the vertical axis, and this reduction is due to the tangent modulus of the parabolic concrete

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Figure I: Stiffness deterioration for a cross section. Reinforcement 1%; $\bar{N} = 0.0269$; $M = R_k EI_k$. Case C4.

Figure II: Values of R_k (stiffness reduction coefficient based on initial tangent E_0). Approximate contours and terminal values of R_k at limiting strain of 0.0035. Reinforcement 1%. $\sigma_{ys} = 20$; $\sigma_{yc} = 0.002$; $\sigma_u = 0.0035$; $\sigma_{ys} = 0.00207$. Strong axis bending.

Figure III: Butler's case C4. $L/h = 40$; $\bar{N} = 0.269$.

stress-strain curve. It is approximated by $1 - \bar{N}^2$, but better by $\sqrt{1 - \bar{N}}$. These values persist, provided M is not large. We use them to get an alternative estimate of the maximum moment. Thus, if $\bar{N} = 0.269$, $R_0 = 0.86$ and, from equation 4, $M_{\text{end}} = 0.062N_{\text{uz}}h = 0.075\sigma_c b h^2$. In the absence of buckling, this moment would reduce R_κ to 0.78 and M_{end} to $0.059N_{\text{uz}}h$. But, with $0.78K_c$,

$$\frac{N}{N_{\mathrm{E}}'} = \frac{N}{N_{\mathrm{uz}}} \frac{N_{\mathrm{uz}}}{N_{\mathrm{E}}} \frac{1}{R_{\kappa}} = \frac{0.269}{0.49 \times 0.78} = 0.704$$

and with $C' = 1.87$ from Figure 5,

$$\frac{N}{N'_{CR}} = \frac{0.704}{1.87} = 0.376$$

so that $M_{\max}$ is predicted by $0.59/(1 - 0.376) = 0.094N_{\text{uz}}h$.

By comparison, for the exact value of N , Butler would have used $R_0 = 0.928$ and would have got $M_{\text{end}} = 0.064N_{\text{uz}}h$, $N/N_{\text{E}'} = 0.591$, $C' = 1.90$, $M_{\text{max}} = 0.64/(1 - 0.311) = 0.093N_{\text{uz}}h$. This excellent result illustrates the compensating features he mentioned but there is much more still unexplained. I am at present in the process of tabulating moment-rotation curves for columns, and case C4 is shown in Figure III. This relates end moment to end rotation, maximum moment and ultimate moment (M_u). Butler's end moment agrees with point a on the diagram and is positive. Had *elastic* stability functions been included, it would have been point b. But it is actually point c, a negative reversed restraining moment. It is here that there is an important compensating feature, derived

from my own work for the Joint Committee for Fully Rigid Steel Frames (reference 9 of the paper, Figures 17 and 18) and which affects 'added moments' in all codes: namely that those curves (reference 9) for extra maximum moments are corrections to end moments which (to please designers) deliberately avoid stability functions (point A) even if with stability functions (point B) the end moment had been reversed. In this case, R_{κ} varies along the column (see Figure II) between 0.86 and 0.22 (at M_{ult}) so that plasticity has reversed the end moment (point C) and to that extent Butler's extrapolation has been a lucky one. Note that, in Figure III, the final tangent at M_u has a gradient almost corresponding to $R' = 0.189$.

For further simplification, this raises important points.

- (1) How do we simplify Butler's approach further to make it into direct design? The above examination gives further clues.
- (2) The added moments in CP 110 are considerably exaggerated and vague in application. Conversely, initial curvature is hardly worth bothering about.
- (3) When buckling takes over, should we switch to a variable-stiffness approach, with very different tangential R values, or risk extrapolating Butler's method further with modifications?
- (4) Added-moment curves for other loading patterns are already published⁽⁹⁾.

Whatever the outcome, Butler's method should change the future face of reinforced concrete column design.

Reply by the author

I would like to thank Dr Wood for his informative contribution which sheds additional light on the matter of the actual behaviour of restrained reinforced concrete columns with deteriorating stiffness. Hi Figure III is in agreement with Cranston's computations for case C4 which showed θ to be approximately 0.042 at maximum load (a stability failure corresponding to the point of tangency of the beam line and the $m-\theta$ curve in Figure III) with $M_{\max}$ at mid-height nearly equal to M_u , and with a negative restraining moment at the ends.

In my proposed method, it was not intended that equations 3 and 4 be used to predict the *actual* values of $M_{\max}$ and M_{end} at failure. As Dr Wood notes, equation 4 gives a purely fictitious value of the end moment and cannot possibly predict the moment reversal which occurs in the case of very slender or highly deteriorated columns. Furthermore, equation 3 is to be used only in connection with the available moment diagram, Figure 7, and is not intended to estimate the actual column moment at mid-height. For instance, in case C4, the actual $M_{\max}$, as noted above, is approximately M_u , which equals $0.148N_{uz}h$ for $\bar{N} = 0.269$. This value differs significantly from the $0.094N_{uz}h$ calculated by Wood and the $0.093N_{uz}h$ attributed to me (the latter calculation differs slightly from the proposed method in that Wood used N_{er}' rather than N_{er} in applying equation

3). The relatively low value of $M_{\max}$ given by equation 3 is therefore a poor estimate of the mid-height moment but, when measured from the base-line 0a, Figure 7, it is useful in estimating the load capacity of the column.

The point of all this is that the explanation for the apparent success of the proposed method is even more involved than Dr Wood suggests. A crucial factor is obviously the establishment of a realistic available-moment diagram, Figure 7, since the value of N_o , and hence the location of base line 0a, immediately differentiates columns prone to buckling failure (low $\bar{N}_o$) from those which are heavier or more adequately restrained. For this purpose, at least, the use of the simply stiffness reduction function $R = 1 - \bar{N}^2$ can be defended on the basis that it provides a reasonable estimate of tangent modulus stiffness in the absence of moments. It is indeed fortuitous that for general combined loading cases this same simple function suffices, but a full interpretation must involve the validity of the available-moment diagram established by this function. Admittedly much still remains to be explained.

Dr Wood's generous comment concerning the significance of this work with respect to the future of reinforced concrete column design is deeply appreciated.