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Collapse load of deep reinforced concrete beams*

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We have studied with interest the way in which Mr Kumar uses the truss model (Figures 1 and 2 of his paper) to obtain a theoretical lower bound on the actual collapse load. He states that "This truss essentially represents an equilibrium picture of the distribution of internal forces. Thus, the calculated collapse load is a theoretical lower bound on the actual collapse load". We are puzzled by this statement. Presumably Mr Kumar has assumed that the upper chord and the inclined struts are each in a state of uniform uniaxial compression, as shown in our Figure I; in this Figure, the elements in the regions A and B are each subjected to a compressive stress in the direction of the respective member axis and to zero stress in the other two principal directions. We wish to point out that, for such a stress distribution, neither the elements in the loading region C nor those in the support region D can possibly be in a state of equilibrium. Therefore, even assuming that limit theory is applicable, a collapse load based on such a model cannot be accepted as a lower bound on the true collapse load.

It is, of course, possible to modify the model and the assumed stress distribution so as to satisfy the equilibrium condition everywhere. There are many possible ways of doing this, one of which is shown in Figure II. That every element in the model is now in

Figure I: Stress distribution corresponding to the author's Figures 1 and 2.

Figure II: An equilibrium stress distribution.

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equilibrium is confirmed by the Mohr's circles in Figure III. If the scale for the Mohr's circles is so chosen that the compressive stress p does not exceed the limiting stress in, say, a square yield locus for plane stress, then the yield condition is also satisfied, both in the plane of the paper and normal to it. The simultaneous satisfaction of the equilibrium and yield conditions guarantees that the calculated collapse load is a lower bound on the true collapse load in the context of limit theory.

Several research workers, notably Nielsen and Brstrup⁽¹⁻³⁾, have recently demonstrated the potential application of limit theory to concrete structures. Admittedly, whether the theory should be applied to concrete structures or not is a controversial matter, but personally we feel that the theory provides the engineer with a simple tool for the rapid calculation of a set of numbers, which are the so-called lower-bound and upper-bound solutions. He can then assign appropriate weightings and apply safety factors to these numbers to obtain an estimate of the likely strength of the structure. The question arises of what weightings and safety factors are appropriate; in this context, laboratory tests are valuable. Of course, it is important that the lower- and upper-bound theorems are applied correctly in the first place: these theorems follow directly from the convexity of the yield locus and the normality of the incremental plastic strain vector to the yield locus. A lower-bound solution is guaranteed only if both the equilibrium and the yield conditions are everywhere satisfied, though whether or not geometric compatibility is satisfied is irrelevant. Similarly, an upper-bound solution is guaranteed only if the condition of geometric

Figure III: Mohr's circles for stresses in regions of Figure II.

compatibility is satisfied by the collapse mechanism, and this time the equilibrium and yield conditions are both irrelevant⁽⁴⁾.

The preceding paragraph reflects the philosophy behind the extensive research programme on deep beams which is now in progress at Cambridge. Briefly, the aim is to add the results of experiments to mathematical predictions by observing actual behaviour and, through such observations, to develop an intuitive feel of how deep beams carry their loads. Figure IV shows one of the large beams after a test to destruction. These tests are, among other things, helping us to assess the way in which the powerful theorems of limit theory can be properly applied to the design of reinforced concrete deep beams.

Figure IV: Collapse mechanism of one of the deep beams tested at the University of Cambridge. Ultimate load 4120 kN.

Reply by the author

I thank Dr Kong and Mr Kubik for their interest in my paper and would like to answer the questions they raise. First of all, they are quite correct in reading that the upper chord and the inclined struts are each in a state of uniform uniaxial compression.

The disputed elements C and D are redrawn in Figure V. The equilibrium of element C can be seen at once, and follows from the symmetry of loading which is always present in the testing arrangement shown in Figure VI. This arrangement is one that has been frequently used by Dr Kong himself in his experimental investigations. An interesting situation can arise in the case of a deep beam subjected to two or more independently applied loads which are unequal, disturbing the symmetry of loading. In this situation, two inclined members may be required to take the unbalanced load. This situation is, however, outside the scope of the paper under discussion.

The equilibrium of the element D is always maintained, provided the anchorage of the reinforcement is sufficient to supply the necessary horizontal pressure. In the experimental investigations, two methods of anchoring the reinforcement, shown in Figure VII, were generally used. The arrangement shown in Figure II is the same as Figure VIIb. If there is not sufficient anchorage, obviously the equilibrium will be upset, leading to an anchorage failure. References 8 and 9 of the paper discuss effective means of anchoring the reinforcement. If these means are followed, anchorage failure will not occur and equilibrium will be maintained until failure is initiated by some other cause.

In brief, the truss system used in the paper under discussion essentially represents an equilibrium picture of the internal forces and furnishes a lower bound on the true collapse load. It is left to each individual to work out the minor details to satisfy himself. Dr Kong and Mr Kubik accept that there are many possible ways of satisfying equilibrium. My method happens to differ from theirs.

The objective of the paper is to shape the famous arch analogy of a deep beam, which has undoubtedly been used by others in various ways for a long time, into a disciplined design method. More rigorous solutions can most certainly be obtained, but only at the expense of practical usefulness.

Figure V

Figure VI

Figure VII

Checking whether a theoretical solution gives a lower or an upper bound to experimental results is very difficult, partly owing to the unpredictable and variable properties of the material concrete, which leads to a scatter of results when plotted. Good correlation between the experimental and theoretical values is the only available check. This can be seen from Figures 3 to 6 of the paper.

Unfortunately I cannot reply to Dr Kong and Mr Kubik regarding their experiments, because no experimental facilities are available to me at present. I still have a significant amount of unpublished theoretical research which is awaiting experimental verification.

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