Plastic analysis of shear in reinforced concrete*

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Mr Bræstrup is to be complimented on a very thorough investigation of the shear strength of reinforced concrete beams according to the plastic theory. It is interesting that equation 4 in the paper by Rajendran and myself published in the same issue of the *Magazine* is identical with the equation for one of Mr Bræstrup's special cases.

There are just two points in the paper which puzzle me. One is the identification of the 'kinematic discontinuity' (the squiggly line in Figures 2 and 3) as a 'crack'. I would regard a 'crack' as a discontinuity in the concrete, usually seen to develop rather early in experiments while the steel is still elastic and well before the ultimate load is reached. On the other hand, a 'kinematic discontinuity' is presumably a narrow region of high plastic strain (like a yield line) which is taken to form part of the collapse mechanism used in the plastic theory. It is by no means certain that the

theoretical kinematic discontinuity will coincide with the experimental cracks.

Secondly, on looking at Figure 5, I wonder what determines the position of the discontinuity in the shear span and whether there could not occur a collapse mechanism which was the superposition of a series of those in Figure 5 with the discontinuities in different places. In the limit, the resulting mechanism would be the one proposed by Calladine⁽¹⁾ for plate girders, with a parallelogram-shaped region of constant plastic strain. In Calladine's mechanism, the principal strain directions are the bisectors of the angles of the parallelogram (cf. the angle $\beta/2$ in Figure 2). The resulting formula for τ/σ_c is the same, but the relationship between the angle θ for Calladine's mechanism and the experimental crack direction is again rather obscure.

Reply by the author

Dr Morley rightly points out that equation 4 of the paper by Rajendran and Morley is identical with equations 5b and 5c of my paper. Indeed, the equation generally governs shear which is constrained in the sense that the direction of either the yield line or the deformation rate is given. In the case of the push-off test, the yield line is fixed by the geometry of the specimen. For shear in beams, the deformation rate must be perpendicular to the non-yielding main reinforcement.

I completely agree on the two points raised by Dr Morley. The term 'crack' was used for historical reasons only. In the literature on beam shear, failure is often reported to have taken place along 'diagonal cracks' or 'shear cracks', although it is obvious that

shear forces are transferred across the so-called cracks. This practice is somewhat confusing and should be abandoned. In the paper by Nielsen and Bræstrup (reference 4 of the paper), which still awaits publication, and in a report⁽²⁾ now in preparation, we try to clarify the matter along the lines proposed by Dr Morley by distinguishing between *cracks* and *yield lines*. *Cracks* develop in the direction of principal compressive stress, and they need not be accompanied by any significant deformation. If they are, then the deformation rate must be perpendicular to the crack direction. The deformations are located in *yield lines*, which are idealizations of narrow regions with many criss-crossing cracks and crushing zones. In plane stress, the relative deformation rate may be at any angle to the yield line, which in general does not follow a principal stress trajectory.

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Figure I: Shear failure mechanism with distributed strain rates.

Figure II: Shear test. Failure of beam No. T6025.

I regret not having stressed in the paper that the kinematical discontinuity (yield line) on Figure 5 is introduced merely to facilitate the upper-bound calculations. The position of the yield line is arbitrary; hence in reality the failure is likely to take place in a parallelogram-shaped region, as suggested by Dr Morley. In the papers mentioned above, we have included the failure mechanism with distributed strain rates, see Figure I. That the mechanism has some resemblance to reality is demonstrated by Figure II, a photograph of one of our test beams at failure.

I am grateful for the reference to Calladine's paper; indeed his approach is quite similar to ours. The rela-

tionship $\varphi = \theta/2$ is supposed to govern the inclination φ of the cracks formed just prior to failure, θ being the inclination of the lines bounding the yield zone (Figure I). On the test beam (Figure II) the extension of the yield zone may be judged from the tensile flange cracks near the support, indicating the formation of yield nodes in the flange. The beam No. T6025 had a shear reinforcement parameter $\psi = 0.35$. Taking the web effectiveness to be $v = 0.85$, we get a theoretical crack inclination given by $\cot \varphi = \sqrt{v/\psi - 1} = 1.2$, i.e. $\varphi = 40^\circ$ and $\theta = 80^\circ$. The experimental evidence of Figure II seems to indicate that these values are somewhat too high.

REFERENCES

1. CALLADINE, C. R. A plastic theory for collapse of plate girders under combined shearing force and bending moment. *The Structural Engineer*. Vol. 51, No. 4. April 1973. pp. 147-154.
2. BRAESTRUP, M. W., NIELSEN, M. P., BACH, F. and JENSEN, B. C. *Shear tests on reinforced concrete T-beams. Series T*. Copenhagen, Technical University of Denmark, Structural Research Laboratory. (In preparation.)