A method for determining the shear strength of reinforced concrete beams with small a_v/d ratios*

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Contribution by Professor G. Brock

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Professor Desayi, in commenting on my discussion⁽⁶⁾ of Kani's paper, feels that "to assume a cylinder of a diameter bigger than the effective depth of the beam itself and a component in the direction of the crack greater than the vertical shear itself is unsatisfactory". It would seem that Professor Desayi's analysis is in error since, in his Figure 4, the horizontal forces necessary to preserve equilibrium are neglected. These forces must be taken into account as indicated in Figure I and it is then clear that the splitting force is $V_u \operatorname{cosec} \theta$ and thus in excess of V_u . Although the exact position of the application of the splitting forces is open to speculation, there is no doubt that the beam splits over the whole distance from the load to the support.

The question of the best constant K to use in the formula $M_u = f'_c b d^2 \times a_v/d \times K$ may be resolved by reference to experimental data from beams which have been observed to fail by diagonal splitting. A value of $K = 0.182$ has been suggested as being somewhat conservative.⁽¹⁾ By incorporating this value into an interaction diagram of the type shown in Figure II, the limits of each type of failure can be defined, including those of splitting, and it becomes possible to predict the ultimate moment (M_c) of any beam and compare it with the experimental value (M_E). Some 1200 tests of this type have been investigated⁽²⁾ and the statistic M_E/M_c for them had a mean value of 1.05 with a coefficient of variation of 19% and a distribution close to the lognormal. Of this number, only 72 tests were identified as splitting failures and for these M_E/M_c had a mean value of 1.16 and a coefficient of variation of 25%. This mode of failure is characterized by the greatest scatter of prediction.

The first eight of Professor Desayi's test beams are predicted as diagonal splitting failures and have a mean value of M_E/M_c of 1.22. The last four are predicted to be shear bond failures for which the mean

Figure I: Suggested mechanics of splitting failures.

Splitting force = $V_u \operatorname{cosec} \theta$, i.e. the resultant of V_u and H_u

Splitting diameter = $D \operatorname{cosec} \theta$

Hence

$$V_u \operatorname{cosec} \theta = \pi/2 \times D \operatorname{cosec} \theta \times b \times f_{t \text{ cyl}}$$

and

$$\frac{M_u}{f_{c \text{ cyl}} b d^2} = \frac{\pi}{2} \times \frac{f_{t \text{ cyl}}}{f_{c \text{ cyl}}} \times \frac{D}{d} \times \frac{a_v}{d} = K \times \frac{a_v}{d}$$

Figure II: Variation of M_u with a_v/d (after reference 2).

*Pages 29 to 38 MCR 86.

value of M_E/M_c is 1.07. For the whole 12 beams, M_E/M_c has a mean value of 1.17 with a coefficient of variation of 18.5%. This interpretation agrees with that of the author in that the last four beams fail by some different mechanism where the dependence upon the ratio a_v/d is very slight.

When web reinforcement is introduced into the analysis of the splitting failure, its effect is statically indeterminate. Professor Desayi has assumed that it will augment the area of concrete resisting splitting in proportion to the modular ratio. However, this effect is slight in comparison with the scatter of the strength of beams without web reinforcement. I therefore discount any effect of web reinforcement in predicting the strength of beams which fail by splitting, and Figure III gives evidence in support of this view.

Figure III: Splitting failure of beams with varying web reinforcement (M_c assumed to be independent of web reinforcement).

Reply by the author

I thank Professor Brock for his interesting and useful comments. Referring to his Figure I, I am doubtful whether the steel at the support (where the bending moment is zero) could be assumed to develop a horizontal force H_u . The analysis given in the paper does not appear to be in error when the equilibrium of forces is considered as shown in Figure IV. The forces acting on the portion of the beam AC which is to the left of the vertical section BC are shown in Figure IV. The forces V_u acting at A and B are resolved into components in the direction of and perpendicular to AB. The forces $V_u \sin \theta$ acting at A and B and along AB are equal and opposite while the forces $V_u \cos \theta$ acting in the two opposite directions at A and B result in a clockwise moment $V_u \cos \theta \times AB = V_u a_v$. On the face AC, the force F_c (compression in concrete) and F_t (tension in the steel) are a result of the bending moment $V_u a_v$ and are such that $F_c = F_t$ and offer an anticlockwise moment of $F_c jd$ which is equal in magnitude to $V_u a_v$. Thus, all the forces and moments are in equilibrium and the force that could cause splitting along AB is $V_u \sin \theta$, as assumed in the paper (Figure 4).

The suggestion by Professor Brock, viz. that M_u should be determined from $f'_c b d^2 (a_v/d) K$, where K is decided from a statistical analysis of the test data, is indeed a novel method of finding a solution to the problem considered, and also to all similar situations where a full understanding of the structural behaviour is not yet available.

Regarding the contribution of shear steel, whilst I agree that neglecting it altogether results in a safe method of design, it should not be overlooked that, in some of the available test results (references 5, 7 and 10 of the paper), the steel has been found to have yielded at failure. Also, as the crack is inclined closer to the

Figure IV: Equilibrium of forces and moments.

vertical in the beams with small a_v/d , the horizontal steel is probably more effective in contributing towards shear resistance than the vertical stirrups. A precise understanding of the behaviour and contribution of steel would be helpful in the proper detailing (and designing) of corbels, brackets and other structural elements where failure due to splitting (owing to small a_v/d ratio) is a possibility.

I would like to point out an omission, due to oversight, which has unfortunately occurred in some of the equations. This happened while A_n was being replaced in terms of A_h and A_v in equation 2. Thus, in equations

3 and 4, the second term in the parentheses requires to be multiplied by $\sqrt{1 + \beta^2}$ and that in the equation 6 by $\sqrt{1 + \beta_1^2}$. This correction would result in a small increase only in the shear contribution of the two steels A_h and A_v and a corresponding marginal increase in

the total shear resistance of the section. Thus the numerical values of the shear forces calculated in the paper are fairly close to the corrected ones, but the correction indicated may be noted by interested readers.

REFERENCES

1. BROCK, G. and DUTTA, S. C. Ultimate strength of reinforced concrete beams in bending and shear. *Proceedings of International Conference on Shear, Torsion and Bond in Reinforced and Prestressed Concrete, Coimbatore, January 1969.*
2. DEPARTMENT OF CIVIL ENGINEERING, LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY. *Results of shear tests on reinforced concrete beams.* Loughborough. pp. 86.