Discussion

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Shear reinforcement of reinforced concrete beams subject to distributed loading*

by R. B. L. Smith, PhD, MScTech, MICE, MStructE

Contribution by Surendra K. Ojha, Dr Ing

Pant College of Technology, Pantnagar (Nainital), U.P., India

Dr Smith must be complimented on a valuable contribution to the study of the shear strength of beams subjected to uniformly distributed loading. As he has rightly observed, very little has been contributed in this important area, although uniformly loaded beams are by far the most common, especially in building construction.

A theoretical study of beams with distributed loading is made difficult because of lack of prior knowledge regarding the location of the critical section for 'shear-compression' failure, which is fixed in the case of two-point loading. It is necessary to estimate the ultimate strength of the shear-compression zone before we can write down the equilibrium of moments at failure, but this requires knowledge of the prevailing normal and shear stresses (or of the moment and shear force). The location of the failure section, therefore, assumes primary importance. It must, moreover, be estimated very accurately, because a small error in this will change the critical moment/shear ratio appreciably since, in simple beams, bending moment and shear force increase in different directions. I should like to restrict my comments on the present paper to the problem of the location of the critical failure section, i.e. of the distance x_1 as defined in the paper.

The method proposed by Dr Smith is based upon the assumption that the diagonal crack is initiated at the section at which $M/Qd_1 = 1.2$ but, before the latter can be calculated, it is necessary to know the cracking

load; this has been obtained empirically. It is obvious that such assumptions, and the empirical bases upon which they are founded, cannot be expected to be universally applicable. Similar conclusions were reached by Whitney⁽¹⁾, who found for x_1/L a value between 0.30 and 0.35; by Bernaert and Siess⁽²⁾, who concluded that the diagonal crack is at a mean distance of $0.39L$ measured from the support on the neutral axis; and by Leonhardt and Walther⁽³⁾, who observed in their tests that x_1/d_1 lies between 2 and 3.5.

In this context, I should like to refer to an earlier work⁽⁴⁾, in which I described the possibility of locating the critical section theoretically. The two-dimensional polynomial solution for the uncracked beam, from the theory of elasticity, enables the concrete stresses σ_x , σ_y and τ_{xy} to be expressed in terms of the co-ordinates x and y and the cracking load p . The failure criterion equation is now written down for a critical fibre of the beam, thus eliminating y (in deeper beams, for example, it is known that the diagonal crack appears first at the mid-depth, where $y = 0$); in this manner, we obtain one equation with two unknowns, x and p . The correct value of x can now be iterated by taking as the starting value for p the flexural failure load of the beam. This fixes the location of the crack for the particular value of p , from which, with a knowledge of the crack inclination⁽⁵⁾, it is easy to fix, theoretically, the potential failure section.

I have since developed this approach further and have used the distortion energy criterion⁽⁵⁾ as a means of writing down the failure equation. Since plastic con-

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ditions prevail in the shear-compression zone at the approach of failure, it is permissible to use average values σ for and τ (rectangular stress blocks). The failure condition gives an eighth-degree equation in x_1 which has only one real root in the region $0 \leq x \leq L/2$. This fixes the failure section and enables one subsequently to obtain, theoretically, the shear-carrying capacity. The method is presently being tested against available experimental evidence and has already been found to give satisfactory results for beams without web steel. It is hoped to publish the method in the near future.

REFERENCES

1. WHITNEY, C. S. Ultimate shear strength of reinforced concrete flat slabs, footings, beams and frame members without shear reinforcement. *Journal of the American Concrete Institute*. *Proceedings* Vol. 54, No. 4. October 1957. pp. 265–298.
2. BERNAERT, S. and SIESS, C. P. *Strength in shear of reinforced concrete beams under uniform load*. Urbana, University of Illinois, June 1956. Civil Engineering Studies, Structural Research Series No. 120.
3. LEONHARDT, F. and WALTHER, R. Stuttgarter Schubversuche 1961. Beiträge zur Behandlung der Schubprobleme im Stahlbetonbau. *Beton- und Stahlbetonbau*. Vol. 56, No. 12. 1961. Vol. 57, Nos. 2, 3, 6, 7 and 8. 1962. English translation: *The Stuttgart shear tests, 1961. Contributions to the treatment of the problems of shear in reinforced concrete construction*. London, Cement and Concrete Association, 1964. pp. 134. Library Translation Cj.111.
4. OJHA, S. K. *Über die Schubtragfähigkeit von Stahlbeton- und Spannbeton-Rechteckbalken mit einer oder zwei Einzellasten*. (The shear-carrying capacity of reinforced concrete and prestressed concrete rectangular beams with one or two point loads.) Thesis presented to the Technische Hochschule Hannover for the degree of DrIng.
5. OJHA, S. K. The shear strength of rectangular reinforced and prestressed concrete beams. *Magazine of Concrete Research*. Vol. 19, No. 60. September 1967. pp. 173–184.

Reply by the author

The problem of shear failure under distributed loading depends, as Dr Ojha says, upon the location of the critical section. It also depends upon the value of the shear-compression moment at this section and this, in turn, depends upon the pattern of diagonal cracking, especially in beams with web reinforcement. This is because of the influence of the dimension c defined in the paper. Thus there is a rather complex interaction between diagonal cracking and shear compression. The independent investigation of this problem described by Dr Ojha is therefore welcome. The purpose of the present paper was to introduce a simple method of

estimating the web reinforcement required to ensure flexural failure, not to calculate the failure load under all circumstances.

It is surely not necessary to know the cracking load in order to determine the section at which $M/Qd_1 = 1.2$. The calculation of web reinforcement requirement is independent of the estimated cracking load, which is only needed in order to predict whether diagonal cracking will occur at all before the flexural capacity of the beam is reached.

I am grateful to Dr Ojha for his contribution.