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Shear in reinforced concrete beams*

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I have studied Dr Regan's paper with interest, but there are two aspects of the theory presented therein which I feel might be examined in more detail. In the first place I believe that the effects of aggregate interlock and dowel action of the reinforcement are of such importance that their contribution towards shear resistance cannot be ignored. This may be apparent if the theory presented in the paper is developed a little further. In Figure 1, which has been reproduced with a few additions in Figure I, the projected trajectory of the flexural compression force is shown together with the transverse tensile and compressive forces, P , which the requirements of equilibrium show must develop between the tension and compression zones of the beam. The existence of these forces was pointed out in the paper but their influence appears to have been overlooked when the strength of the compression zone was assessed. A possible distribution of stresses arising from these forces is shown in Figure I.

From the requirements of equilibrium of the zone of a beam containing flexural cracks, and if shear transfer across the crack is neglected, the following equations may be derived:

$$\begin{aligned}\Delta TL_{a2} &= (T_1 - T_2)L_{a2} \\ &= Qs_r - F_{cc1}(L_{a1} - L_{a2}) \dots \dots (I)\end{aligned}$$

$$\Delta Th_c = Pr \dots \dots \dots (II)$$

where the subscripts 1 and 2 refer to the cracks shown in Figure I. With the suggested distribution of stresses the term Pr may be rewritten as $0.28 bs_t f_t \times 0.65 s_t$.

Thus equation II becomes:

$$\Delta Th_c = 0.18 bs_t^2 f_t \dots \dots \dots (IIa)$$

As this equation is based upon an assumed distribution of stress, it may only be used to assess the order of magnitude of these stresses. However, even if a rectangular stress distribution is used, the numerical coefficient only increases to 0.25.

The crack pattern and test results of a beam tested at the University of Canterbury (New Zealand) are shown in Figure II. Strain measurements were made at many points on the reinforcing bar, enabling the distribution of the flexural tension force to be found. Strains were also measured on the compression zone of the beam, but these measurements were made with a 203 mm strain gauge allowing only the mean trajectory of the compression force to be found. The experimental results show that over a large section of the span the internal lever arm did not vary in magnitude. Thus equation I simplifies to:

$$\Delta T = \frac{Qs_r}{L_a}$$

and, by combining this with equation IIa, an expression for the tensile stress may be obtained, viz:

$$f_t = \frac{Qs_r h_c}{0.18 bs_t^2 L_a}$$

Substituting in this equation the values of crack spacing, crack height and internal lever arm measured in the above test, a shear of 50.6 kN would induce a tensile stress of 12.4 N/mm². This result is not an exceptional one. The theory described above may be applied to the results of tests published in the literature

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Figure 1: Internal forces in a beam.

on beams with shear span to depth ratios of 4 or greater. Only the crack pattern of such beams needs to be known, as the position of the compression force and hence the magnitude of the internal lever arm may be located within certain well defined limits in the zone of the beam containing the flexural cracks. Clearly, tensile stresses of the order of 12.4 N/mm² could not be sustained by the concrete, and hence modes of resistance other than those considered in the paper must operate.

I have carried out a number of beam tests, including that depicted in Figure II, which were designed to investigate the mechanism of shear resistance. From these tests it was found that, before diagonal cracking of the beam occurred, a very considerable portion of the shear was resisted by aggregate interlock action acting across the cracks. When this action was reduced by the insertion of crack initiators in the tensile zone of the beam, the diagonal cracking and ultimate strengths were reduced to 56 and 74% respectively.

From direct tests on aggregate interlock action, a relationship was established between the crack width, shear displacement and shear stress transmitted across a crack for the particular type of concrete used. This relationship, together with extensive measurements of crack width and shear displacement in beam FA4, were used to assess the shear forces transmitted across the cracks. The results of these calculations are shown in Figure II. The cracks in this beam were located by casting 25.4 mm wide strips of plastic in the sides of the beam, and dowel action of the reinforcement was reduced by covering the bars in the vicinity of the cracks with soft wrappings. This beam has been described in greater detail elsewhere⁽¹⁾. It may be seen from Figure II that about 60% of the total shear was resisted by aggregate interlock forces acting across the flexural cracks. This result, which shows that only a relatively small portion of the shear is resisted by the compression zone, is in agreement with Taylor's findings⁽²⁾.

The second aspect of the paper I would like to discuss is the compatibility condition used by Dr Regan to relate the depth of the concrete above the diagonal crack to the displacements in the zone containing this crack. The analysis has been based upon

Figure II: Experimental results for beam FA4.

(a) Crack pattern ($u_w = 41.1 \text{ N/mm}^2$).

(b) Strain distribution.

(c) Tension force in reinforcement.

(d) Distribution of shear at sections containing a crack ($Q = 50.6 \text{ kN}$).

Q_a = aggregate interlock shear across crack.

Q_c = shear resisted by compression zone above crack.

Q_d = dowel shear.

the assumption that the sections A-A and B-B located at the ends of the diagonal crack remain plane (see Figure III). The relationship between the depth of the neutral axis at the end of the crack and the strains in this zone is given as:

$$\frac{\Delta_{cc}}{\Delta_{st}} = \frac{\bar{n}}{1 - \bar{n}} \dots \dots \dots (IV)$$

where Δ_{cc} is the deformation of the top fibre between A_t and B_t

Δ_{st} is the deformation of the reinforcement between A_t and B_r

$\bar{n}$ is the neutral axis factor at A-A.

The derivation of this equation has unfortunately not been given. However, it would appear to be only an approximation which will over-estimate the magnitude of $\bar{n}$ even if the sections A-A and B-B do remain plane.

Strain measurements which have been made in the region of a beam containing a diagonal crack⁽³⁾ have shown that the line of action of the compression force follows an inclined path similar to that shown on Figure III. Typical strain distributions are shown on this Figure. The displacement of A_t from B_t and of A_n from B_n may be found by summing the strains between these points. It may be seen from the Figure that A_t and A_n are both displaced towards the line B-B. Thus the value of $\bar{n}$ given by equation IV must be greater than the depth of the neutral axis at the end of the diagonal crack, and it would appear to correspond to an average value of neutral axis depth for the zone of the beam containing the crack.

Figure III: Compatibility condition in zone of beam containing a diagonal crack.

Contribution by N. S. Bhal, BSc, BTech(Hons), MTech, Dr Ing, MASCE, AMIE(Ind), MIABSE
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I wish to confirm Dr Regan's finding that the unit shear resistance for otherwise apparently similar beams is affected by the absolute magnitude of d_1 . A study^(4,5) I made of the influence of beam depth upon the shear strength of reinforced concrete beams with and without web reinforcement also showed that the effect of beam depth is most noticeable in beams without web reinforcement. From the tests on such beams up to a depth of 1.2 m, it appears that this effect reaches a limiting value for beams with $d_1 > 1$ m. Fortunately, for beams with web reinforcement, the effect of absolute beam depth upon the ultimate shear strength was found to be negligible. This is probably due to the favourable effect of slip at the stirrup anchorage being

distributed over a larger length of stirrup in case of a beam with larger depth. However, for beams both with and without web reinforcement, the width of shear cracks was found to increase with increasing beam depth. I found an exponential relationship between Q_{cr}/bd_1 and d_1 from an analysis of over 300 tests on beams without web reinforcement.

I would also like to refer to the work⁽¹⁾ by Fenwick and Paulay on the shear carried by the interlocking of aggregate across the cracks and by the dowel action of the reinforcement. It appears that the shear carried by these two actions may even be the major portion of the total shear and hence cannot be neglected.

Reply by the author

I would like to thank Dr Fenwick and Dr Bhal for their interesting contributions.

The influence of the absolute magnitude of d_1 upon unit shear resistance is, as noted by Dr Bhal, important and should be taken into consideration when design formulae are derived from relatively small-scale laboratory tests. Since the effect upon strength is indeed much more pronounced in members without shear reinforcement, it is of significance primarily for slabs. It does appear to have been taken into account in recent CEB proposals⁽⁶⁾ in which the permissible shear stress in a slab without shear reinforcement is related to the effective depth. I would imagine that the proposals were influenced by the work at Stuttgart

referred to by Dr Bhal, and would therefore meet with his approval.

I am not conversant with Dr Bhal's work, but I would think that the effect of scale upon crack widths is largely a matter of the effect of stirrup spacing. In recent tests at Imperial College, we have found the widths of shear cracks to be virtually linearly dependent upon the spacing of shear reinforcement.

Dr Fenwick questions the derivation of the compatibility equation used to determine the depth of the concrete zone at the head of the shear crack. It is indeed only an approximation, but not as poor a one as he implies.

Figure IV shows the same part of a beam as his

Figure IV: Zone of beam containing a crack.

Figure III. If we use the symbols of the Figure,

$$\begin{aligned}\Delta_{cc} &= \Delta_{ca} + \Delta_{cb} \\ \Delta_{st} &= \Delta_{sa} + \Delta_{sb} \\ \frac{\bar{n}}{1 + \bar{n}} &= \frac{\Delta_{cc} - \Delta_{cb}}{\Delta_{st} - \Delta_{sb}}\end{aligned}$$

If flexural behaviour is more or less preserved and Δ_{ca}/Δ_{cb} is approximately equal to Δ_{sa}/Δ_{sb} , this reduces to:

$$\frac{\bar{n}}{1 - \bar{n}} = \frac{\Delta_{cc}}{\Delta_{st}}$$

If the slope of the thrust line is large, Δ_{cb} and Δ_{sb} will be very small in comparison with Δ_{ca} and Δ_{sa} and again the equation will reduce to the same form. In other cases, there is some error involved in using the reduced form but, if we take reasonable values for $\bar{n}$ and assume (somewhat unfavourably) that $\Delta_{ca}/\Delta_{cb} = n^*/\bar{n}$, the maximum error in $\bar{n}$ for any value of n^* is only approximately 15%. The percentage error is considerably reduced by the time it appears in the final expression for shear strength and is thus, to my mind, acceptable.

The other issue raised by both contributors is that of interlock and dowel actions, which was admittedly rather skated over in the paper. The main reason for this is that in design one is not much concerned with beams without stirrups, and it appears that these actions can contribute little to the strength of a web-reinforced beam failing in shear. The shear cracks at which failure occurs in such cases fall into two categories:

- (1) cracks at about 30° to the axis of the beam at which failure occurs by a primarily vertical movement—a type typical of T beams and rectangular beams with excessive compression reinforcement;
 - (2) cracks at any angle, at which failure is accompanied by crushing of the compressive zone and rotation at the crack—typical of normal rectangular beams.
- The angle between the former type and the shear force

is too large for aggregate interlock to be effective, whilst in the latter case rotation involves no interlock.

The equations of the paper take account of a certain amount of dowel action by assuming the number of vertical stirrups resisting shear to be equal to the horizontal projection of the crack length divided by the spacing of the stirrups. The number of stirrups necessarily crossing a crack can be one less than this, and the equations thus include a dowel action stressing one stirrup just beyond the tail of the crack. That this is a reasonable estimate of dowel action seems to be confirmed by Baumann's⁽⁷⁾ dowel tests.

This treatment is, of course, less appropriate in the case of very short beams in which a crack at an angle of more than 30° traverses the whole shear span. The behaviour of such beams is now being studied at Imperial College and we hope to have a fuller understanding of their behaviour in the near future.

Returning to the case of members without stirrups (or to shear cracking in other beams), it would seem from Fenwick and Paulay's paper that interlock must be significant in at least some cases. Further evidence of this is afforded by Sayani's⁽⁸⁾ work on the shear strengths of beams subjected to axial tension. In many of his tests, the axial tension produced vertical cracks extending through the entire depth of the beam, without materially affecting shear strength. Separate tests showed that the available dowel forces were far too small to account for this. Thus, in the absence of a compressive zone, the shear must have been carried by interlock.

To go from a general agreement that interlock exists to its incorporation in usable equations is, however, a difficult step. The proportions of the total shear carried by the three actions must depend upon their relative stiffnesses, and it would seem that, if the flexural cracks are wide, the rigidity of the interlock action must be reduced and shear must be concentrated towards the compressive zone. I think I have shown that this zone is capable of supporting a shear force of the magnitude Q_{cr} given in the paper. There is evidence for this view in that, if interlock were really very important, the maximum shear stress should have the conventional value Q/bI_a and shear cracking should not occur until this reached the tensile strength of the concrete. In fact, as is well known, shear cracking occurs much earlier than this.

Another point is that the ratio of tensile reinforcement is an important parameter influencing shear cracking resistance. It seems that this could be explained in terms of interlock only by saying that the interlock is reduced at the wider flexural cracks permitted by lower ratios of main steel, but this is contradicted by the fact that it would necessarily predict a reduction in shear resistance if the external moment were increased, whilst no notable reduction is observed if the 'shear-span/effective-depth' ratio of a point-loaded beam is increased from 3 to 6.

The evidence that the support of shear by the compressive zone, as assumed in the paper, would necessitate high vertical tensile stresses in the teeth between flexural cracks is indeed an indication of the existence of some interlock, but the tension in the teeth can also be considerably reduced by even a very small over-all slope of the compression above the cracks, which redistributes bond back towards the less cracked zones of low moment. This possibility is examined in considerable detail by Lorentsen.⁽⁹⁾

A final thought on this subject is that one would expect interlock action to be considerably reduced in lightweight-aggregate concretes, because the aggregate

would tend to fracture at the flexural cracks. In tests, lightweight beams seem to have diagonal tension strengths comparable with those of normal beams, though I would suspect that they might be more sensitive to the effect of axial tension.

In view of the above, I do not feel too apologetic about the shear cracking equation given in the paper but, if some further excuse is required, it can be said that it works well and gives good agreement with a very wide range of test results. If this is not due to its theoretical accuracy, then it must be because all three actions are dependent in much the same way upon the same parameters.

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