The flexural behaviour of slabs at ultimate load*

by D. L. Parkhill, ME, MASc, AMIE(Aust), MASCE

Contribution by K. O. Kemp, BSc, PhD, AMICE

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Mr Parkhill's interesting method of elastic-plastic analysis of slab is one that has also attracted the writer. The idea of obtaining an elastic solution by numerical methods for the segments of the slab bounded by the yield lines of a trial collapse mechanism and the boundaries of the slab appears to be a general and powerful approach. Essentially, it is analogous to the statical moment check used in the limit analysis of rigid-plastic frames to determine a lower-bound solution from a trial collapse mechanism.

In reality, however, there are serious difficulties which prevent the determination of a unique collapse load or even a valid lower-bound solution in all but a few symmetrical problems such as the square slab considered by the author. These difficulties arise from the classical plate flexure problem of not being able to specify more than two conditions along any boundary. Thus, along a yield line, any two quantities chosen from the normal bending moment M_n , the twisting moment M_{nt} , the vertical shear force Q_n or the

Thomson and Tait vertical force $V_n = Q_n - dM_{nt}/dt$ may be specified and continuity of these two will be maintained across the yield line. Wherever the segments of slab adjacent to a yield line are non-symmetrical, however, it will not be possible to insist in the computation that continuity of all three M_n , M_{nt} and Q_n is maintained. The solutions obtained will not, therefore, necessarily satisfy equilibrium and their relationship to the bound theorems of limit analysis cannot be specified. In the problem examined by Mr Parkhill, the normal bending and twisting moments have been specified as boundary conditions and, because of symmetry, the vertical shear force will also be zero, but it must be recognized that this is a special case.

It is important, therefore, to appreciate that the method cannot be extended to more general problems as readily as Mr Parkhill suggests. Solutions in general cannot be incorporated into the theory of limit analysis; all that can be claimed is that they may provide a guide to the distribution of moments in the slab away from the yield lines. How reliable these predictions of moments will be seems impossible to state.

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Contribution by G. M. McNiece, BSc, AMEIC

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Mr Parkhill's approach to the collapse analysis of plates is an interesting one and constitutes a genuine departure from the usual methods of lower-bound analysis. However, for the problem of the square slab under a uniform pressure, the pressure causing first yield at the centre of the slab is approximately 85% of the limit analysis collapse load. This represents very little redistribution of moments in the elastic-plastic behaviour and consequently the moment field at collapse should not be too dissimilar to a scaled-up elas-

tic distribution except close to and along the yield lines. This is not the case in the analytical results of Figure 5. The lines of constant principal moment that intersect the central axis ON of the slab indicate that both principal moments are equal at any one point on ON. This condition can only exist where the bending moments M_x and M_y are equal in magnitude and sign. The writer finds it difficult to accept that this is a unique solution and not simply another lower bound obtained by a numerical method.

Figure I: Contours of principal moments at collapse.

The writer has participated in the development of an elastic-plastic analysis by the finite element method for certain slab bending problems including the square slab being discussed. The resulting moment field at collapse (Figure I) is very different from Mr Parkhill's. A paper is presently being prepared with a complete elastic-plastic solution of the above problem along with others.

Reply by the author

I thank Dr Kemp and Mr McNeice for their contributions to the paper.

Dr Kemp expresses concern over the difficulties in

specifying boundary conditions for slab segments where non symmetry in the yield pattern exists. The argument overlooks the fact that, with the Johanssen criterion, bending moments across yield lines are principal moments and thus twisting moments are absent on yield line boundaries, whether symmetry exists or not.

Mr McNeice's intuitive feelings as to what the moment field should look like are appreciated. However, there can surely be no dispute over the fact that a moment field, such as the one presented, which satisfies both 'static' and 'kinematic' requirements must represent a unique solution.