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Thermal stresses in reinforced concrete slabs*

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Dr Hannant's paper on thermal stresses in reinforced concrete slabs is of great value in confirming some previous experimental investigations into the relevance to reinforced concrete of the elastic analysis of thermal stresses in building units in cases of prolonged heating. Experiments at King's College, London⁽¹⁾ have also shown that effects of differential and total shrinkage in concrete caused by moisture movements within the concrete and total loss of moisture to the surroundings can distort and indeed completely reverse the stress picture. Practical designers apparently put almost as much steel on the "compression" side as on the "tension" side but for the different reason of the possibility of a reversed temperature gradient in the relevant heating cycle.

In some of the experiments described in Dr Hannant's paper, the temperature of the "cool" surface must, in some cases, have been of the order of 80°C or higher. Although Demec mechanical extensometers have moving parts made of Invar, it would be helpful to know whether it was established that the heating of the gauge resulting from its proximity to the warm surface had a negligible effect on the measurements. It also appears that the heating portion of the cycle in all the tests was for at least three days. It would have been interesting to know how relevant elastic analysis without inelastic effects is to a 24-h cycle.

Tables 1 and 2 in the Appendix of the paper demonstrate the effect of a reduced modulus of concrete on the steel stress, and they are derived from equations 1 and 3. The derivation of these equations appears to be questionable. Presumably the process was as follows. (a) Uncracked section, taking concrete in tension into account: the thermal strain is considered to be linear, following the thermal gradient as shown in Figure 18a of the paper. Considering w as an "average strain",

$$w(A_c + mA_s) = A_c \alpha_c \frac{Tm}{2} + mA_s \alpha_s r Tm$$

r , incidentally, is given in the notation as $(x-d)/d$, but should strictly be $(x-d)/x$.

$$\text{Therefore, } w = \frac{Tm(A_c \alpha_c + 2mA_s \alpha_s r)}{2(A_c + mA_s)}$$

$$\begin{aligned} \text{Steel stress, } t &= E_s(w - \alpha_s T_s) \\ &= Tm E_s \left(\frac{A_c \alpha_c + 2mA_s \alpha_s r}{2(A_c + mA_s)} - \alpha_s r \right) \end{aligned}$$

$$\begin{aligned} \text{where } T_s &= r T_m \\ &= GE_s(B - \alpha_s r x) \dots \dots \dots (1) \end{aligned}$$

The fallacy of the above approach derives from the fact that while, for a given section, free thermal strain is a function only of the coefficients of expansion of mild steel and concrete, the position of the neutral axis is a function of the moduli.

If we proceed differently and consider the curvature produced by free thermal expansion (without connecting bolts), employing the same notation as in the paper,

$$\text{curvature} = \alpha_c G$$

The bending moment, M , required to flatten the slab $= E_c I_c \alpha_c G$, where I_c is the second moment of area of the equivalent concrete section.

$$\text{Steel stress } t = \frac{E_s M(d-z)}{E_c I_c} = E_s \alpha_c G(d-z)$$

Neutral axis z is given by

$$\begin{aligned} z &= \frac{A_c x/2 + mA_s d}{A_c + mA_s} \\ d - z &= \frac{A_c(2d - x)}{2(A_c + mA_s)} \end{aligned}$$

$$\text{Therefore } t = GE_s \left(\frac{\alpha_c A_c(2d - x)}{2(A_c + mA_s)} \right) \dots \dots \dots (I)$$

*Pages 91 to 98 in *Magazine* No. 41.

(b) By similar reasoning, the steel stress for an uncracked section, neglecting concrete in tension, is given by:

$$t = \frac{E_c I_c \alpha_c G}{A_s (d - z/3)} \dots \dots \dots (II)$$

where
$$z = \frac{-mA_s + \sqrt{m^2 A_s^2 + 2mA_s y d}}{y}$$

and
$$I_c = \frac{yz^3}{3} + mA_s (d - z)^2$$

The values $A_c = 204 \text{ in}^2$, $A_s = 0.77 \text{ in}^2$, $d = 7.375 \text{ in.}$, $x = 8.5 \text{ in.}$, $r = (x - d)/d = 0.1526$, $E_s = 34 \times 10^6 \text{ lb/in}^2$, $y = 24 \text{ in.}$ substituted in equation 1 of Dr Hannant's paper give the values in the first column of Table 1. These same values used in equations I and II give the stresses in Table I. Comparison with Table 1 shows differences which can be substantial at high thermal gradients; at the ultimate sustained modulus, equation 3 breaks down completely. Equations I and II would appear to be both simpler and more correct.

Lastly, the investigation seems to have been confined to the central area. Theoretical considerations and photoelastic analysis by the contributor appear to show, however, that the maximum principal stress occurs very near the ends, and experiments to check this in reinforced concrete would have been very valuable.

TABLE I

α_c $\times 10^6$ per °C	α_s $\times 10^6$ per °C	E_c (lb/in ² $\times 10^6$)	t (lb/in ²)			
			Concrete uncracked		Concrete tension neglected	
			Eq. 1	Eq. I	Eq. 3	Eq. II
12.0	12.0	3.92	1,162G	1,238G	2,113G	2,290G
13.5	12.0	3.92	1,347G	1,390G	2,413G	2,580G
12.0	12.0	0.70	1,028G	1,077G	1,503G	409G

REFERENCE

- ROSS, A. D. and ENGLAND, G. L. Reinforced concrete under thermal gradients. *Magazine of Concrete Research*. Vol. 14, No. 40. March 1962. pp. 5-12.

Reply by the authors

In reply to the query regarding the effects of heat on the Demec extensometer, it was found by frequent checks of the gauge on the cool Invar calibration bar that the zero of the gauge did not change during any of the tests.

Mr Ogunlesi is correct in pointing out that r as defined in the paper should equal $(x - d)/x$. This was in fact the value used throughout the theoretical work and the definition given in the paper is an error.

The theory given by Mr Ogunlesi is a restatement of the theory given in the paper and gives identical results when the steel and concrete coefficients of expansion are the same.

The shortage of space in the original paper did not allow the values used in the calculations to be stated; they are

$$A_c = 204 \text{ in}^2 \quad A_s = 0.774 \text{ in}^2 \quad d = 7.57 \text{ in.}$$

$$x = 8.5 \text{ in.} \quad r = \frac{x - d}{x} = 0.1093$$

$$E_s = 30 \times 10^6 \text{ lb/in}^2 \quad y = 24 \text{ in.}$$

When these values are substituted in Mr Ogunlesi's equations I and II, results identical with those given in Table 1 of the original paper are obtained.

The apparent differences mentioned by Mr Ogunlesi were due to the fact that the values he used in his formulae were not the same as those used in our paper.

However, it is difficult to understand how Mr Ogunlesi obtained values for steel stresses from his formulae for the cases where steel and concrete expansion coefficients differ since he does not include the coefficient of expansion for steel in his analysis.