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The relation between the strains in the concrete and the steel in reinforced and prestressed concrete beams*

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NOTATION

e_c' = strain in concrete at compression edge	} at ultimate stage, defined by beginning of yield of steel or crushing of concrete
e_s' = tensile strain in steel	
$n_1 = \frac{\text{depth of neutral axis}}{\text{effective depth}}$	

The detailed analysis of strain and slip distribution by X-ray techniques reported in the paper provides valuable data for use in ultimate bending strength formulae which include a parameter for the influence of bond strength and slip. There is one point, however, in regard to the definition of the factor F which requires clarification. In my paper quoted as reference 6 by Professor Evans and Dr Williams, I intended that F should be defined by the equation

$$F = \frac{e_s' n_1}{e_c'(1 - n_1)}$$

so that, in design formulae for bending strength, the value of n_1 could be based on values of e_c' , e_s' , and the bond slip parameter F . The virtual tensile strain in the concrete is then defined as $e_c'(1 - n_1)/n_1$, which is not necessarily equal to the average strain in the concrete adjacent to the tensile steel.

Experimental values of F were to be, and have been, obtained by measuring strain in the steel, the strain in the compression edge of the concrete, and the depth of the neutral axis. The essential purpose of the parameter F , therefore, was to provide an adjustment of linear strain distribution across the depth of the section

to allow for the influence of bond slip on the position of the neutral axis. To obtain experimental values of F by measuring the strain in the concrete at the level of the steel does not necessarily accord with the above definition, and it would be interesting to see whether values of F obtained from measured values of e_c' and n_1 are different from those given in Table 2.

The values of F in Table 3 have a wide scatter which might be reduced by reverting to the above definition of the parameter, since the values of e_c' and n_1 are not nearly so influenced by local yield and slip, as is the strain in the concrete at the level of the steel. The value of n_1 is also influenced by the stress distribution in the compression zone of the concrete and by the value of the total compression, neither of which are greatly influenced by local strains and slips at the level of the tensile steel. Moreover, the value of n_1 is also influenced by strains in the uncracked sections adjacent to the critical section, since there is a limited rate at which n_1 can vary along the length of the beam under the influence of cracks. In order to have a good basis for n_1 values, it is, therefore, necessary to define F in terms of average strain values over a short length of beam, equal, say, to about one-third or one-half of the depth. It is also necessary to ensure that, in tests at the ultimate stage of loading, high yield strains have not developed locally at the critical section; otherwise the measured average strain in the steel may be inflated without an adequate corresponding reduction in n_1 . This will result in inflated experimental values of F .

Since the value of ultimate bond strength in regard to very short lengths is bound to have wide variations, it will not be possible to avoid some scatter in F values, and minimum values must be adopted when F values are used in design formulae. It is to be hoped, however, that the measured values of n_1 and e_c' will be

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Figure I

Figure II

published, together with any alterations to the values of F in Table 3 that may result if F is calculated by substituting those values in the definition formula.

Figures I and II show the variation of F values for 13 in. $\times$ 7 in. $\times$ 12 ft reinforced concrete beams loaded

at their third points for various percentages of ultimate bending moment obtained by test. The curves have not been continued beyond the calculated bending moment for yield point of the steel (0.2% proof stress for Square Grip), since this is defined as the failing bending moment and test results beyond this

value are sporadic. The curves have been prepared by A. J. Ashdown from tests carried out by R. Wildt at Imperial College. The following trends are evident:

- the value of F approaches 1 as ultimate load is approached;
- values of F increase as diameters of bars decrease;
- values of F for Square Grip bars (Figure I) are greater than for mild-steel bars (Figure II) at the same bending moment.

Contribution by A. J. Ashdown, A.M.I.Struct.E.
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The X-ray technique employed by Professor Evans and Dr Williams, although it can only be used for small test specimens, reveals some very interesting results.

The difference between the strains in the concrete at the steel-concrete interface and at the surface is what might be expected in reinforced concrete beams, since the tensioned concrete recoils back upon itself at cracking, and drags upon the steel, reducing the tension in the steel between cracks. The difference reveals the capacity of the concrete to take considerable shear strain; for instance, in Table 2, the difference in strains at interface and surface is $177 - 145 = 32 \times 10^{-4}$, presumably acting along 3 in.; from a central point the movement is $1\frac{1}{2} \times 32 \times 10^{-4}$ in.; the distance from the steel to the surface is 0.531 in., whence the shear strain is $0.0048/0.531 = 0.009$, which is greater even than that which concrete can take in compression.

Whether there are any hidden cracks occurring at this strain is unknowable.

The process of making pre-formed cracks with metal initiators would reduce the amount of drag upon the steel, since the concrete at the initiators would exhibit little tension before cracking.

It appears that the value of F , as defined by $e_s'n_1/e_c'1(1-n_1)$, need not be the same as that obtained by the methods given in the paper, since no values of e_c' or n_1 are given, and possibly none were measured.

A more realistic approach, which would include all bonding effects and slip occurring throughout the length of the beam and during its loading life, could be made by replacing F by another parameter, say F' , and substituting stresses for strains.

$F' = t_f/t_s$, $t_f = M/(1-\gamma n_1)dA_t$, and t_s can be found from the stress-strain curve of the steel, the strain being given by $e_c'(1-n_1)/n_1$. Typical values of F' calculated by this method, and branching off to F values as the stress departs from linearity with strain, are given in Figure III for two of a series of beams tested at Imperial College by R. Wildt. The results compare two beams with different kinds of steel having approximately the same area, the concrete being of much the same strength.

The effect of slip along the beam is reflected in the F' curve, and usually occurs in the Square Grip bar at a later stage of loading, although the F' value for the mild steel at the ultimate is higher than for the Square Grip.

The smaller the percentage of steel, the steeper the

Figure III

Figure IV: Stress-strain curve for steel.

F' curve becomes. For prestressed beams, Professor Evans and Dr Williams define e'_s as the ultimate strain less pre-strain. The ultimate strain has now been substituted for the yield strain, as defined for reinforced concrete beams, presumably because the pre-strain is so large. Actually, where the steel is still gripped by the concrete towards the ends of the beam, at the ultimate condition of loading, the negative pre-stressing moment Pe must still be affecting the critical section, but since the new arm $(1-\gamma n_1)d$ at the critical section is greater than e , the original pre-strain is reduced in the proportion $e/(1-\gamma n_1)d$, giving a large residual strain and stress, which may be used for the

purpose of calculating the value of n_1 at the ultimate bending moment.

It is interesting to note that in the prestressed beams SP1 and M1, there is no difference between surface and interface strains in the concrete, whilst the slip is much greater for the larger bar in M1 than for SP1.

Reply by the authors

The authors wish to thank Professor Baker and Mr Ashdown for their contributions which are both welcome and stimulating.

Both contributors raise the point that the values of F given in the paper need not necessarily be identical with values obtained from the expression

$$F = \frac{e'_s n_1}{e'_c (1 - n_1)}$$

From Figure 9 in the paper it is clear that, provided plane sections remain plane after bending, the measured virtual strain in the concrete at the level of the reinforcement is defined by the geometrical relation

$$\frac{e'_c (1 - n_1)}{n_1}$$

and the F values obtained with measured or calculated values for the virtual strain must be identical. In Table I the measured and calculated values for the virtual strains in the concrete at the level of the steel are tabulated for six beams in which the average strain readings were recorded over the depth of the beam in the central 8 in. With the exception of beam M1 the calculated values are lower than the measured values, the average difference being 12%. Hence, the values of F found if Professor Baker's expression is used are rather higher than those given in the paper.

The fact that Professor Baker and Mr Ashdown record decreasing values of F as the ultimate load is

TABLE I: Summary of recorded strains, depth of neutral axis, and values of F .

1	2	3	4	5	6	7	8	9	10
Beam mark	Percentage of ultimate load	e'_s ($\times 10^{-4}$)	n (in.)	n_1	e'_c ($\times 10^{-4}$)	$\frac{e'_c (1 - n_1)}{n_1}$ ($\times 10^{-4}$)	Measured average strain in concrete ($\times 10^{-4}$)	Ratio 3/7 = F	Ratio 3/8 = F
SP1	100	226	0.65	0.144	13	77	82	2.9	2.7
LS1	100 89	222 73	0.95 1.25	0.211 0.278	23.5 12.5	88 32	92 42	2.5 2.3	2.4 1.8
LS2	100 87	180 57	0.90 1.05	0.200 0.233	20 12	80 40	90 43	2.25 1.4	2.0 1.3
LS3	100 93 89	170 88 65	0.70 0.85 0.90	0.155 0.189 0.200	20 14.5 11	109 62 44	111 67 54	1.6 1.4 1.5	1.5 1.3 1.2
LS4	100 93 89	233 131 71	0.90 1.10 1.15	0.200 0.244 0.256	36 21 17	144 65 49	145 85 60	1.6 2.0 1.4	1.6 1.5 1.2
M1	100	100	1.35	0.300	41	96	83	1.0	1.2

approached whereas the authors record increasing values of F must be due to the technique used in measuring the strain in the steel. The authors' results were obtained by means of the X-ray technique which permits the strain in the steel to be measured without disturbing the bond characteristics of the reinforcement. The technique employed by Professor Baker is not known, but probably involves forming openings in the concrete to expose the steel. This suffers from the disadvantage that the bond is destroyed over a considerable length of reinforcement so that bond slip occurs as the stress in the steel increases. Also, sufficient closely spaced gauge points must be provided to permit a complete picture to be obtained of the variation of the strain in the steel near the section of failure. If this condition is not fulfilled the maximum strain in the steel cannot be obtained but only the average strain between isolated gauge points. Both the destruction of bond and the provision of insufficient gauge points contribute to the measured strain in the steel being lower than the actual strain and the estimated value of F being lower than the true value and decreasing as the loading increases.

Another factor contributing to differences in the results obtained by Professor Baker and the authors is the interpretation of the expression "failing bending moment" for under-reinforced beams. Professor Baker defines this for steel not possessing a definite yield point as the bending moment at which the reinforcement reaches the 0.2% proof stress. The authors, on the other hand, define this as the bending moment at which the reinforcement reaches its ultimate strength, that is the ultimate flexural capacity. It is suggested that this is the more realistic definition as the ultimate strength of the reinforcement must be developed in any under-reinforced beam.

The values of F recorded in Table 3 do not exhibit a wide scatter. Rather do they illustrate the fact that the value of F depends on the type of loading and the diameter and type of reinforcement used, the surface condition of the reinforcement, and the stress in the reinforcement. Table II has been prepared to clarify this and gives average values of F at the ultimate load.

TABLE II: Variation of F with type of loading and type and surface condition of reinforcement.

Reinforcement		Type of loading	F
type	surface condition		
No. 2 gauge	polished	central point	0.67
$\frac{1}{2}$ in. deformed Macalloy	"ex-works"	central point	1.2
$\frac{3}{8}$ in. Square Grip	polished	two point	1.55
$\frac{3}{8}$ in. Square Grip	polished	central point	2.2
$\frac{1}{2}$ in. Square Grip	polished	two point	1.5
$\frac{1}{2}$ in. Square Grip	polished	central point	2.6
$\frac{1}{2}$ in. Square Grip	"ex-works"	central point	2.8

The expression for F' proposed by Mr Ashdown is interesting but, unfortunately, does not provide a means for obtaining the depth of the neutral axis. This was the original purpose for which the expression for F was intended. However, with slight modification, the expression may be adapted to obtain the value of F from test results when the strain in the steel is not measured. The maximum strain in the steel at the ultimate moment is given by

$$e_s' = \frac{M_u}{(1 - \gamma n_1) d A_s} \times \frac{1}{E_s}$$

where M_u = measured ultimate moment

E_s = secant modulus of elasticity of the steel at its ultimate stress

A_s = area of tensile reinforcement

$\gamma n_1 d$ = depth of the centre of compression

This expression gives values for the maximum strain in the steel which are higher than those measured since, for deformed bars of small diameter, the theoretical stress in the steel at the ultimate moment is invariably higher than the ultimate strength of the steel as obtained by a tensile test. Table III compares values of F calculated by this method with values of F obtained from measured strains. The ultimate strengths of the $\frac{1}{2}$ in. and $\frac{3}{8}$ in. Square Grip bars, as obtained in tensile tests, were 75,000 and 80,000 lb/in² respectively. The value of γ has been taken as 0.42.

TABLE III: Calculated values of F at the ultimate load.

1	2	3	4	5	6	7	8	9	10
Beam mark	M_u	$(1 - \gamma n_1) d$	A_s	$\frac{M_u}{(1 - \gamma n_1) d A_s}$	E_s	Ratio 5/6	Measured strain	F	F
	(lb/in.)	(in.)	(in ²)	(lb/in ²)	(lb/in ²)	($\times 10^{-4}$)	($\times 10^{-4}$)	(from 7)	(from 8)
SP1	23,700	4.23	0.063	89,000	3.1×10^6	287	226	3.1	2.7
LS1	35,850	4.10	0.098	89,000	3.5×10^6	254	222	2.8	2.4
LS2	36,600	4.12	0.098	90,500	3.5×10^6	258	180	2.9	2.0
LS3	35,650	4.21	0.098	86,500	3.5×10^6	247	170	2.2	1.5
LS4	35,650	4.12	0.098	88,000	3.5×10^6	251	233	1.7	1.6
M1	114,900	3.93	0.196	138,000	12.6×10^6	109	100	1.3	1.2