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The ultimate strength of reinforced concrete beams in shear*

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Contribution by Kamal N. Ghali, B.Sc.
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Dr Jones has ably reported on his excellent investigation of the ultimate strength of reinforced concrete beams in shear. The report was of special interest to the writer since he is preparing a thesis on the same subject. The following remarks are based on the outline of that thesis.

THE EXPERIMENTAL WORK

Dr Jones states that "After a shear span with stirrups at 9 in. centres had failed, this span was strapped up by bedding heavy steel plates in cement mortar on the top and bottom of the beam, and then drawing these together by yokes. In this manner the section which had failed was sufficiently reinforced to carry load while the shear span with stirrups at 6 in. was tested to failure". This method of testing the same beam first until the right-hand side fails and then until the left-hand side fails suggests the following comments.

(1) Was Dr Jones able to test the shear span with stirrups at 6 in. on the same day as the other side? This seems unlikely since the cement mortar must have been left for some time before reloading the beam and, if this was the case, then the values of f_c' in the series "45° stirrups at 6 in. centres" should have been higher than those initially reported.

(2) It appears that the failure of one side of a beam widens seriously the inclined cracks and thus damages the other side, especially if it is taken into consideration that shear failures are brittle, and that the ultimate load required for the failure of the left-hand side was only about 1.25 times that required for the right-hand side.

(3) For the left-hand shear span, the experimental procedure used by Dr Jones necessitates its loading up to 0.8 of the ultimate strength, relaxing the load suddenly and then reloading once more up to failure. This is in the nature of a fatigue test and it would be surprising if this did not weaken the beams.

Such a discrepancy can be seen from Table I, in which M_{t9} and M_{t6} are the ultimate strengths of shear spans

having stirrups at 9 in. and 6 in. centres respectively. It can be seen that the ratio M_{t6}/M_{t9} varies between 1.04 and 1.69, while $M_{t6} - M_{t9}$ (the actual difference in strength) varies widely.

TABLE I

Beam	M_{t9} (lb.in. $\times 10^3$)	M_{t6} (lb.in. $\times 10^3$)	$\frac{M_{t6}}{M_{t9}}$	$M_{t6} - M_{t9}$ (lb.in. $\times 10^3$)
J12	604.8	716.8	1.18	112.0
J7	761.6	878.1	1.14	116.5
J5	761.6	940.8	1.24	179.2
J17	627.2	918.4	1.47	291.2
J6	896.0	985.6	1.10	89.6
J13	896.0	1,232.0	1.38	336.0
J15	851.2	1,164.8	1.37	313.6
J8	873.6	1,209.6	1.39	336.0
J16	896.0	1,164.8	1.30	268.8
J11	985.6	1,164.8	1.18	179.2
J14	1,254.4			
J1	846.6	1,187.2	1.40	340.6
J3	1,075.2	1,411.2	1.315	336.0
J2	1,164.8	1,344.0	1.155	179.2
J4	1,097.6	1,366.4	1.245	268.8
J18	851.2	1,433.6	1.69	582.4
J10	1,164.8	1,254.4	1.075	89.6
J9	1,187.2	1,232.0	1.04	44.8

The writer wishes that Dr Jones had used a special procedure which could have thrown light on the above remarks.

It is known that previous investigators have stated that the presence of stirrups does not change the value of initial diagonal tension cracking load. Dr Jones's experimental work should give the perfect answer to this question and he is urged to include in his closure the values of loads at which initial diagonal cracks appeared, whether these cracks appeared simultaneously on both sides of the beams and whether these values were deducted from the load deflexion curve, or by visual examination.

*Pages 69-84.

THE ANALYTICAL WORK

I wish to comment on three aspects of the analytical work:

- (1) the validity of the term $(1 + 2rf_{yw}/10^3)$ as a representative of the effect of web reinforcement;
- (2) whether Dr Jones's equation is more reliable than Dr Laupa's for concrete strengths varying between 2,000 and 6,000 lb/in²;
- (3) the theory of shear moment failures and the rupture of T beams.

(1) As can be seen from Table I, experimental values for M_{t6}/M_{t9} range from 1.04 to 1.69; according to either Laupa's or Jones's equations this ratio should have been 1.11. It is to be noted too that about 50% of the beams reported give divergences greater than 20%.

If we refer to Laupa's paper, which included the analysis of all the experimental results that can be found in the literature, it can be seen that his equation is less reliable for beams with stirrups than it is for beams without stirrups.

In my opinion the introduction of the empirical term $1 + 2rf_{yw}/10^3$ merits further discussion. The results for Dr Jones's divided beams furnish an excellent background for such a study.

(2) Referring to Figure 4, it can be seen that for f_c' values between 2,000 and 6,000 lb/in², Dr Jones amends Laupa's equation by about 20%. It might be interesting to study Jones's equation in the light of other experimental work.

Table II presents the results of Clark's experiments⁽¹⁾ using both Laupa's and Jones's equations. Series A_o , B_o , C_o and D_o had no web reinforcements, whilst all the other beams were provided with web reinforcements. It is beyond the scope of this discussion to make a comparison between all the tests that can be found in the literature but, from Clark's tests and even from J_{12} to J_{16} in Dr Jones's series, it seems that for concrete strengths less than 6,000 lb/in², Laupa's equation might be more reliable. For values of f_c' higher than 6,000 lb/in², Laupa's equation should not be used, Dr Jones's equation being the more representative.

(3) Numerous tests have been carried out in the past on reinforced concrete beams of rectangular cross-section, and it seems that the failure of such beams can be considered as some sort of weak compression bending failure. Very few experiments have been carried out on beams of T section, but it seems that there are fundamental differences between the two modes of failure.

(a) For rectangular beams the major inclined crack that causes failure exists always near the load. For T beams, the major inclined crack exists midway between the load and the support and it seems that the governing factor for T beam failures is the separation of the web from the compressed slab.

(b) Whilst a rectangular beam fails through the crushing of the compressed concrete fibres, T beams generally fail while the concrete stresses in the compressed zone are well short of the concrete crushing strength.

TABLE II

Beam	f_c'	M_c (lb.in. $\times 10^3$) (Jones)	M_t	M_t/M_c (Jones)	M_t/M_c (Laupa)*
8 $\times$ 18 in. beams with no web reinforcement. Span 6 ft 0 in.					
Ao1	3.12	1,020	720	0.713	0.86
Ao2	3.77	1,105	873	0.788	0.96
Ao3	3.435	1,078	964	0.895	1.10
Bo1	3.42	1,115	817	0.733	0.93
Bo2	3.468	1,095	636	0.582	0.72
Bo3	3.41	1,024	866	0.845	0.99
Co1	3.58	1,068	940	0.872	1.05
Co2	3.405	1,095	958	0.875	1.10
Co3	3.42	1,098	902	0.823	1.03
Do1	3.75	1,098	896	0.817	0.98
Do2	3.80	1,117	1,045	0.940	1.15
Do3	3.765	1,105	902	0.817	0.99
8 $\times$ 18 in. beams with web reinforcement. Span 6 ft 0 in.					
$r = 0.0038$					
A11	3.575	2,310	1,800	0.779	0.97
A12	3.43	2,270	1,690	0.742	0.92
A13	3.395	2,240	1,800	0.804	0.99
A14	3.59	2,340	1,980	0.846	1.06
$r = 0.0037$					
B11	3.388	2,230	1,880	0.843	1.04
B12	3.680	2,380	1,730	0.727	0.92
B13	3.435	2,250	1,922	0.806	1.05
B14	3.380	2,250	1,800	0.800	1.00
B15	3.570	2,290	1,620	0.707	0.88
$r = 0.0073$					
B21	3.370	2,790	2,030	0.727	0.90
B22	3.820	2,980	2,160	0.726	0.90
B23	3.615	2,900	2,250	0.777	0.96
$r = 0.0037$					
B61	6.110	2,260	2,560	1.135	1.18
$r = 0.0034$					
C11	3.720	1,955	1,490	0.767	0.94
C12	3.820	1,975	1,665	0.837	1.03
C13	3.475	1,895	1,318	0.666	0.85
C14	4.210	2,060	1,535	0.745	0.91
$r = 0.0069$					
C21	3.430	2,350	1,560	0.666	0.81
C22	3.625	2,430	1,620	0.666	0.82
C23	3.500	2,380	1,740	0.732	0.90
C24	3.910	2,520	1,550	0.617	0.78

*Values taken from Laupa's paper, reference (6) of the original paper.

From these and from other considerations, it seems that the shear moment theory might not be applicable for T beam shear failures, but still many more data are required in this field. The writer wishes to open the question in this discussion and urges wide research programmes on T beams, especially since shear spans are generally of T section in practical designs.

REFERENCE

1. CLARK, A. P. Diagonal tension in reinforced concrete beams. *Journal of the American Concrete Institute*. Vol. 23, No. 2. October 1951. pp. 145-156.

Contribution by A. J. Ashdown

On 18th May 1956 at the Symposium on the Strength of Concrete Structures, Mr R. Elstner showed pictures of the catastrophic shear failure of beams carrying a large warehouse roof at Shelby, U.S.A. Although this failure was partly due to thermal stress, Mr Elstner said that in the U.S.A. lack of understanding of the nature of shear was one of the most serious gaps in knowledge of the behaviour of beams. No doubt Mr Elstner was well aware of the latest theories of his compatriots, E. M. Zwoyer and A. Laupa, the theories which Dr Royston Jones adjusts in his article in an effort to attain better agreement with test results.

It is interesting to note that the shear device in the pre-formed separated beam shows that very little shear is transmitted across the top of the crack, so that, apart from the contribution from the stirrups, the bulk of the shear must be carried to the support through the bottom of the beam below the main diagonal crack, in the region of the tension steel, and in the first few inches from the support.

I have seen a diagonal crack in a beam without stirrups extend to $\frac{1}{4}$ in. from the top before failure, and without any sign of crushing in this region, so that practically the whole of the final shear resistance must occur along the first few inches.

It appears then, that in a beam with a normal "shear span" without stirrups, after the diagonal crack has formed, final failure occurs when a nearly horizontal crack forms near the steel level in the first few inches or along the soffit.

Figure 1

In the portion of the beam below the diagonal shear crack, the shear stress below the neutral axis is normally assumed to be split into a principal compressive stress acting diagonally towards the support, with a tensile stress acting normal to this direction but, owing to the formation of the diagonal crack or cracks, the latter stress cannot act in this plane in a beam without stirrups throughout most of the depth, and becomes concentrated across the first few inches (Figure I).

The principal diagonal compressive stress acts parallel to the cracks, and is taken up by bond along the horizontal steel, building up the tension in the steel; the little shear that is transmitted over the top of the crack can easily be taken by the uncracked concrete. Consequently there is no tension built up in the steel on the support side of the crack. This is evident in the diagram given by C. B. Wilby where the tension appears to be zero at about 7 in. from the support. At the final stages of loading, according to J. R. Janney, the bond stress appears to increase towards the end of the beam, presumably short of the tearing length.

The mechanics of shear failure for a beam without stirrups, illustrating the foregoing remarks, is shown in Figure II.

The average direction θ of thrust C_d is given by $\tan \theta = d/2a_1$.

$$R = (K) f_{as} a_s b \sec \theta$$

$$S = (K) f_{as} a_s b (1 + \frac{2}{3} \tan^2 \theta)$$

The factor f_{ao} is the maximum tensile stress available

Figure 11

in the concrete after allowing for shrinkage stress, and K is a distribution factor of stress along a_o , and is affected by the bond characteristics of the steel used.

The length a_o is affected by the tensile value of the concrete, the amount of longitudinal steel, the concrete cover and the shrinkage.

It can be readily understood that, as the resistance to shear of a beam without stirrups depends upon the tensile strength of the concrete, which is notoriously variable, any calculation of the ultimate shear failure can only be approximate.

In beams with vertical stirrups, the main diagonal shear crack will be straighter and flatter, tending to follow a straight line drawn from the edge of the load-bearing plate to the edge of the support plate.

Diagonal stirrups appear to lift the crack still further, owing to the top ends being nearer the support.

The term "shear span" frequently used in describing shear failure, is clear enough for singly or two-point loaded beams, but to cover all cases of loading, I take it to mean the maximum bending moment divided by the end shearing force. The formulae developed by E. M. Zwoyer are for point-loaded beams failing by crushing at a point of maximum bending moment and therefore they would be difficult to apply to, say, a uniformly loaded beam where the point of breaking off does not coincide with the point of maximum bending moment and maximum concrete strain.

As the shear span shortens, more shear force is transferred over the top of the crack and more diagonal compression is evident on the support side until, with a very short shear span, failure may occur by crushing in the web alongside the crack. At a particular shear span, the crack forms so steeply that it may be considered to be a pure shear or near punching shear failure; the steepness of the angle beyond which it cannot form can be termed the "shear wall", and depends upon the quality of the concrete.

Figure III shows a typical variation of shear resistance with increasing shear span over depth ratio, based upon the foregoing formulae. The calculated flexural failure bending moment is shown by the broken line. With a stronger concrete, the point A will have a smaller a/d value and a greater shear resistance. With a larger percentage of steel, the space between the curves will increase and point B will move further out. At large a/d values, it is possible with smooth bars that bond failure may occur, owing to the flattening of the diagonal thrusts; such failures will be sudden and catastrophic.

Stirrups cutting the diagonal portion of the crack each take up at failure a shearing resistance equivalent to the force they have attained at the yield strain; stirrups cutting the tearing length add little to the resistance, and indeed may weaken the resistance by reason of the tensile stress induced by shrinkage in the concrete surrounding the stirrups in their direction, especially if they are closely spaced.

Regarding Dr Royston Jones's article, it would be

interesting to know if the force in the end stirrup (Figure 15) without an e.r.s. gauge was included in the total resistance, and what assumption was made for this force.

A comparison of beams J4 and J18 in Table 1, which have much the same concrete strengths, shows that J4 is over 25% stronger than J18, an artificially divided beam; a similar comparison can be made between J5 and J17. The additional shear strength of J4 and J5 means either that the dowel action resistance should be doubled (which is difficult to admit) in order to bring balance, or that the shear force in the stirrups is increased in some way, perhaps by part of the work done in tensioning the concrete across the stirrups to cracking being transferred to and strengthening the steel; in this connexion I have found compression both in stirrups and in bent-up bars which increased with loading, until a crack formed in the concrete across them; the strains were measured with protected e.r.s. gauges attached at half height.

I find difficulty in accepting the fundamental assumptions given for the dowel action in equation (16), which is similar to the wave distribution of pressure acting on the wall of a circular tank in the vertical direction, where the end is rigidly held in position and direction: nor can I see how the compression steel can help by dowel action, since so little shear can pass over the top of the crack.

The usual assumption of a linear distribution of stress in a concrete section arising from the drag of the concrete on the steel during shrinkage implies a slip occurring between the steel and the concrete, and does not accord with the end deformation measured by W. H. Glanville (Building Research Technical Paper No. 11) at the end of a centrally reinforced 8 in. diameter cylinder 3 ft long, where the central steel rod and the adjacent concrete moved together whilst the concrete at the circumference lagged behind by 0.001 in. after shrinking for 28 days. The force in the bar was found to be approximately 1,600 lb.

The centre of average pressure on the cylinder is situated at $\frac{2}{3}r = 2\frac{2}{3}$ in. where the average pressure will be $1,600/50 = 32 \text{ lb/in}^2$.

Figure III

Figure IV

If E_c is assumed to be 2.7×10^6 lb/in² which is about right for this concrete at 28 days, then the increase of stress at the bar from the end deformation may be $0.00067/18 \times 2.7 \times 10^6 = 100$ lb/in² making the total stress at the bar 132 lb/in²: by extrapolation of deformation, zero stress occurs at about $3\frac{1}{2}$ in. from the centre of the bar. However, the total of such a stress distribution is not compatible with the force applied on the bar, being somewhat high.

Whatever may have been the average value of E_c developed during shrinkage when in the semi-plastic state at setting, it seems certain that the tension in the concrete at the bar would be much greater than the average tension on a radius of $3\frac{1}{2}$ in. and that the stress does not necessarily follow the end distortion, since the value of E_c of the concrete near the bar would tend to diminish as it is stretched in its semi-plastic state. An assumption that the maximum tensile stress due to shrinkage around a bar on a radius of $3\frac{1}{2}$ in. is twice the average for force compatibility seems not unreasonable. Such a distribution is shown by the broken line in Figure IV, and implies that the average value of E_c at the bar is nearly half its unrestricted value at 28 days, the restriction diminishing to zero at about 1 in. from the bar.

The tension in the concrete produced by shrinkage on stirrups could be written as $f_s = \frac{2E_s\sigma}{m + A_c/A_s}$, where A_c is the area of a block of concrete equal to the breadth of the web, by the pitch of the stirrups, but not exceeding $7\frac{1}{2}$ in. W. H. Glanville recommends m to be 15 for all qualities of concrete.

The value of A_c for shrinkage tension in the concrete surrounding the longitudinal steel is approximately $b[h - d + (3\frac{1}{2} \text{ in. or } d - n \text{ for very small beams})]$.

Contribution by R. B. L. Smith, B.Sc.(Tech.) and K. Subbiah, B.E.

(Manchester College of Science and Technology)

The treatment of shear effects on an ultimate load (i.e. shear-compression moment) basis, as initiated by Laupa and Zwoyer, is an important development in reinforced concrete theory and Dr Jones has rightly decided to follow this approach. His modified form of Laupa's

shear-compression moment equation seeks to extend the range of concrete compressive strength up to about 12,000 lb/in², but his own tests on which this modified equation is based were on beams with about 9% tensile reinforcement. The other Leeds tests against which he checked his results were likewise on very heavily reinforced beams specially designed to fail in shear. In order to provide a check against more normal beams, Table III was calculated, using several published series of tests.

The average ratios of test moment to calculated moment are:

	Laupa	Jones
beams with web reinforcement	98%	81.6%
beams without web reinforcement	78.7%	81.4%

From this it is evident that Dr Jones's equation would not be applicable in a general theory of shear-moment failure.

Dr Jones has accepted Laupa's factor $(k + mp')$ for beams with compression reinforcement, while acknowledging that Wilby's evidence raises doubt concerning the validity of the compression steel term. Laupa⁽⁶⁾ himself accepted the fact that compression steel does not usually yield in shear failures, and this is a weakness in his theory since he finds it difficult to reconcile this fact with his concept of shear-compression failure. Furthermore, it is fundamentally illogical to use the elastic value of the neutral axis factor k in an ultimate load theory.

Neither Laupa nor Jones include the effect of the nominal shear stress s , since it is assumed that the shear-compression moment is independent of this factor over the range considered. Research in this college has revealed that it plays an important part in the value of shear-compression moment over an extended range.

For these reasons we suggest that a more fundamental revision of Laupa's approach is necessary and we hope to report such work in the near future.

On more detailed matters we should like to make these comments.

Divided beams. The analysis of the shear contributions of stirrups, concrete compression zone and dowel action of the main steel is sound and realistic, but we cannot accept Dr Jones's suggestion that design should be based

TABLE III

Test reference	Beam serial No.	p	f_c' (lb/in ²)	$\frac{M_t}{M_L}$	$\frac{M_t}{M_J}$	Test reference	Beam serial No.	p	f_c' (lb/in ²)	$\frac{M_t}{M_L}$	$\frac{M_t}{M_J}$	
Beams with web reinforcement						Beams without web reinforcement						
Moretto ^(1, 2) 1944	1-V $\frac{1}{4}$ 1	3.98	3,880	0.93	0.77	Richart ⁽⁵⁾ 1917	16B20 1	3.71	4,770	1.04	0.89	
	2		3,040	1.05	0.86		2			0.99	0.84	
	2-V $\frac{1}{4}$ 1		4,680	1.00	0.85		16B1 1	3.69		1.07	0.91	
	2		4,930	0.98	0.84		2			0.97	0.83	
	1-I $\frac{1}{4}$ 1		4,020	0.99	0.83		16B2 1	2.74		0.99	0.84	
	2		3,570	0.95	0.79		2			1.09	0.93	
	2-I $\frac{1}{4}$ 1		4,840	1.06	0.91	Richart ⁽⁵⁾ 1922	221 1	2.33		4,076	1.09	0.92
	2		4,560	0.89	0.75		2			3,696	1.13	0.93
	1-D $\frac{1}{4}$ 1		3,360	0.98	0.81		222 1			4,522	1.17	1.00
	2		3,590	1.00	0.83		2			4,337	0.91	0.77
	2-D $\frac{1}{4}$ 1		3,370	1.21	1.00	Moretto ^(1, 2) 1944	1-N 1	3.98	3,550	0.76	0.63	
	2		3,540	1.15	0.95		2		3,620	0.94	0.78	
Clarke ⁽³⁾ 1951	A1	3.10	3,575	0.97	0.80		2-N 1		4,340	0.78	0.66	
	2		3,430	0.92	0.76		2		4,660	0.88	0.75	
	3		3,395	0.99	0.82	Moody ⁽⁴⁾ 1953	A1	2.17	4,400	0.82	0.69	
	4		3,590	1.06	0.88		2	2.15	4,500	0.88	0.75	
	B1		3,388	1.04	0.86		3	2.22	4,500	0.97	0.83	
	2		3,680	0.92	0.76		4	2.37	4,570	0.88	0.75	
	3		3,435	1.05	0.87		B1	1.62	3,065	0.97	0.79	
	4		3,380	1.00	0.83		2	1.63	3,125	1.01	0.83	
	5		3,570	0.88	0.73		3	1.60	2,785	0.99	0.80	
Moody ⁽⁴⁾ 1953	30	4.25	3,680	0.80	0.66		4	1.66	2,430	1.05	0.84	
	31		3,250	0.73	0.60	C1	0.81	920	0.95	0.71		
M_L = shear-compression moment according to Laupa's formula						2	0.83	880	1.14	0.86		
						3	0.80	1,000	1.10	0.83		
						4	0.82	980	1.08	0.81		

M_L = shear-compression moment according to Laupa's formula

M_J = shear-compression moment according to Jones's formula

M_t = test failing moment

on a preliminary calculation of the moment-shear relation followed by a subsidiary calculation of the incidence of splitting etc. Enough empirical evidence should be produced to evolve an equation meeting all circumstances.

Shrinkage. This is also best determined empirically from tests, since it is a factor of general application and not peculiar to certain beams only, but we agree on the importance of shrinkage effects, as engineers in the U.S.A. know very well from recent failures.

Finally, apropos of Dr Jones's conclusions (and those of several other writers) may we ask whether CP 114:1948 is really too conservative? Is it not in need of a new outlook based on reliable evidence?

REFERENCES

- MORETTO, O. Thesis presented to the University of Illinois for the degree of M.S. 1944.
- MORETTO, O. An investigation of the strength of welded stirrups in reinforced concrete beams. *Journal of the American Concrete Institute*. Vol. 17, No. 2. November 1945. pp. 141-162.
- CLARK, A. P. Diagonal tension in reinforced concrete beams. *Journal of the American Concrete Institute*. Vol. 23, No. 2. October 1951. pp. 145-156.
- MOODY, K. G. Thesis presented to the University of Illinois for the degree of Ph.D. 1953.
- RICHART, F. E. University of Illinois Engineering Experiment Station Bulletin. No. 166. 1927.
- LAUPA, A., SIESS, C. P. and NEWMARK, N. M. University of Illinois Engineering Experiment Station Bulletin. No. 428. 1955. pp. 73.

Contribution by D. M. Criticos, B.Sc., D.I.C., M.Sc.
(Consulting Engineer, Athens)

The ultimate shear strength of reinforced concrete beams has been a great challenge to research workers. Dr Royston Jones's modification of Laupa's formula, although it is semi-empirical and based on a limited number of experiments, appears to be the best answer to the challenge that has been suggested up to the present time.

An important point in this paper is the differentiation

Figure V: Prestressed T beam tested by the writer at Imperial College, showing crushing of the web but no crushing at the compression zone.

of the splitting failure from the usual shear failure; often, especially when no web reinforcement has been provided, splitting accompanies the shearing of the critical section and in such cases it is difficult to decide whether the splitting is a secondary effect, or whether it actually caused the failure.

Dr Jones's formula does not appear to hold good for small a/d ratios. The writer applied the formula to a number of beams of small a/d ratios that have failed in shear for beams with neither web nor compression reinforcement (Table IV⁽¹⁾), beams with compression reinforcement but without web reinforcement (Table V⁽²⁾), and beams with compression and web reinforcement (Table VI⁽²⁾).

It has been established that for small a/d ratios, the shear resistance is provided by truss (or arch) action within the beam; it is not surprising, therefore, that Dr Jones's

TABLE IV: Beams with neither web nor compression reinforcements.

Third-point loading. Moist-cured up to the time of testing.
 $b = 8$ in.; $d = 21$ in.; $a = 32$ in.; $a/d = 1.52$; $L = 96$ in.
 Reinforcement: six 1 in. diameter bars
 $(f_y = 36,000 \text{ lb/in}^2, p = 2.8\%)$

Beam No.	f'_c (lb/in ²)	Q_j	M_c (lb.in. $\times 10^3$)	M_t (lb.in. $\times 10^3$)	$\frac{M_t}{M_c}$
1	4,760	0.412	3,200	2,280	0.71
2	4,620	0.419	3,180	2,550	0.78
3	4,290	0.440	3,120	2,430	0.78
4	3,860	0.470	3,020	2,140	0.71
5	2,230	0.600	2,420	1,690	0.70
6	2,630	0.567	2,630	1,865	0.71

formula does not agree with experimental results of beams of small a/d ratios, as the suggested formula is based on the hypothesis that: "... whatever may be the distribution of shear resistance between the beam components the criterion of failure is the limiting concrete strength in the compression zone". The mechanism of shear failure of thin-web, prestressed T beams tested by the writer at the Imperial College⁽³⁾ showed that the failure of the

TABLE V: Beams with compression reinforcement but without web reinforcement.

Third-point loading.

$b = 5.5$ in.; $d = 18.25$ in.; $a = 32$ in.; $a/d = 1.75$; $L = 96$ in.
 Reinforcement: tension: four 1 in. square deformed bars
 $(f_y = 48,000 \text{ lb/in}^2, p = 3.98\%)$

compression: two $\frac{1}{2}$ in. square deformed bars.

Beam No.	f'_c (lb/in ²)	Q_j	M_c (lb.in. $\times 10^3$)	M_t (lb.in. $\times 10^3$)	$\frac{M_t}{M_c}$
1N1	3,550	0.495	1,790	1,120	0.63
1N2	3,620	0.489	1,830	1,410	0.77
2N1	4,340	0.437	1,870	1,260	0.67
2N2	4,640	0.418	1,910	1,450	0.76

TABLE VI: Beams with both compression and web reinforcement.

Third-point loading.

$b = 5.5$ in.; $d = 18.25$ in.; $a = 32$ in.; $a/d = 1.75$; $L = 96$ in.
 Reinforcement: tension: four 1 in. square deformed bars
 $(f_y = 48,000 \text{ lb/in}^2, p = 3.98\%)$

compression: two $\frac{1}{2}$ in. square deformed bars

$(p = 0.005, D = 17 \text{ in.})$

web: $\frac{1}{4}$ in. and two $\frac{3}{8}$ in. diameter bars as indicated in the Table, at 45° to the horizontal at 6.5 in.

Beam No.	f'_c (lb/in ²)	Q_j	M_c (lb.in. $\times 10^3$)	M_t (lb.in. $\times 10^3$)	$\frac{M_t}{M_c}$
1D1/41	3,360	0.509	2,280	1,848	0.81
1D1/42	3,590	0.490	2,340	1,950	0.83
2D1/21	3,370	0.508	2,280	2,280	1.00
2D1/42	3,540	0.495	2,330	2,220	0.95
1D3/82	2,940	0.543	2,660	2,110	0.79
1D3/82	2,770	0.555	2,570	2,040	0.71
2D3/81	3,790	0.474	2,880	2,240	0.78
2D3/82	3,580	0.491	2,840	2,360	0.83

compression zone may be of secondary importance, the shear failure being initiated by the crushing of the web.

Therefore the limits of the range of application of the suggested formula should be established by a specifically designed research programme and particularly in relation to beams with thin webs, for which the cause of shear failure (web crushing) is totally different to the criterion of shear-compression failure (crushing of the compression zone).

REFERENCES

1. RICHART, F. E. and JENSEN, V. P. University of Illinois Engineering Experiment Station Bulletin. No. 237. 1931.
2. MORETTO, O. An investigation of the strength of welded stirrups in reinforced concrete beams. *Journal of the American Concrete Institute*. Vol. 17, No. 2. November 1945. pp. 141-162.
3. CRITICOS, D. *Experimental investigation of the shear strength of plastic hinges of pretensioned concrete beams*. Thesis presented to the University of London for the degree of M.Sc. 1956.

Reply by the author

The most important comment that arises out of the discussion is that the modification of Laupa's formula predicts higher failing loads in the region of concrete strengths from 2,000 to 6,000 lb/in². At a concrete strength of 5,000 lb/in², for example, the modified formula predicts a failing load 18% greater than that predicted by the original formula. Laupa made no claim for his formula for concrete strengths above 6,000 lb/in² and it is plainly not applicable for high concrete strengths. The concrete strength for beam J9 was 11,810 lb/in² and the failing load was 5.4 times the failing load predicted by Laupa's formula, but only 4% above the failing load predicted by the modified formula. The modified formula is based on my test results and is supported by the results of Sunderland and Wilby. The average ratios of test failing moment to computed failing moment for these results were 0.95 and 1.08 respectively; the supporting details are given in Table VII. I think it should be emphasized that the tests of beams J2 to J18 were designed primarily to examine the effect of one variable, namely concrete strength. There are obvious difficulties in this approach; for example, it was necessary to provide heavy compression reinforcement to avoid bending failures at low concrete strengths, but at high concrete strengths the same amount of compression steel was excessive. Nevertheless, these results should provide a sound basis for obtaining an expression to record the relation of the shear strength of beams to concrete strength. If, however, the expression is to be combined with Laupa's terms $(1 + 2rf_{yw}10^{-3})$ and $(k + mp')$ it will be necessary to embark on a systematic investigation of their validity. It is the second of these terms which is most suspect and I think that as a result of further investigation it may be found that the line for

TABLE VII

Beam	f'_c (lb/in ² × 10 ³)	$\frac{M_t}{M_j}$	Beam	f'_c (lb/in ² × 10 ³)	$\frac{M_t}{M_j}$
SUNDERLAND			WILBY		
2A	4.92	1.22	21a	7.94	0.99
2B	5.06	1.16	21b	7.94	0.97
2C	4.69	1.25	22a	8.61	1.09
2D	4.02	0.97	22b	8.61	1.10
2E	4.50	0.80	23a	8.25	1.20
			23b	8.25	1.20
5A	5.99	0.92	24a	7.53	1.23
5A	5.99	1.05	24b	7.53	1.06
5B	5.61	0.91	25a	8.09	0.98
5B	5.61	0.96	25b	8.09	1.08
5C	6.01	0.72	26a	7.57	1.08
			26b	7.57	1.19
7A	6.60	1.05	27a	7.84	1.07
7A	6.60	0.96	28a	7.67	0.95
7B	4.63	1.08	28b	7.67	0.95
7B	4.63	0.95	29a	7.59	0.83
			30a	7.04	0.98
8A	5.03	1.00	30b	7.04	1.01
8B	4.33	1.01	31a	8.09	0.82
8C	4.57	1.05	31b	8.09	0.85
8D	4.93	0.58	32a	7.80	1.22
8E	4.56	1.03	32b	7.80	1.05
			33a	8.02	1.25
9A	6.49	0.88	33b	8.02	1.03
9B	5.66	0.91	34a	8.02	1.53
9C	5.76	0.90	34b	8.02	1.08
9D	4.73	0.80	35a	7.07	1.45
9E	4.94	0.71	35b	7.07	1.39
9E	4.94	0.84	36a	8.09	1.08
			36b	8.09	0.95
			37a	6.97	0.93
			37b	6.97	1.01

Q_j shown in Figure 4 of the original paper may have to be a little lower. This would involve a reduction of the constant 0.79 in equation (5). It should be recalled that the reinforcement was constant for all the beams except J1. Thus, excluding the constant 0.79, the remainder of the expression $(0.79 - 9.74f'_c \cdot 10^{-5} + 4.02f'_c \cdot 10^{-9})$ should remain valid whatever variations are necessary for the terms $(1 + 2rf_{yw}10^{-3})$ or $(k + mp')$.

Since a great deal of the discussion involves questions of testing technique it will be convenient to give answers and make comments in the same order as that in which most of the points were raised.

It is suggested by Mr Ghali that there would be a variation of concrete strength if both halves of the test beams were not tested to their ultimate loads on the same day. Most of the beams were tested after they had been maturing for more than three months and none was

tested before a month after casting. The cement used was either rapid-hardening or aluminous cement so that any concrete strength variations over two or three days would be insignificant.

The technique of placing different web reinforcement in the two halves of each beam was purely an economy measure to get two results from each beam. Mr Ghali rightly points out the danger that the failure of one half of the beam might affect the subsequent behaviour of the other half. But an examination of photographs and diagrams of crack patterns for all the beams of series J, as given in the original thesis, should convince him that the crack patterns for each half are separated by the middle section, which is in pure bending.

The "brittle" fractures, mentioned by Mr Ghali, may be typical of the shear failure of beams without web reinforcement, but the failure of beams with adequate web reinforcement is a relatively slow process. It is surprising to see my tests described as fatigue tests; there were often more than thirty loading cycles, but this is a trivial number of cycles compared with the number demanded by a fatigue test. The number of loading cycles may have influenced the rate of failure, but it is unlikely to have affected seriously the value of the ultimate load. On approaching failure, it was found possible to re-apply loads frequently without causing severe distress, yet a small increase of load would initiate a rapid collapse. It was also found that a load could be sustained for 24 hours at a value close to failing load, yet again a small increase would cause collapse. The evidence of previous investigators also confirms that cyclic loading has little effect on the ultimate load.

Although I have agreed that Laupa's term for the influence of web reinforcement needs further investigation, deductions should not be made from Mr Ghali's Table I until the results of beams J17 and J18 are excluded. These were the divided beams and the low failing load of the sections of these beams that incorporated pre-formed gaps is discussed later. Thus the ratio M_{f6}/M_{f9} shows a range of values of 1.04 to 1.40 compared with the ratio of 1.11 obtained from Laupa's expression $(1 + 2rf_{yw}10^{-3})$. A more consistent result would have been pleasing, but it would also have been unusual for tests involving shear failures in beams of these dimensions.

Mr Ghali asks for the loads at which initial diagonal tension cracks were discovered. As far as the observations were considered reliable, these figures have already been given in Table 3 of the original paper. The difficulties of crack observations are well known. Briefly, the beams were coated with a thin plaster wash and strongly illuminated while observations were made with a hand microscope. It was frequently possible to detect cracks of one-third of a thousandth of an inch in width. Visual observations were closely correlated with deflexions and strain readings. It was not possible to observe the whole of both sides of the beams in one of the testing machines used, but diagonal tension cracks were observed to appear simultaneously on both sides of the beams in some of

the tests. It is wise of Mr Ghali to draw attention to the different mode of failure for T-beams, but these provide a field of investigation beyond the scope of this discussion.

Mr Ashdown's comments on the mechanics of shear failure of beams without stirrups are interesting, but it would need more experimental evidence to convince me of the existence of compressive stresses in the bottom tension reinforcement.

I am asked if the force in the end stirrup of Figure 15 was included in the total resistance, and what assumptions were made for this force. It was pointed out that the forces measured were those transferred by the reinforcement across the pre-formed gap. The object was to obtain a positive measurement of forces that would involve no conjectures about the distribution of forces between steel and concrete. Thus no assumptions were made about the force in the end of the stirrup. The shear force transmitted by means of the longitudinal tensile reinforcement at the gap was calculated from the strain measurements in the steel at this point. I would have liked to extend the gap so that it would cross the last stirrup; it is possible to do this, but the experimental difficulties had not been overcome at the time of these tests.

The behaviour of beams J17 and J18 is discussed at some length in the original thesis. Beam J18 failed at a load of 6 tons less than failing load of similar but undivided beams. Beam J15 failed at a load so close to the predicted failing load that it is thought that the different method of fixing the shear device may have caused the low failing loads for J17 and J18. The shear device was fitted to beam J15 after the beam had matured, so that when the device was grouted and forced into position there was little loss of continuity in the compression zone. The shear device fitted to the other beams had to be threaded over the compression bars, and remained in position during casting of concrete. It is possible that shrinkage caused a small gap between the concrete and the end plates of the shear device. This was not evident from inspection but was suspected when no horizontal load registered in the device until a load of 6 tons had been applied. The implications of this are also discussed at length in the original thesis but, briefly, it was believed that the initial slackness aided the premature development of cracks and resulted in premature failing loads.

It is agreed that, in relation to dowel action, there are difficulties in making assumptions which lend themselves to mathematical analysis, but I am satisfied that the attempt should have been made. The value of the resulting expressions certainly needs examining in relation to further tests, but the interesting correlation so far obtained makes the investigation worth recording. Mr Ashdown cannot see how the compression steel can help the dowel action; in my opinion, this is possible, if there are sufficient stirrups to restrain the steel deflexions. The fact is that dowel action was measured in the compression steel and furthermore near failing load; it amounted to about one-third of the total shear force.

With regard to Mr Ashdown's comments on shrinkage,

I am well aware of the work of Dr Glanville and others on bond, shrinkage and creep. The shrinkage analysis assuming linear strain distributions may appear an oversimplification of the problem to those who are involved in the study of bond, shrinkage and creep. But the simple analysis should draw attention to a hitherto neglected aspect of the phenomenon of shear failures. I should emphasize again that these tests were not devised to deal with this aspect, but evidence cannot be ignored, and I hoped that the simple calculations would encourage others to give attention to this problem. I suspect there may be serious consequences if shrinkage stresses are ignored when shear stresses are being investigated in continuous frames.

Mr Criticos remarks that the modification of Laupa's formula does not hold good for small a/d values or for beams of small breadth to depth ratio. A reference to item (3) of the summary of conclusions will show that I made a point of mentioning that the limits of application need establishing in relation to both these conditions. Laupa has given some consideration to the shear failures of beams with low a/d values. I am also aware of the problem of failures which are initiated by the crushing of thin webs in prestressed concrete beams, but it would be necessary to draw on more experimental data to discuss this aspect of shear.

Messrs Smith and Subbiah, too, have shown from the results of others that, in the range of concrete strengths up to 6,000 lb/in², the modification of Laupa's formula predicts failing moments about 20% above the test results. An examination of their Table III shows that the average ratio of failing moment to computed failing moment for the various groups varies from 0.63 to 0.90. A reference to the original data for Moody's two beams which give the ratio 0.63 will show that the cross-section of the beams was 24 × 7 in. I have already pointed out that Sunderland obtained low failing moments for beams 20 × 5 in.; Moody's beams therefore belong to the category of beams of small breadth to depth ratio which have been specifically excluded from the range of validity of the suggested formula. There are other results which give both higher and lower values for the ratio M_f/M_j . Incidentally, Laupa's 1953 tests of beams without web reinforcement give a ratio of M_f/M_j equal to 0.92. My own check of the average ratio of test failing moment to computed failing moment, based on an analysis of a 100 results from different investigators excluding Sunderland and Wilby, gave a figure of 0.80. I have hinted earlier that, if the scope of the test were widened, a general expression might be obtained which would give somewhat lower failing moments. Nevertheless, there are so many factors which affect shear that it is most unlikely that a general

expression will be produced that can predict shear failures within very close limits. This opinion is supported by the discussion on shrinkage, in which it was stated that the mode of failure may be radically changed by the position of initial shrinkage cracks. An element of chance was mentioned; fortunately there are reasons to believe that this will usually lead to higher failing loads than might be expected. I would agree that any code recommendations should provide a generous allowance for possible freak results at low loads. Apropos of this discussion, it should also be remembered that research workers concerned with reinforced concrete are usually well satisfied if they obtain a set of results within ±15% of the predicted values.

Messrs Smith and Subbiah remark that it is fundamentally illogical to use an elastic value for the neutral axis factor k in an ultimate load theory. I have much sympathy with this comment and have discussed the term $(k + mp')$ at some length in the original thesis. There I stated that it is difficult to justify this term theoretically, though a possible physical interpretation arising out of Laupa's discussion is that the depth of neutral axis for a shear-compression failure is equal to the product of some constant and the depth of the neutral axis determined from the elastic theory. Further discussion is of little value without more experimental data. I think it is still less logical to use the nominal shear stresses unless Messrs Smith and Subbiah have a meaning for s which is different from the customary meaning, i.e. shear force divided by the product of the lever arm and the breadth.

They also state "We cannot accept Dr Jones's suggestion that designs should be based on a preliminary calculation of the moment-shear relation followed by a subsidiary calculation of the incidence of splitting etc.". Here they have misunderstood my suggestion. I think it should be possible to establish basic conditions, for design purposes, so that a simple check would ensure that a splitting failure is avoided. It will probably be in the form of a minimum amount of web reinforcement. I shall be surprised but delighted to see an equation which will meet all the circumstances.

In regard to their final remarks, I still think, in spite of my earlier comments about suitable load factors, that CP114 (1948) is too conservative for normal simply supported beams. A new outlook may be desirable but, since the measured stirrup stresses are between 30 and 60% of those predicted by the Code formula, the existing evidence gives scope for permitting more economical designs. If the early pioneers in reinforced concrete had waited for the final answer to many of their problems, there would have been no reinforced concrete structures.