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Moment redistribution in continuous steel-fibre-reinforced concrete slabs

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The phenomenon of moment redistribution in steel-fibre-reinforced concrete (SFRC) members has not been adequately studied. In this study, the phenomenon was investigated by conducting point-load tests on seven two-span, one-way continuous slabs without traditional reinforcement. Significant redistribution of moments from the elastic values was observed at the critical mid-span and interior support sections. An analytical approach to predict the moment–curvature response of SFRC is presented, with which the predicted variation of moments with increasing load compared to the observed trends with reasonable accuracy. Also, the analytical approach predicts the load-carrying capacity of the slabs within 8% of the average observed values, with a standard deviation of 8%. In addition, the load–deflection curves of the slabs were well predicted. The results are beneficial for the analysis and design of a SFRC slabs-on-pile system by considering moment redistribution due to slab continuity.

Notation

C	area of compressive stress block
D_f	diameter of steel fibres (mm)
E_i	slope of descending portion of compressive stress–strain relation (MPa)
E_{SFRC}	modulus of elasticity of steel-fibre-reinforced concrete (SFRC) (MPa)
f'_c	cylinder compressive strength of concrete (MPa)
f_{ct}	tensile strength of SFRC (MPa)
f_{cu}	cube compressive strength of concrete (MPa)
f_{Ftu}	residual tensile strength of SFRC at 1.5 mm crack width (MPa)
f_{R3}	residual flexural strength of SFRC corresponding to a crack mouth opening displacement of 2.5 mm (MPa)
H	section height (mm)
L_f	length of steel fibres (mm)
M_{ACT}	observed moment
M_{cr}	moment at first crack (kNm)
M_{EL}	elastic moment
M_n	negative moment at interior support (kNm)
M_{po1}	positive moment at mid-span 1 (kNm)
M_{po2}	positive moment at mid-span 2 (kNm)
P	load-carrying capacity (kN)
T	area of tensile stress block
V_f	volume fraction of steel fibres
w_{u2}	crack width at which the residual tensile stress of SFRC becomes zero (mm)
x	neutral axis depth at mid-span 2 corresponding to a given loading stage (mm)
z	distance between centroids of the compressive and tensile stress blocks
Δ	total deflection
Δ_{cr}	elastic deflection up to first cracking (mm)
Δ_p	plastic deformation due to rigid body rotation (mm)

η_1	reduction factor accounting for fibre orientation correction in flexural members
η_2	reduction factor accounting for influence of notch
ψ	factor accounting for size effect of fibres

1. Introduction

In continuous flexural members, the ductility of sections at continuous supports allows the redistribution of moments, leading to higher load-carrying capacities (Beeby, 1997; CEB-FIP, 1998; Eligehausen and Langer, 1987). The advantage of moment redistribution is evident in members reinforced with traditional steel bars, by which the amount of reinforcement at the supports could be reduced accordingly in design (Bagge *et al.*, 2014; CEB-FIP, 1998; Eligehausen and Langer, 1987).

In recent times, steel-fibre-reinforced concrete (SFRC) flat slab systems without traditional steel bar reinforcement have gained popularity owing to the ease in construction and achievable load-carrying capacities (Destrée and Mandl, 2008; Hedebratt and Silfwerbrand, 2014; Ward, 2010). The analysis and design of continuous SFRC slab systems without traditional steel bars have been mainly carried out using the yield line theory assuming an elastic–perfectly plastic moment–curvature (M – ϕ) relation of SFRC sections, in accordance with the recommendation given by international standards (ACI 544-6R (ACI, 2015); SCA, 1997). This assumption is reasonably accurate in the prediction of the peak load-carrying capacity of continuous slab panels (ACI 544-6R (ACI, 2015); Destrée and Mandl, 2008; Hedebratt and Silfwerbrand, 2014). However, the assumption is applicable only for the M – ϕ relation plotted up to curvatures 40 to 60 times that at the first crack (ACI 544-6R (ACI, 2015); Soranakom and Mobasher, 2007). Recent studies which have established the M – ϕ relation of SFRC up to curvatures equal to 175 times that at the first crack indicate that

the full moment–curvature relation is far from rigid-plastic (Venkateshwaran and Tan, 2018). In fact, it has been noted that the moment drops to nearly 40% of its peak value at very large curvatures (Venkateshwaran and Tan, 2018). Therefore, the general assumption is inadequate in representing the M – ϕ relation of SFRC sections; hence, the redistribution of moments that occurs in continuous slabs at the yield lines at various stages of loading.

Previous studies on moment redistribution have been extensively carried out in reinforced concrete (RC), fibre-reinforced polymer (FRP) and prestressed concrete continuous slabs (Aiello *et al.*, 2007; Casadei *et al.*, 2003; Cohn and Lounis, 1991; Lou *et al.*, 2015; Oehlers *et al.*, 2004; Rebentrost *et al.*, 1999; Santos *et al.*, 2013; Tajaddini *et al.*, 2016). Limited studies on moment redistribution in SFRC slabs have been carried out (Abas *et al.*, 2013). Abas *et al.* (2013) conducted tests on six two-span continuous SFRC slabs fabricated using deep trapezoidal steel sheeting (W-deck) to represent a composite bridge deck with a trapezoidal section. The purpose of the steel sheeting was to resist the tensile forces due to positive moment at the mid-span sections. At the interior support section, steel mesh reinforcement was provided at the top of the slab in three out of the six SFRC slabs. In all the slabs, the moment at the interior support dropped after first cracking, whereas those at the mid-span sections increased due to moment redistribution. However, the presence of steel mesh reinforcement in three slabs ensured that the drop in load was gradual compared to the slabs with only steel fibres. The trend in moment redistribution in these slabs was similar to RC slabs. Nevertheless, no literature is available for SFRC slabs devoid of traditional reinforcement.

In this study, the moment redistribution in continuous one-way SFRC slab panels was investigated by conducting tests on seven two-span concrete slabs reinforced with varying fibre dosages and fibre end configurations. An analytical approach using the complete M – ϕ relation of SFRC proposed by Venkateshwaran and Tan (2018) is presented to determine the moment redistribution at different stages of loading. Load–deflection curves of the test slabs are also presented and compared with the predicted results.

2. Moment redistribution in SFRC flexural members

2.1 Theoretical considerations

In SFRC slabs and beams without traditional reinforcement, test results show that the ultimate failure generally occurs by the formation and propagation of primary flexural cracks at the critical sections (Michels *et al.*, 2012; Venkateshwaran and Tan, 2018). It has also been noted that any loading after the occurrence of first cracking at the critical sections leads to rigid-body rotation of the portions of the slab or beam about those cracked sections, which behave as hinges (Ahmadii *et al.*,

2012; Michels *et al.*, 2012; Venkateshwaran and Tan, 2018). In a simply supported beam, only one crack forms and propagates near the mid-span (Venkateshwaran and Tan, 2018). In continuous SFRC slabs and beams, dominant primary cracks at multiple sections carrying the largest negative and positive moments can be expected. The moment capacities at these hinges can be established from the M – ϕ relation of SFRC for a given stage of loading. In this section, the phenomenon of moment redistribution in SFRC members without traditional reinforcement is first examined using the M – ϕ relation of SFRC by considering different loading stages in a one-way slab fixed at both its ends.

2.2 Moment–curvature relation of SFRC without traditional reinforcement

Figure 1 shows the constitutive stress–strain relation for SFRC in compression and tension extending up to large strains (Model Code (Fib, 2010); Venkateshwaran and Tan, 2018). In the figure, f'_c is the cylinder compressive strength of concrete (MPa) and f_{ct} is the tensile strength of SFRC (taken as equal to $0.3 (f'_c)^{2/3}$, according to the Model Code (Fib, 2010)) (MPa). It is observed that the fibre content does not have a significant influence on the compressive strength of SFRC (Thomas and Ramaswamy, 2007)

In Figure 1, f_{Ftu} is the residual tensile strengths of SFRC at 1.5 mm crack widths given by the Model Code (Fib, 2010); Venkateshwaran and Tan, 2018)

$$1. \quad f_{Ftu} = \frac{f_{R3}}{3}$$

in which f_{R3} denotes the residual flexural strengths of SFRC corresponding to a crack mouth opening displacement (CMOD) of 2.5 mm, measured by conducting three-point bend tests on notched prisms in accordance with EN 14651 (CEN, 2007).

Figure 1. Constitutive stress–strain relations for SFRC in compression and tension

Also, in Figure 1, H is the section height (mm), η_1 is a reduction factor accounting for the fibre orientation correction in flexural members with depths different from the notched prisms (Michels *et al.*, 2012; Venkateshwaran and Tan, 2018); η_2 is a reduction factor accounting for the influence of the notch on pre-defining the crack path in prisms (Amin *et al.*, 2015; Htut, 2010; Venkateshwaran and Tan, 2018); RI is the reinforcing index ($= V_f L_f / D_f$); V_f , L_f and D_f denote the volume fraction, length (mm) and diameter (mm) of the steel fibres, respectively; w_{u2} is the crack width at which the residual tensile stress of SFRC becomes zero (see Figure 1) ($= (5 - 0.2875 V_f) / (1 - 0.1845 V_f)$ (mm)) (Venkateshwaran and Tan, 2018); E_{SFRC} is the modulus of elasticity of SFRC ($= 4.58(f_{cu})^{0.5} + 0.42(f_{cu})^{0.5} RI + 0.39 RI$ (GPa)) (Thomas and Ramaswamy, 2007); f_{cu} is the cube compressive strength of concrete (MPa); and E_i is the slope of the descending portion of the compressive stress-strain relation ($= 1930 RI^{-0.7406}$ (GPa)) (Nataraja *et al.*, 1999).

According to the Model Code (Fib, 2010), the crack width w_t in SFRC members is due to the strain spread over a length equal to the height of the crack. For SFRC flexural members without traditional reinforcement, this length is assumed to be approximately equal to the section depth (Model Code (Fib, 2010)). Subsequently, the tensile strain in SFRC can be defined as the corresponding crack width divided by the section depth (Model Code (Fib, 2010)), as shown in Figure 1.

In this study, f_{R3} was determined based on the empirical equations developed by Venkateshwaran *et al.* (2018) and given as follows

$$2. \quad f_{R,3} = \psi [0.300(f'_c)^{0.5} + 7.629(RI) + 0.373N^2]$$

where ψ is a factor accounting for the size effect of fibres ($= (1 + L_f/100)^{0.5}$) and N is the number of end-hooks in the steel fibres. From Equations 1 and 2, it is noted that the residual tensile strength of SFRC is significantly influenced by the fibre content. An enhancement of 120 to 160% in the residual tensile strength is observed with an increase in the fibre content from 0.25 to 1.5% (Venkateshwaran and Tan, 2018).

As externally vibrated prisms used in EN 14651 (CEN, 2007) produced a higher tensile strength of about 15% (Shaaban and Gesund, 1993), the residual tensile strengths of SFRC in this study are further multiplied by a factor of 0.85 to account for the disturbance in fibre orientation due to internal vibration of the slab specimens.

Figure 2 shows the crack width, strain and stress distributions across a SFRC section in bending with a crack width, w_t , at the extreme tensile fibre. The corresponding moment and curvature are M_{wt} and ϕ_{wt} , respectively. Using the constitutive

relations in Figure 1 and considering strain compatibility, the bending moment can be established as

$$3. \quad M_{wt} = Cz = Tz$$

where C is the area of the compressive stress block; T is the area of the tensile stress block; and z is the distance between the centroids of the compressive and tensile stress blocks.

A typical piece-wise linear moment-curvature ($M-\phi$) relation for an SFRC section without traditional reinforcement is shown in Figure 3. There are six milestone points. Point A corresponds to first cracking, while points B, C, D, E and F correspond to extreme tensile crack widths, w_t , of 0.25 mm, 0.5 mm, 1.5 mm, 5 mm and w_{u2} mm, respectively. The peak moment occurs at a w_t value of 1.5 mm, after which, there is a significant drop in the moment up to an extreme tensile crack width of w_{u2} mm. The $M-\phi$ relation shown in Figure 3 has previously been

Figure 2. Crack width, strain and stress distributions across a section corresponding to an extreme tensile crack width, w_t

Figure 3. Moment-curvature relation for an SFRC section without traditional reinforcement

validated using results from 24 four-point bending tests on SFRC beams (Venkateshwaran and Tan, 2018).

For the compressive strength of concrete and the range of fibre contents considered in this study, the first crack strength of SFRC, f_{ct} , is always higher than the residual tensile strength, f_{Ftu} , as shown in Figure 1. Although f_{Ftu} is lower than f_{ct} , the moment capacity of SFRC at ultimate limit state is much higher than that at the first crack. This is owing to the large lever arm developed by the tensile stress block at the ultimate limit state compared to first crack (see Figure 2), leading to the robustness of the continuous member. Bending tests conducted on SFRC beams show that there is no sudden reduction of moment due to the reduction of stress at first crack (Susetyo, 2009; Venkateshwaran and Tan, 2018). However, there is a change in the linear elastic behaviour of SFRC after the first crack, as shown in Figure 3.

2.3 Moment redistribution in a one-way slab fixed at its ends

To illustrate the phenomenon of moment redistribution, consider a one-way slab fixed at both ends and subject to uniformly distributed loading, as shown in Figure 4(a). The elastic bending moment diagram for such a loading case is shown in Figure 4(b). The loading on the slab can be classified

Figure 4. Elastic and plastic deformations of a one-way slab fixed at both ends: (a) one-way slab fixed at its ends; (b) elastic bending moment diagram; (c) bending moment diagrams at loading stages 1 and 2; (d) rigid body rotation of two halves

into six stages (denoted by numbers 1 to 6) following the $M-\phi$ relation (denoted by milestone points A to F) in Figure 3.

The bending moment diagrams for the first two loading stages are shown in Figure 4(c). At stage 1, the concrete at the supports experiences first cracking. The moments at the supports are equal to M_{cr} while the span of the slab remains elastic, with the mid-span moment being $M_{cr}/2$.

At stage 2, a crack is formed at the mid-span section as well. The support moment increases by δ_M due to the additional rotation that occurs at this section from stage 1 to stage 2. It can be established that δ_M is very small, thereby leading to the moments at the support regions still being approximately equal to M_{cr} .

When the slab is further loaded, existing cracks would propagate, without formation of new cracks owing to the lesser energy requirement in the former (Ahmadii *et al.*, 2012; Michels *et al.*, 2012; Venkateshwaran and Tan, 2018). Subsequently, the cracks at the support and mid-span sections are expected to propagate further, leading to rigid body rotation of the slab portions between the cracks, as shown in Figure 4(d). The cracks at the mid-span and support sections behave as plastic hinges, with the entire energy being absorbed by rotation at the plastic hinges. Thus, the bending moment diagrams in Figures 4(b) and 4(c) are no longer valid.

Owing to the ductile crack-bridging action of steel fibres, it is expected that the slab sections at the mid-span and the support regions continue to hold substantial moments even after formation of plastic hinges. Figures 5(a) and 5(b) show the moments in the mid-span and support regions, respectively, during different stages of loading after formation of the three plastic hinges in stage 2.

Stages 3 to 6 represent the phases at which the extreme tensile crack widths at the mid-span section, w_t , are 0.25 mm, 0.5 mm, 1.5 mm and w_{u2} mm, respectively. By geometry of deformation, the extreme tensile crack widths and curvatures at the mid-span section at the corresponding stages of loading are approximately twice those at the support sections. The moments at the mid-spans and support corresponding to these extreme tensile crack widths can be obtained from Figure 5. The deflection of the slab at these stages can be established from geometrical relations (Ahmadii *et al.*, 2012; Michels *et al.*, 2012; Venkateshwaran and Tan, 2018).

It can be deduced that the maximum load-carrying capacity of the slab occurs at stage 5, wherein the mid-span and support sections attain the maximum moment capacities. Figures 6(a) and 6(b) show the deviation of moments from their elastic values at the mid-span and support sections, respectively, at different stages of loading, for the one-way slab under consideration. It is seen that the moments are redistributed

Figure 5. Moments and curvatures at (a) mid-span and (b) support section at different stages of loading

Figure 6. Expected moment redistribution at different loading stages for a one-way slab fixed at both its ends: (a) mid-span section; (b) support section

from the support sections to the mid-span section. At stage 6, which signifies the final failure, there is a reduction in the degree of redistribution due to a drop in the moment at larger curvatures.

To verify the phenomenon of moment redistribution in SFRC slabs without traditional reinforcement, as presented above, an experimental programme on seven two-span, one-way continuous slabs was carried out.

3. Experimental programme

Table 1 shows the details of the seven two-span continuous one-way SFRC slab specimens tested in this study. Each slab had a cross-section measuring 600 mm wide and 120 mm thick. The spans were each equal to 1050 mm. All slabs were cast with hooked-end fibres that were 60 mm long and 0.9 mm dia. The concrete mixture proportion consisted of 758 kg/m³ of coarse aggregate, 871 kg/m³ of fine aggregates, 400 kg/m³ of cement and 240 kg/m³ of water, with the concrete class targeted at C_{35/45}. The actual cylinder compressive strength of the concrete ranged from 29 MPa to 40 MPa, as shown in Table 1. The slabs were designated S_{xy}, where x denotes the type of fibres (3 for 3D, 4 for 4D and 5 for 5D) and y denotes twice the fibre volume fraction in per cent. The fibres were named based on the number of hook-ends, that is, 3D fibres consisted of a single hook-end while 4D and 5D fibres had 1.5 and two hook-ends, respectively (Venkateshwaran and Tan, 2018; Venkateshwaran *et al.*, 2018).

Figure 7(a) shows the test set-up with the SFRC slab subject to centre-point loading on each span. The load was applied using a 50 t hydraulic jack at a deflection-controlled rate of 0.1 mm/min up to the peak load and 0.3 mm/min thereafter. Horizontal movement was allowed at the end supports by means of rollers, as shown in Figure 7(a). Two 30 t load

cells were positioned below the internal support to measure the reaction. Owing to safety concerns, load cells were not placed over the roller supports. A curvature measuring device was mounted at each of the the mid-span sections of both spans and at the interior support section of the slab, with two displacement transducers positioned in the horizontal direction at the top and the bottom to measure the strains over a length

of 150 mm. Linear variable displacement transducers (LVDTs) were also positioned at the two mid-span sections to measure the deflections. The tests were carried out up to a point where the load dropped substantially or until a deflection of 15 mm, whichever occurred earlier, to capture the complete behaviour of the slab.

4. Test results and discussion

As the loading was designed to be symmetrical in all the seven slabs, it was theoretically expected that the mid-span deflections in both the spans would be equal throughout the loading phase. However, after formation of the first crack at the two mid-span sections, only one of them propagated further, as shown in Figure 7(b). This was due to the lesser energy required in the propagation of a single crack (i.e. the weaker one) than in simultaneous extension of both cracks (see Figure 7(b)). Consequently, no further deflection was observed at the middle of one of the spans (denoted herein as mid-span 1)

Table 1. Details of test slabs

S no.	Label	f_c MPa	Fibre type	V_f %
1	S32	32.1	3D	1
2	S33	30.5		1.5
3	S41	38.8	4D	0.5
4	S42	35.8		1
5	S43	29.6		1.5
6	S52	29.5	5D	1
7	S53	30.5		1.5

Figure 7. Test set-up of two-span continuous SFRC slab: (a) schematic diagram of test setup and elastic bending moment diagram; (b) slab after cracking

after first cracking. Thus, after first cracking, span 1 is considered to act as a rigid body and is ignored in the analysis. The moment redistribution in span 2 alone is investigated.

The reaction V_B was measured using two 30 t load cells. Slight deviation from elastic behaviour was recorded in the initial stages of loading owing to the low load applied. The load at first cracking was only approximately 10% of the maximum capacity of the load cells. The changes in the observed positive moments at mid-span 2, M_{po2} , as well as the negative moment at interior support, M_n , beyond the cracking moment, are plotted with respect to the applied total load, P , in Figures 8(a)–8(g). By statics, span 2 sustains half of the total load P applied by the actuator. The observed moments at various loads are compared with the elastic values typically given as $5PL/64$ and $3PL/32$ at the mid-span and the support sections, respectively, for a two-span continuous slab (see Figure 7(a)), where L is the length of each span.

The behaviour of the two-span continuous RC slabs with an elastic–plastic moment–curvature relation suggests that the moment at the interior support is expected to reach its peak value first (1, 3). Subsequently, the moments are redistributed to the mid-span equally until all three critical sections attain their maximum values (1, 3). However, unlike RC slabs, the moment redistribution in SFRC slabs follows the moment–curvature relation as shown in Figure 3. After first cracking, the moments are no longer elastic and mid-span 2 attains its peak moment first, as shown in Figures 8(a)–8(g). On further loading, the moments were redistributed to the interior support section until both the interior support and the mid-span 2 sections attained their maximum values. Subsequently, as observed from Figures 8(a)–8(g), the moments at both the mid-span 2 and the interior support drop gradually after the peak load, following the M – ϕ relation in Figure 3. Owing to the crack-bridging action of fibres, the slab sections were capable of sustaining moments substantially. The final deflection at stoppage of the test was approximately 5 to 6 times that at peak load.

Table 2 shows that there is only a marginal amount of moment redistribution at mid-span 2 and interior support at peak load. However, greater moment redistribution can be observed at stages other than peak load, as shown in Figures 8(a)–8(g).

At a given stage of loading, the degree of redistribution (DOR) at a critical section can be determined as follows (Lou *et al.*, 2015)

$$4. \quad \text{DOR} = \left(1 - \frac{M_{\text{ACT}}}{M_{\text{EL}}}\right)$$

where M_{ACT} is the observed moment and M_{EL} is the elastic moment.

Figures 9(a)–9(g) show the degree of redistribution of moments at mid-span 2 and interior support with respect to increasing load. High degrees of redistribution were observed at the interior support and the mid-span 2 sections with increasing load. Stage 3, which indicates the state when the surface crack width at mid-span 2 is 0.25 mm, experienced the highest the degree of redistribution in the range of 20–40%. Slab S41, however, showed an anomaly in the moment redistribution trend, which could be due to a lower fibre content. The moment redistribution at the mid-span 2 and the interior support sections at peak load was generally independent of the fibre configuration.

5. Assessment of moment redistribution in continuous slabs

The analytical approach presented in Section 2 was used to determine the moment redistribution at various stages of loading in the continuous slabs tested in this study. The following loading stages were considered.

- Stage 1. At elastic limit – the moment at the interior support reaches M_{cr} while those at the mid-span sections are equal to $5M_{cr}/6$.
- Stage 2. Cracking occurs at the mid-span sections 1 and 2.

Since the deflection at mid-span 1 is negligible, span 2 can be assumed to deflect as in Figure 5; that is, the extreme tensile crack width and the curvature at the mid-span 2 section are assumed to be twice those at the interior support section. Thus, in loading stages 3 to 6, the moments at mid-span 2, M_{po2} , and the interior support, M_n , are respectively, given by Figures 5(a) and 5(b). In addition to the six stages explained in Section 2, stages 5a and 5b have been added in this section between stages 5 and 6 for better illustration of moment redistribution. Stages 5, 5a and 5b represent the stages wherein the moments at the mid-span 2 section correspond to extreme tensile crack widths of 1.5 mm, 3 mm and 5 mm, respectively. At the same time, the moments at the interior support section at these stages correspond to crack widths equal to half that at the mid-span 2 section, that is, 0.75 mm, 1.5 mm and 2.5 mm, respectively.

Applying the virtual work method to span 2 gives the load-carrying capacity as follows

$$5. \quad P = \frac{4}{L} (M_n + 2M_{po2})$$

The predicted moments at mid-span 2 and the interior support are plotted against the total load, P , in Figures 8(a)–8(g). Except for slab S41 and the slabs containing 5D fibres, the proposed approach predicts the redistribution of moments quite accurately. The general trend is well predicted for most slabs. The predicted degrees of redistribution at mid-span 2 and at the interior support are shown in Figures 9(a)–9(g).

Figure 8. Load plotted against moment at mid-span and interior support sections: (a) S32; (b) S33; (c) S41; (d) S42; (e) S43; (f) S52; (g) S53 (continued on next page)

Figure 8. Continued

Table 2. Observed and elastic values of moments at peak load

Slab label	Peak load, P : kN	$M_{n,TEST}$: kNm	$M_{n,EL}$: kNm	$M_{n,TEST}/M_{n,EL}$	$M_{po2,TEST}$: kNm	$M_{po2,EL}$: kNm	$M_{po2,TEST}/M_{po2,EL}$
M32	86.0	9.73	8.5	1.15	6.42	7.05	0.91
M33	114.9	11.24	11.3	0.99	9.46	9.42	1.00
M41	63.7	3.88	6.27	0.62	6.43	5.23	1.23
M42	72.1	7.50	8.8	0.85	8.00	7.35	1.09
M43	142.5	14.41	14.0	1.03	11.50	11.69	0.98
M52	89.5	8.96	8.8	1.02	7.26	7.34	0.99
M53	119.7	16.24	11.8	1.38	7.58	9.82	0.77
			Ave	1.01			1.00
			Std dev.	0.24			0.14

Note: Subscripts TEST and EL indicate observed and elastic values, respectively

Figure 9. Degree of moment redistribution: (a) S32; (b) S33; (c) S41; (d) S42; (e) S43; (f) S52; (g) S53 (continued on next page)

Figure 9. Continued

6. Load–deflection relation

The moments at the mid-spans and support sections at various loading stages are used to determine the deflections of individual spans in continuous SFRC slab systems. With the deformed shape of span 2 being similar to the rigid body rotation of the slab halves shown in Figure 4(d), the total deflection at mid-span 2 for a given extreme tensile crack width, w_t , is determined as (Venkateshwaran and Tan, 2018)

$$6. \quad \Delta = \Delta_{cr} + \Delta_p = \Delta_{cr} + \frac{w_t L}{4(H - x)}$$

where Δ_{cr} is the elastic deflection up to first cracking (mm); Δ_p is the plastic deformation due to rigid body rotation (mm); and x is the neutral axis depth at mid-span 2 corresponding to a given loading stage (mm).

The observed and predicted load–deflection curves using Equations 5 and 6 are shown in Figures 10(a)–10(g). The figures also show the predicted and observed load plotted against curvature relations for the mid-span 2 and interior support sections. The analytical approach accurately predicts the load–deflection profile of most slabs. As shown in Table 3, the analytical method predicts the average peak load-carrying capacity of the slabs

Figure 10. Deflection and curvatures: (a) S32; (b) S33; (c) S41; (d) S42; (e) S43; (f) S52; (g) S53 (continued on next page)

within 8% of the observed values with a standard deviation of 8%. As expected, an increase in the load-carrying capacity of the continuous slabs is observed with an increase in fibre content. The fibre configuration did not have a significant influence on the load-carrying capacity for the given range of parameters.

7. Conclusion

The phenomenon of moment redistribution that occurs in continuous SFRC slabs without traditional reinforcement was investigated by conducting centre-point load tests on seven two-span, one-way slabs with varying fibre content and fibre

Figure 10. Continued

Table 3. Load-carrying capacity of test slabs

Slab label	P_{TEST} : kN	P_{PRED} : kN	P_{TEST}/P_{PRED}
S32	86.0	93.7	0.92
S33	114.9	125.4	0.92
S41	63.7	69.1	0.92
S42	89.5	101.0	0.89
S43	142.5	130.8	1.09
S52	89.5	106.9	0.84
S53	119.7	139.3	0.86
		Ave	0.92
		Std dev.	0.08

Note: Subscripts TEST and PRED indicate observed and predicted values, respectively

configuration. Considerable moment redistributions at critical sections were noted after first cracking. An analytical approach was presented which predicts the redistribution of moments well at various stages of loading for most slabs. The load–deflection behaviour was also plotted accurately for most test slabs. With test results validating the analytical approach, the moments in internal panels of a SFRC flat slab system, for example, can be predicted and expected to follow the trend shown in Figure 6.

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