Non-linear elastic model incorporating temperature effects

Hong Liu MEng, BEng (Hons)

PhD candidate, School of Civil Engineering, Chongqing University, Chongqing, China

Hanlong Liu PhD, MEng, BEng (Hons)

Professor and Chair, School of Civil Engineering, Chongqing University, Chongqing, China

Yang Xiao PhD, MEng, BEng (Hons), SMASCE

Associate Professor, China Key Laboratory of New Technology for Construction of Cities in Mountain Area (Chongqing University), Ministry of Education, Chongqing, China; School of Civil Engineering, Chongqing University, Chongqing, China (corresponding author: hhxyanson@163.com)

Qingsheng Chen PhD

Research Fellow, Department of Civil and Environmental Engineering, National University of Singapore, Singapore

Yufeng Gao PhD

Professor and Director, School of Civil and Transportation Engineering, Hohai University, Nanjing, China

Jing Peng BEng (Hons)

Lecturer, School of Civil Engineering, Chongqing University, Chongqing, China

The objective of this paper is to present a non-linear elastic model, considering temperature effects, that engineers can readily use to predict the mechanical behaviours of soils in geotechnical applications. Instead of using hyperbolic and exponential models, a non-linear equation is first formatted to describe the tangent moduli for saturated clays by introducing only one additional parameter (i.e. θ). By extending the non-linear equation, a new simplified non-linear elastic model is obtained that is capable of capturing well the stress–strain relationship of saturated clays at room temperature. Thereafter, based on experimental results, the relationships between cohesion, internal friction angle, index, tangent bulk modulus and temperature are developed and incorporated in the non-linear elastic model. A revised semi-regression method is also developed to determine the relationship between the additional model parameter θ and temperature. In this model, all of the eight parameters for the model have clear physical meanings and could be readily obtained by performing temperature-controlled triaxial tests. The accuracy and general applicability of the proposed method was checked by comparing its predictions with experimental results on saturated clay under various stress-path and temperature conditions as well as existing solutions proposed elsewhere.

Notation

$a, b, k, n,$	constants
$\gamma_c, \gamma_\phi, \gamma_\theta, k_b,$	
γ_k, m_o and γ_m	
B_t	tangent bulk modulus: kPa
c	cohesion at target temperature: kPa
c_0	cohesion at room temperature: kPa
E_i	initial tangent modulus: kPa
E_{te}	tangent modulus in the exponential model: kPa
E_{th}	tangent modulus in the hyperbolic model: kPa
E_{tp}	tangent modulus in the power function model: kPa
O_{CR}	overconsolidation ratio
p_a	atmospheric pressure: kPa
p'	effective confining pressure: kPa
R_f	failure ratio
T	target temperature: °C
T_0	initial ambient room temperature: °C
ε_a	axial strain: %
θ	index in the power function model
σ_1	maximum principal stress: kPa
σ_3	minimum principal stress: kPa
$(\sigma_1 - \sigma_3)$	deviatoric stress: kPa
$(\sigma_1 - \sigma_3)_f$	deviatoric stress at the failure state: kPa
$(\sigma_1 - \sigma_3)_u$	deviatoric stress at the ultimate state: kPa

ϕ	internal friction angle at target temperature: °
ϕ_0	internal friction angle at room temperature: °

Introduction

Over recent years, the influence of temperature on the engineering properties of soils has received increasing attention from researchers, due to its wide applications in the constructions of various energy-related infrastructures such as nuclear waste repositories (Gens *et al.*, 2009), high-voltage electric cables (Brandon *et al.*, 1989; Rao *et al.*, 2017; Sun *et al.*, 2011), energy piles (Abdelaziz and Ozudogru, 2016; Knellwolf *et al.*, 2011; Saggi and Chakraborty, 2016) and geothermal structures (Brandl, 2006), highway pavements (Bianchini *et al.*, 2011; Kertesz and Sansalone, 2014), thermally active embankments (Coccia and McCartney, 2013) and thermally active retaining walls (Stewart *et al.*, 2014).

The volume change for saturated soils performs a transition from contractive to expansive behaviour during drained heating or the temperature cycle with increasing overconsolidation ratio (O_{CR}) (Abuel-Naga *et al.*, 2006; Baldi *et al.*, 1988; Cekerevac and Laloui, 2004; Romero *et al.*, 2005) or relative density (Ng *et al.*, 2016). Some experimental test results showed that the shear strength of saturated clays under both drained and undrained conditions would increase with increase in temperature (Abuel-

Naga *et al.*, 2006; Cekerevac and Laloui, 2004), while some other studies revealed a reduction in the shear strength with increasing temperature (Bruyn and Thimus, 1996; Kuntiwattanakul *et al.*, 1995).

Meanwhile, many advanced thermo-mechanical constitutive models have been proposed to evaluate the mechanical behaviours of saturated and unsaturated soils undergoing temperature changes (Coccia and McCartney, 2016; Modaressi and Laloui, 1997; Najari and Selvadurai, 2013; Wang *et al.*, 2016). Hueckel *et al.* (1998) put forward one of the first thermo-mechanical constitutive models for saturated clays considering thermal softening within the framework of the critical state theory. By employing the framework of the modified Cam-clay model, Cui *et al.* (2000) proposed a temperature-dependent model, which could be capable of predicting well the plastic strains at higher O_{CR} values. Graham *et al.* (2001) revised the Cam-clay model to consider the effects of temperature on the volume changes, pore water pressures and shear strengths for both normally consolidated and overconsolidated clays. Hamidi *et al.* (2014) used the relationships of the isotropic compression curves for saturated clays at various temperatures to find a new thermal-elastic-plastic mechanical model incorporating the stress history influence.

However, most of these temperature-dependent constitutive models mentioned previously are based on the elasto-plastic Cam-clay or modified Cam-clay model in the critical state framework. These models often include a number of parameters which may not have physical meanings or that have values difficult to determine due to the complicated soil properties, particularly considering the effects of temperature. Compared with these elastic-plastic models, it is easier to understand the theory of non-linear elastic models, and the parameters could also be easily determined by performing triaxial compression tests (Hicher and Chang, 2006, 2008). In addition, the non-linear elastic models (e.g. Duncan-Chang model) could be readily implemented in finite-element analysis software without a singular matrix (Chen and Liu, 2007; Taciroglu and Hjelmstad, 2002; Xiong and Fang, 2008; Zheng and Binienda, 2008). Therefore, there is a pressing need to present a non-linear elastic model that considers temperature effects, that engineers can readily use to predict the mechanical behaviours of soils in geotechnical applications.

The main objective of this paper is to propose a new non-linear elastic model to capture the mechanical behaviours of saturated clays incorporating effects of temperature based on the general framework of the power function and the regression analysis method. In this model, the effects of temperature on the cohesion, internal friction angle and index for saturated clays are taken into account. The accuracy and general applicability of the proposed method is checked by comparing its predictions with experimental results and existing solutions proposed elsewhere.

Power function non-linear elastic model

The stress-strain relationship of saturated soils can be expressed by the following equations

- hyperbolic model (Duncan and Chang, 1970)

$$1. \quad \sigma_1 - \sigma_3 = \frac{\varepsilon_a}{a + b\varepsilon_a}$$

- exponential model (Gitau *et al.*, 2006)

$$2. \quad \sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_u \left\{ 1 - \exp \left[-\frac{E_i \varepsilon_a}{(\sigma_1 - \sigma_3)_u} \right] \right\}$$

where $\sigma_1 - \sigma_3$ is the deviatoric stress, $(\sigma_1 - \sigma_3)_u$ is the deviatoric stress for soils at the ultimate state, ε_a is the axial strain and E_i is the initial tangent modulus.

The tangent moduli of these non-linear models can be estimated from the derivation of Equations 1 and 2 and can be described as follows

$$3. \quad E_{th} = E_i \left[1 - \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_u} \right]^2$$

$$4. \quad E_{te} = E_i \left[1 - \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_u} \right]$$

where E_{th} and E_{te} are the tangent moduli in the hyperbolic and exponential models, respectively.

It can be seen that the tangent moduli in Equations 3 and 4 are identical in formation except for the indices. The index for the tangent modulus in the hyperbolic model is 2, while its counterpart is 1 in the exponential model. Due to this, the power function model is a traditional type of non-linear elastic model; it may be reasonable to employ a unique power function model instead of using the hyperbolic and exponential models to describe the tangent modulus, and it can be given as follows

$$5. \quad E_{tp} = E_i \left[1 - \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_u} \right]^\theta$$

where E_{tp} is the tangent modulus in the power function model and θ is the index of the power function non-linear elastic model.

Then, the non-linear stress-strain behaviour of saturated soils at room state in the power function can be rewritten as

$$6. \quad \sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_u \left\{ 1 - \left[1 + \frac{(\theta - 1)E_i}{(\sigma_1 - \sigma_3)_u} \varepsilon_a \right]^{1/(1-\theta)} \right\}$$

in which

$$7. \quad R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_u}$$

$$8. \quad (\sigma_1 - \sigma_3)_f = \frac{2c \cos \phi + 2\sigma_3 \sin \phi}{1 - \sin \phi}$$

$$9. \quad E_i = kp_a \left(\frac{\sigma_3}{p_a} \right)^n$$

where $(\sigma_1 - \sigma_3)_f$ is the deviatoric stress for soils at the failure state; R_f is the failure ratio, which is the ratio between the deviatoric stresses at the failure and ultimate states; c is the cohesion; ϕ is the internal friction angle; p_a is the atmospheric pressure of 101 kPa; and k and n are the material constants, respectively. The derivation of the relationship described by Equation 6 can be seen in the Appendix.

The combination of Equations 6–9 gives

$$10. \quad \sigma_1 - \sigma_3 = \frac{2c \cos \phi + 2\sigma_3 \sin \phi}{(1 - \sin \phi) R_f} \times \left\{ 1 - \left[1 + \frac{(\theta - 1)(1 - \sin \phi) R_f}{2c \cos \phi + 2\sigma_3 \sin \phi} kp_a \left(\frac{\sigma_3}{p_a} \right)^n \varepsilon_a \right]^{1/(1-\theta)} \right\}$$

As shown in Equation 10, only one additional parameter (i.e. θ) is required in the proposed power function model, compared with the traditional hyperbolic model or exponential model. Although there are some slight variations (Guo *et al.*, 2014), these three parameters (i.e. R_f , k and n) remain relatively constant with change in temperature for saturated soils. It may be reasonable to take the average values of these parameters determined by the Duncan and Chang (1970) equations at each target temperature. The determination of the thermal related parameters (i.e. c , ϕ and θ) is presented in the following section.

Determination of model parameters

Parameters c and ϕ

The results of tests conducted by Guo (2014) on Ningbo soft clay, Uchaipichat and Khalili (2009) on Bourke silt, Soleimanbeigi *et al.* (2014) on recycled asphalt shingle mixtures and Cekerevac and Laloui (2004) on kaolin clay are depicted in Figures 1 and 2 as normalised temperature plotted against cohesion and internal friction angle. It should be taken into account that all the test data used in these figures are calculated in the saturated condition. In the figures, the solid lines stand for the simulation results which

Figure 1. Influence of temperature on the cohesion of soils. RASBA, recycled asphalt shingle–bottom ash mixture

Figure 2. Influence of temperature on the internal friction angle of soils

are determined from the test data. Interestingly, it can be seen that the relationships of cohesion, internal friction angle and temperature are essentially linear and can be expressed by the following equations

$$11. \quad c = c_0 [1 + \gamma_c (T/T_0 - 1)]$$

$$12. \quad \phi = \phi_0 [1 + \gamma_\phi (T/T_0 - 1)]$$

where c_0 and ϕ_0 are the cohesion and internal friction angle of soils at room temperature, respectively. T and T_0 are the target and room temperatures, respectively. γ_c and γ_ϕ are the material constants.

Parameter θ

The determination of index θ can be classified into two categories.

The first category is the full regression method. Xu and Jin (2000) took the test data (i.e. $\sigma_1 - \sigma_3$ and ε_a) into Equation 6 and produced the value of the deviatoric stress at the ultimate state $(\sigma_1 - \sigma_3)_u$, initial tangent modulus E_i and index θ by regression analysis directly. In this method, there are two knowns (i.e. $\sigma_1 - \sigma_3$ and ε_a) and three unknowns (i.e. $(\sigma_1 - \sigma_3)_u$, E_i and θ), so it may be not easy to achieve an exact solution during the regression analysis process, thereby possibly leading to an unacceptable deviation between the predictions and test data.

The second category is the semi-regression method. Cai *et al.* (2012) calculated the deviatoric stress at the ultimate state $(\sigma_1 - \sigma_3)_u$ and the initial tangent modulus E_i using Equations 7–9 and then put these values as well as the test data (i.e. $\sigma_1 - \sigma_3$ and ε_a) into Equation 6 to obtain a result for index θ through semi-regression analysis. In this method, there is only one unknown (i.e. θ), the exact value of which could be readily determined, thereby leading to a very good agreement of the model predictions with the test data. However, compared with the first method, the values of $(\sigma_1 - \sigma_3)_u$ and E_i are determined from the experimental data in the second method; the predictions become very sensitive to the accuracy of the test data.

To overcome the issues in the preceding methods, in this work, a revised semi-regression method is proposed through combination of the two regression methods mentioned earlier to evaluate the index θ . In this method, the deviatoric stress at the ultimate state $(\sigma_1 - \sigma_3)_u$ and initial tangent modulus E_i are calculated by using Equations 7–9, 11 and 12 in terms of the predicted results (rather than the test data). In addition, the predicted values of $(\sigma_1 - \sigma_3)_u$ and E_i and the test data for $\sigma_1 - \sigma_3$ and ε_a at different confining pressure and temperature conditions are used for Equation 6 to reach a solution for index θ by performing semi-regression analysis. In the revised semi-regression method, there are four knowns (i.e. $(\sigma_1 - \sigma_3)_u$, E_i , $\sigma_1 - \sigma_3$ and ε_a) and only one unknown (i.e. θ), so it may be easy to get the solution with precision compared to the full regression method. Otherwise, two knowns (i.e. $(\sigma_1 - \sigma_3)_u$ and E_i) are the predicted results, so the revised regression method may be not sensitive to the test data compared to the semi-regression method. As indicated in Figure 3, the index θ with regard to the temperature can be linearly expressed as follows

$$13. \quad \theta = \theta_0 + \gamma_\theta(T/T_0 - 1)$$

where θ_0 is the index θ in the power function model for soils at room temperature and γ_θ is the material constant.

Parameter B_t

The tangent bulk modulus B_t of the power function model is similar to that of the hyperbolic model and can be described by the following function

Figure 3. Influence of temperature on the index θ

$$14. \quad B_t = k_b p_a \left(\frac{\sigma_3}{p_a} \right)^m$$

in which

$$15. \quad k_b = k_{b0} + \gamma_k(T/T_0 - 1)$$

$$16. \quad m = m_0 + \gamma_m(T/T_0 - 1)$$

where k_b and m are the material parameters related to the temperature, k_{b0} and m_0 are the initial values of k_b and m at room temperature and γ_k and γ_m are the material constants.

Combining Equations 14–16, the tangent bulk modulus B_t incorporating the effects of temperature changes can be given as follows

$$17. \quad B_t = [k_{b0} + \gamma_k(T/T_0 - 1)] p_a \left(\frac{\sigma_3}{p_a} \right)^{m_0 + \gamma_m(T/T_0 - 1)}$$

Validation of the proposed model

In this section, the newly proposed thermo-mechanical constitutive model is validated against the temperature-controlled triaxial test data reported by Guo (2014) and Cekerevac and Laloui (2004). The material parameters of soils are listed in Table 1. In addition, the performance of this model is compared with the predictions of the model proposed by Yao and Zhou (2013), which was developed based on the unified hardening theory but requires more model parameters.

Undrained triaxial test

Guo (2014) reported the results of temperature-controlled triaxial compression tests on the Ningbo soft clay. The clay specimens

Table 1. Material parameters of saturated soils for numerical study

Experimental data	γ_c	γ_ϕ	O_{CR}	γ_θ	γ_k	γ_m	k	n	R_f
Guo (2014)	-0.37	-0.10	1.0	0.31	—	—	34.00	0.70	0.74
Cekerevac and Laloui (2004)	0.19	0.03	1.0	0.02	4.08	-0.11	93.86	0.41	0.79
			1.2	-0.09					
			1.5	0.05					
			2.0	-0.04					
			3.0	0.12					
			6.0	0.29					

were initially consolidated under three isotropic confining pressures (i.e. 50, 100 and 200 kPa) at room temperature. For each confining pressure, four temperatures (30, 45, 60 and 80°C) were applied under drained conditions. Further, after drained heating, specimens were sheared under undrained conditions to investigate the effects of temperature on the shear strength of saturated Ningbo soft clay. According to the test data, it can be evaluated that the cohesion and internal friction angle of saturated Ningbo soft clay decrease with an increase in temperature as shown in Figures 4 and 5. However, as the temperature increases, the index θ also increases to a high value as shown in Figure 6. Figure 7 shows the measured and predicted stress-strain relationships of clays at different confining pressure and temperature conditions. A reduction in strength for Ningbo soft clay during the undrained shearing phase with increasing temperature is predicted by the power function model and is in good agreement with the laboratory data.

Drained triaxial test

Cekerevac and Laloui (2004) carried out a series of temperature-controlled triaxial tests to evaluate the thermal mechanical behaviour of saturated, overconsolidated kaolin clay. The specimens were initially pre-consolidated to 600 kPa at 22°C and then unloaded to 600, 500, 400, 300, 200 and 100 kPa, thereby leading to different overconsolidation ratios ($O_{CR} = 1.0, 1.2, 1.5,$

Figure 4. Calibration for model parameter c (data from Guo (2014))

Figure 5. Calibration for model parameter ϕ (data from Guo (2014))

Figure 6. Calibration for model parameter θ (data from Guo (2014))

2.0, 3.0, 6.0). After completion of consolidation, the specimens were heated up to 90°C at a rate of 3.33°C/h under drained conditions. These specimens were sheared in the drained condition to 30% of the axial strain at two different constant temperature conditions (i.e. $T = 22$ and 90°C).

Figure 7. Model predictions against experimental data (data from Guo (2014)): (a) $p' = 50 \text{ kPa}$; (b) $p' = 100 \text{ kPa}$; (c) $p' = 200 \text{ kPa}$

The stress–strain relationships of kaolin clay are predicted by the power function non-linear elastic model and compared with the test results as shown in Figure 8. The predicted responses using the thermal–elastic–plastic model developed by Yao and Zhou (2013) are also included in Figure 8. It can be observed that an increase in temperature leads to an increase in the strength of soils for all O_{CR} values, which is predicted well by the non-linear elastic model. In addition, the softening behaviour during shearing for heavily overconsolidated specimens is not predicted by this non-linear model. However, the general trend of predictions seems acceptable.

Conclusions

A new isotropically thermal mechanical constitutive model for normally and lightly overconsolidated clays was developed in present work by extending the existing hyperbolic model (Duncan and Chang, 1970) and exponential model (Gitau *et al.*, 2006). Instead of using the hyperbolic and exponential functions, a unique power function model is employed to describe the tangent modulus with only one additional parameter (i.e. θ). In terms of experimental

results, linear equations are proposed to capture the effects of temperature on the cohesion and internal friction angle of soils. In addition, a revised semi-regression method was used to determine the accurate value of the index, with an attempt to reach a very good agreement of the model predictions with the test data. All of the nine model parameters for the model have clear physical meanings and could be readily obtained by performing temperature-controlled triaxial tests. The accuracy and general applicability of the proposed method was checked by comparing its predictions with experimental results and existing solutions proposed elsewhere.

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Figure 8. Model predictions against experimental data on remoulded kaolin clay (data from Cekerevac and Laloui (2004)): (a) $O_{CR} = 1.0$; (b) $O_{CR} = 1.2$; (c) $O_{CR} = 1.5$; (d) $O_{CR} = 2.0$; (e) $O_{CR} = 3.0$; (f) $O_{CR} = 6.0$

Appendix

The tangent modulus can be assumed in a power function, given as follows

$$18. \quad E_{tp} = E_i \left[1 - \frac{(\sigma_1 - \sigma_3)_u}{(\sigma_1 - \sigma_3)_u} \right]^\theta$$

In addition, the tangent modulus of saturated soils during the conventional triaxial test can be defined as

$$19. \quad E_{tp} = \frac{d\sigma_1}{d\varepsilon_a} = \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon_a}$$

Equations 18 and 19 can be rewritten as

$$20. \quad \left[1 - \frac{(\sigma_1 - \sigma_3)_u}{(\sigma_1 - \sigma_3)_u} \right]^{-\theta} d(\sigma_1 - \sigma_3) = E_i d\varepsilon_a$$

Integration of Equation 20 gives

$$21. \quad \left[1 - \frac{(\sigma_1 - \sigma_3)_u}{(\sigma_1 - \sigma_3)_u} \right]^{1-\theta} (\sigma_1 - \sigma_3)_u = (\theta - 1)(E_i \varepsilon_a + C)$$

Reduction of the fraction to a common denominator gives

$$22. \quad [(\sigma_1 - \sigma_3)_u - (\sigma_1 - \sigma_3)]^{1-\theta} = (\theta - 1)(E_i \varepsilon_a + C)[(\sigma_1 - \sigma_3)_u]^{-\theta}$$

If $\theta \neq 1$ the stress and strain behaviour of soils can be written as follows

$$\begin{aligned} & \sigma_1 - \sigma_3 \\ &= (\sigma_1 - \sigma_3)_u - [(\sigma_1 - \sigma_3)_u]^{-\theta/(1-\theta)} [(\theta - 1)(E_i \varepsilon_a + C)]^{1/(1-\theta)} \\ &= (\sigma_1 - \sigma_3)_u - (\sigma_1 - \sigma_3)_u [(\sigma_1 - \sigma_3)_u]^{-1/(1-\theta)} [(\theta - 1)(E_i \varepsilon_a + C)]^{1/(1-\theta)} \\ &= (\sigma_1 - \sigma_3)_u \left\{ 1 - \left[\frac{\theta - 1}{(\sigma_1 - \sigma_3)_u} (E_i \varepsilon_a + C) \right]^{1/(1-\theta)} \right\} \end{aligned}$$

23.

If $\theta = 1$ it can be replaced by 0.99999 or 1.00001, and the stress–strain relationship also can be described by Equation 23.

During the conventional triaxial test, the test data should pass through the point (0, 0) on the $(\sigma_1 - \sigma_3) - \varepsilon_a$ plane. And then, the constant can be obtained as follows

$$24. \quad C = \frac{(\sigma_1 - \sigma_3)_u}{\theta - 1}$$

Therefore, the stress–strain relationship of soils using the power non-linear elastic model at room state can be given as follows

$$25. \quad \sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_u \left\{ 1 - \left[1 + \frac{(\theta - 1)E_i}{(\sigma_1 - \sigma_3)_u} \varepsilon_a \right]^{1/(1-\theta)} \right\}$$

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