Settlement of a pile under cyclic lateral loads in dry sand

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It is found through centrifuge model tests that the cyclic lateral load on a pile reduces the shaft friction and induces additional pile settlement. A theoretical model using the load-transfer method was proposed for the settlement prediction of cyclic laterally loaded piles in dry sand. A simple formula was established to quickly predict the pile settlement in practical engineering. The theoretical model provided a reasonable estimate of the pile settlement, while the predictions from the proposed quick prediction formula were relatively conservative. A new concept of ‘settlement-controlled design’ was proposed to advance the methodology for the design of a pile by considering potential settlement after many cycles of lateral loads. According to the design plane derived from this study, it was suggested that the design-state points should be limited to the convergent zone.

KEYWORDS: cyclic lateral load; geotechnical engineering; piles & piling; pile settlement; quick prediction; settlement; settlement-controlled design; theoretical analysis; theoretical model

INTRODUCTION

Pile foundations have been widely adopted for the construction of a variety of infrastructures, such as viaducts, high-rise buildings and offshore structures (Zhang *et al.*, 2005; Kong *et al.*, 2019). As the basic components that support superstructures, piles are inevitably subjected to cyclic lateral loads (e.g. the starting and braking forces of vehicles acting on a viaduct, earthquake forces acting on a high-rise building and wind or wave loads acting on an offshore wind turbine) during their service lives.

In recent decades, many innovative studies on laterally loaded piles have been documented in the literature (Broms, 1964; Matlock, 1970; Reese *et al.*, 1974; Baguelin *et al.*, 1977; Banerjee & Davies, 1978; Meyerhof *et al.*, 1981; Randolph, 1981, 2003, Randolph & Schneider, 2018; Budhu & Davies, 1987; Sun & Pires, 1993; Meyerhof, 1995; Poulos *et al.*, 1995; Rollins *et al.*, 1998; Zhang, 2009; Leblanc *et al.*, 2010; Zhang *et al.*, 2016; Hong *et al.*, 2017). Relatively complete industry standards have also been released for the design of pile foundations (BSI, 1997; MOHURD, 2008; API, 2015). However, attentive engineers may notice that these documents are focused on the deformation and horizontal capacity of the pile, whereas the vertical response of a pile under cyclic lateral loads has barely been mentioned.

Pile response analyses have confirmed that pile-supported buildings exhibit obvious settlement during earthquakes (Tokimatsu & Asaka, 1998; Stringer & Madabhushi, 2011). However, it is difficult to obtain a consensus on the reasons for pile settlements. One mainstream opinion was that the seismic failure of soil gives rise to the subsidence of piles, which emphasises the loss of soil strength. The most typical case would be the liquefaction of the soil, which has been proposed as the cause of excessive pile settlement during

earthquakes, for example, Niigata in 1964, Hanshin in 1995 and Kobe in 1995 (Hamada & O’Rourke, 1993; AHG, 1995; EQE, 1995). A survey of the 1985 Mexico earthquake deviated from this point of view (Mendoza & Auvinet, 1988). Most obviously, buildings with piles driven in clay (typical non-liquefiable soil) also exhibited substantial subsidence. According to Mendoza & Auvinet (1988), significant settlement mainly occurred at friction piles, whereas point-bearing (or end-bearing) piles performed much better during the earthquake. This finding inspired another hypothesis that the adhesion or friction at the pile–soil interface degraded under seismic loading. The force equilibrium was disrupted to generate additional settlement. However, there has been little research that attempts to validate and clarify this possibility, which means that the truth behind this theory is yet to be revealed.

To reveal the mechanism of the vertical displacements of a pile under cyclic lateral loads, a series of centrifuge model tests was conducted by Zheng *et al.* (2021). These tests provided evidence for the transmission of the shaft friction lost on the pile tip, which caused additional pile settlement. In this study, a theoretical model using the load-transfer method was proposed to predict the settlement of a pile under cyclic lateral loads in dry sand. A simple formula was established to predict pile settlement quickly in practical engineering. A new concept of ‘settlement-controlled design’ was proposed to advance the methodology for the design of a pile by considering potential settlement after many cycles of lateral loads.

BRIEF REVIEW OF THE CENTRIFUGE MODEL TESTS

Zheng *et al.* (2021) conducted centrifuge model tests at 100g to reveal the mechanism of the vertical displacements of a pile under cyclic lateral loads. Cyclic lateral loads were applied to a pile model with a diameter of 6 mm by way of a self-designed loading system, as shown in Fig. 1. The lateral load was applied by the displacement-controlled method with the amplitude of horizontal displacement denoted as a . Standard weights were installed on the top of the pile to simulate the vertical load, F_v . More details of the centrifuge modelling process can be found in Zheng *et al.* (2021).

The results of a typical test with $F_v = 60$ N and $a = 1.6$ mm are shown in Fig. 2, where s_v and f_b denote the pile settlement and the measured force acting on the pile base, respectively.

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Fig. 1. Schematic diagram of the centrifuge model

The pile settlement increased non-linearly with increasing cycle number and stabilised after a certain number of cycles. The measured force acting on the pile base f_b provided further insight into the settlement process. The curve of the measured force acting on the pile base f_b plotted against the number of cycles n shows a trend that is highly consistent with the s_v plotted against n curve. This implies an intrinsic relationship between the pile–settlement behaviour and the base force. Since no vertical loads were applied to the pile during cyclic loading, it is reasonable to conclude that the mobilised shaft friction of the pile was transmitted to the pile base, which caused an increase in both f_b and s_v (Zheng *et al.*, 2021).

THEORETICAL MODEL

Load-transfer model

Under the framework of the classic load-transfer method, a theoretical model was proposed to calculate the pile settlement under cyclic lateral loading. The soil continuum around the pile shaft was discretised into a series of infinitesimal areas. Each area of soil had its own load-transfer curves in three directions – namely, the radial and tangential directions in the cross-section and the tangential direction in the longitudinal section. These curves defined the relationship between the stress (cf. normal pressure σ_n , horizontal shear stress τ_h and vertical shear stress τ_v in Fig. 3) applied at the pile shaft and the local displacement of

the soil relative to the pile (radial and tangential displacement). Similarly, the soil underneath the pile base was characterised by one load-transfer curve in the vertical direction (cf. σ_b in Fig. 3). According to Li *et al.* (2012), the load-transfer curves adopted the form of a parabola. More details on the load-transfer curves are given in Appendix 1. (For ease of description, half a pile shaft contacting the compacted soil was defined as the front side, while the other half was defined as the back side, as shown in Fig. 3. In Fig. 3, D_R is the depth of the rotation centre, RC, at which no horizontal displacement is anticipated. It should be noted that the front side and back side are not constant because the lateral moving direction of the pile under cyclic loads may reverse.)

When the pile head moves horizontally at a distance of da , the normal pressure σ_n changes to σ_{n_da} . On the back side of the pile, the limit shaft friction would decrease due to the decreased normal pressure. If the limit shaft friction was smaller than the mobilised shaft friction, the mobilised shaft friction thereupon would decrease and be equal to the value of the limit shaft friction. In this way, in the da movement, the loss in the mobilised shaft friction, dF_{f_da} , can be calculated as

$$dF_{f_da} = \int_S \alpha(\tau_v - \tan \delta \times \sigma_{n_da}) dS \quad (1)$$

where δ is the friction angle of the pile–soil interface; S is the surface of the pile shaft; τ_v is the vertical shear stress (i.e. mobilised shaft friction) before da movement; and α is a sign function expressed by

$$\alpha = \begin{cases} 1, & \tau_v > \tan \delta \times \sigma_{n_da} \\ 0, & \tau_v \leq \tan \delta \times \sigma_{n_da} \end{cases} \quad (2)$$

The loss in the mobilised shaft friction does not disappear but is transmitted to the pile base and pile shaft with a redundant bearing capacity that can carry the vertical loads. The forces that were transmitted to the pile base, $dF_{f_da,b}$, can be calculated by

$$dF_{f_da,b} = dF_{f_da} \frac{K_{b_da}}{K_{b_da} + K_{s_da}} \quad (3)$$

where K_{b_da} and K_{s_da} represent the contribution of the pile base and shaft, respectively, to the stiffness of the pile against vertical displacement.

In this da -movement, the settlement of the pile, ds_v , can be obtained by

$$ds_v = \frac{dF_{f_da,b}}{K_{b_da}} \quad (4)$$

Fig. 2. Experimental results of a typical test

Fig. 3. Schematic diagram of the load-transfer model

The settlement of the pile after many cycles of lateral loads, s_v , can be expressed by

$$s_v = \sum_{i=0}^i \left(\int_0^{a_i} ds_v \right) \quad (5)$$

where i is the number of half-cycle movements and a_i is the horizontal displacement of the pile at the i th half-cycle movement. (In this study, when the pile moved from the left-most position to the right-most position (or vice versa), the pile was considered to have completed one ‘half-cycle movement’.)

Validation

To verify the accuracy of the proposed theoretical model, it is used to predict the data collected from centrifuge model tests conducted by Zheng *et al.* (2021). The settlement plotted against cycle number curves from the centrifuge model test (denoted CT in the figure) and theoretical model (denoted TM in the figure) are shown in Fig. 4. Good agreements have been achieved. The comparison for the cases in which the amplitude of the horizontal displacement, a , was 1.6 mm, while the vertical loads, F_v , were 40 N, 60 N and 80 N is shown in Fig. 4(a). The test data and calculated results show that the pile settlement was greater when F_v was larger. The cases for an F_v of 60 N and the values of a of 0.2 mm, 0.4 mm, 0.8 mm, 1.6 mm and 2.4 mm are shown in Fig. 4(b). Similarly, both the test data and calculated results indicated that the pile settlement could be greater when the amplitude of horizontal displacement, a , was larger.

Predictions of the mobilised shaft friction in the vertical slice abcd (see Fig. 5(a)) after the first cycle for the case of $a = 1.6$ mm and $F_v = 60$ N are shown in Fig. 5(a). The mobilised shaft friction on the front side exhibited steady growth, while the mobilised shaft friction on the back side gradually vanished during the movement of the pile. The mobilised shaft friction on the back side in the far end (the part away from the RC) could quickly decrease to zero. However, a delay in the reduction in the mobilised shaft friction was observed near the RC (as shown in Fig. 5(a)). The mobilised shaft friction near the RC remained almost unchanged when the horizontal displacement was less than $a/4$.

After many cycles of lateral loads, most of the mobilised shaft friction on both the back and front sides is lost except in

a region near the RC, in which the mobilised shaft friction is finally maintained at a certain level (cf. Fig. 5(b)). In the present study, this region is called the friction core.

It should be noted that the response of the soil around a pile under cyclic lateral loads in dry sand can be very complicated. For instance, both soil densification and strain ratcheting were found in similar studies (Ng *et al.*, 1998a, 1998b). However, for ease of analysis, these effects were not considered in the theoretical model presented in this study.

Fig. 4. Comparison of the test data and the calculation results: (a) tests with different F_v values; (b) tests with different a values

Fig. 5. Variations in the mobilised shaft friction during cyclic lateral loading (all dimensions in model scale): (a) pile at different positions in the first cycle; (b) upright pile under different cycles

A quick prediction formula

Although the theoretical model provided satisfactory predictions for pile settlement under cyclic lateral loads, it is still too complicated for engineers to use in engineering practice. A simple formula was proposed to achieve a quick prediction of the pile settlement induced by cyclic lateral loads. The following simplifications are made to derive the quick prediction (QP) formula.

- (a) The load-transfer curves adopt a linear form. The slopes of the load-transfer curves, k_{n_QP} , k_{h_QP} and k_{v_QP} , corresponding to σ_n , τ_h and τ_v , respectively, can be calculated by (Li *et al.*, 2012)

$$k_{n_QP} = \frac{8}{\pi(1-\nu)D} G_{s,ini} \quad (6)$$

$$k_{h_QP} = \frac{G_{s,ini}}{\zeta} \quad (7)$$

$$k_{v_QP} = \frac{G_{s,ini}}{\zeta} \quad (8)$$

where ν is the Poisson's ratio of the sand; $G_{s,ini}$ is the initial shear modulus of the soil; and ζ is a dimensionless coefficient. The determinations of these variables are consistent with those in the theoretical model, which can be found in Appendix 1.

The slopes of the load-transfer curves, k_{b_QP} , corresponding to σ_b , can be calculated by

$$k_{b_QP} = \frac{8}{\pi(1-\nu)D} G_{s,ini} \beta \quad (9)$$

where β is a coefficient that can be calculated by

$$\beta = \frac{1}{2} \left(\sqrt{\frac{\sigma_{b,max} - \sigma_{b0}}{\sigma_{b,max}}} + \sqrt{\frac{\sigma_{b,max} - \sigma_{b0} - \Delta F_{M_QP}}{\sigma_{b,max}}} \right) \quad (10)$$

where $\sigma_{b,max}$ is the maximum base pressure of the pile; σ_{b0} is the initial stress acting on the pile base; and ΔF_{M_QP} is the loss of the shaft friction of the pile during the lateral loading process. More details for the calculation of ΔF_{M_QP} can be seen in simplification (f) below.

As the distribution of the normal pressures acting on the pile in each cycle are the same, the depths of points K (K') and M (M') (see Fig. 5(b)) are

$$D_K = \frac{2AD_{RC} + K_0^2\gamma^2 - \sqrt{K_0^4\gamma^4 + 4AD_{RC}K_0^2\gamma^2}}{2A} \quad (11)$$

$$D_M = \frac{2AD_{RC} + K_0^2\gamma^2 + \sqrt{K_0^4\gamma^4 + 4AD_{RC}K_0^2\gamma^2}}{2A} \quad (12)$$

Fig. 6. Simplification of the friction core: (a) friction core according to the theoretical model; (b) friction core after simplification

$$A = \frac{4m^2\sigma_r\gamma a^2(1+2K_0)}{3\pi^2 D^2(1-\nu^2)^2(D_{RC} + e_L)^2} \quad (13)$$

where D_{RC} is the depth of the RC; and e_L is the eccentricity of the lateral load.

The linear load-transfer curves also result in a constant stiffness ratio, K_{b_QP}/K_{s_QP} , where K_{b_QP} and K_{s_QP} are the stiffness contributions of the pile base and pile shaft against the vertical displacements, respectively. The lost shaft friction is transmitted to the pile base at a constant ratio, r , which can be calculated by

$$\eta_{QP} = \frac{K_{b_QP}}{K_{b_QP} + K_{s_QP}} \quad (14)$$

where K_{b_QP} can be calculated by

$$K_{b_QP} = \pi r^2 k_{b_QP} \quad (15)$$

where r is the radius of the pile.

- (b) The stiffness of the pile shaft resisting the vertical displacement considers the contribution of only the front side. K_{s_QP} can be calculated by

$$K_{s_QP} = \frac{\pi r m \sigma_r^{0.5} L^{3/2} \sqrt{(1+2K_0)\gamma}}{12\sqrt{3}(1+\nu)} \quad (16)$$

- (c) The shaft friction of the pile in the friction core was assumed to be fully mobilised, which means that the mobilised shaft friction in the friction core is equal to the limit shaft friction and proportional to the normal pressure.

- (d) The morphology of the friction core with an accurate profile from the theoretical model, as shown in Fig. 6(a), is simplified to that shown in Fig. 6(b).
 (e) The distribution of the normal pressures acting on slice abcd of the pile in the friction core is assumed to be linear, as shown in Fig. 6(b). The vertical shear stress acting on slice abcd at depth D_1 , τ_{v,D_1-0} , can be calculated by

$$\tau_{v,D_1-0} = \left(\frac{D_1 - D_K}{D_{RC} - D_K} \right) \tau_{v,D_{RC}} \quad (17)$$

where $\tau_{v,D_{RC}}$ is the shaft friction at depth D_{RC} and can be calculated by

$$\tau_{v,D_{RC}} = \tan \delta \times K_0 \gamma D_{RC} \quad (18)$$

- (f) The distribution of the normal pressures acting on the pile shafts at the same depth was assumed to be cosinusoidal, as shown in Fig. 6(b). The vertical shear stress acting on the pile at depth D_1 is defined as $\tau_{v,D_1-\theta}$, which can be calculated by

$$\tau_{v,D_1-\theta} = \tau_{v,D_1_ini} - (\tau_{v,D_1_ini} - \tau_{v,D_1-0}) |\cos \theta| \quad (19)$$

where τ_{v,D_1_ini} can be calculated by

$$\tau_{v,D_1_ini} = \tan \delta \times K_0 \gamma D_1 \quad (20)$$

The settlement of a pile under cyclic lateral loads for quick prediction after j half-cycle movements, S_{j_QP} , can be expressed as

$$S_{j_QP} = \frac{\Delta F_{M_QP} \eta_{QP}}{2K_{b_QP}}, \quad j = 1$$

$$S_{j_QP} = \frac{\Delta F_{M_QP} \left\{ \eta_{QP} + (2 - \eta_{QP}) [1 - (1 - \eta_{QP})^{j-1}] \right\}}{2K_{b_QP}}, \quad j \geq 2 \quad (21)$$

where ΔF_{M_QP} can be calculated by

$$\Delta F_{M_QP} = F_{M0} - r \tan \delta K_0 \gamma \left\{ (D_{RC} - D_K) [\pi D_{RC} + (\pi - 2) D_K] + [\min(L, D_M) - D_{RC}] [\pi D_{RC} + (\pi - 2) \min(L, D_M)] \right\} \quad (22)$$

where F_{M0} is the mobilised shaft friction of the pile before cyclic lateral loading.

The results from the quick prediction are also presented in Fig. 4. It can be found that the results of the quick prediction were slightly larger than those from the centrifuge model test and the theoretical model. The predictions from the proposed quick prediction formula are relatively conservative.

SETTLEMENT-CONTROLLED DESIGN

As discussed in earlier section 'Validation', the transmitted shaft friction to the pile base may exceed the redundancy of the vertical bearing capacity of the pile base if F_v or a is very large. The settlement of the pile may become divergent with respect to the increase in cycle number. Two sets of calculations were performed to show this possibility. Three cases were considered with an initial tip resistance, f_{b0} , of 60 N; an initial mobilised shaft friction to the initial tip resistance ratio, $(F_v - f_{b0})/f_{b0}$, of 0.43; and values of a of 0.6 mm, 0.7 mm and 0.8 mm. The divergent settlements plotted against cycle number curves are shown in Fig. 7(a). When a was 0.6 mm, the settlement of the pile was convergent. When a was 0.8 mm, the settlement of the pile was divergent. When a was 0.7 mm, the pile was in a limit

Fig. 7. Divergent settlement due to increasing F_v or a : (a) increasing a ; (b) increasing F_v

state, in which a very small increment in a would induce divergent pile settlement. The settlements against cycle number curves with a of 0.7 mm, f_{b0} of 60 N and $(F_v - f_{b0})/f_{b0}$ of 0.39, 0.43 and 0.47 are shown in Fig. 7(b). When $(F_v - f_{b0})/f_{b0}$ was 0.39, the settlement of the pile was convergent. When $(F_v - f_{b0})/f_{b0}$ was 0.47, the settlement of the pile was divergent. Similarly, when $(F_v - f_{b0})/f_{b0}$ was 0.43, the pile was in a limit state, in which a very small increment in F_v would induce divergent pile settlement.

Inspired by the above-mentioned problem, a new concept, 'settlement-controlled design', for a pile under cyclic lateral loads, was put forward (see Appendix 2). A three-dimensional coordinate system is first established as shown in Fig. 8. The three coordinates are set as a/D , $(F_v - f_{b0})/f_{b0}$ and s_v/D where $(F_v - f_{b0})/f_{b0}$, a/D and s_v/D represent the apportionment of the vertical loads on the pile, the magnitude of the horizontal displacements and the settlement of a pile after many cycles of lateral loads, respectively. In Fig. 8, $l_{0.017}$, $l_{0.033}$, $l_{0.050}$ and $l_{0.067}$ are the contour lines when s_v/D is 0.017, 0.033, 0.050 and 0.067, respectively. The contour lines were obtained from the theoretical model with an f_{b0} of 60 N, and the other parameters were the same

Fig. 8. Contour lines of s_v/D

as those in the centrifuge model tests. All settlement contour lines are on surface OABCD. Line CB on surface OABCD is the contour line of the maximum s_v/D . Therefore, on line CB, the pile is in a limit state, and any increments in a or F_v can lead to divergent settlement.

A design plane could be obtained by projecting the surface in Fig. 8 on plane xOy , as shown in Fig. 9. On the design plane, point O on the $(F_v - f_{b0})/f_{b0}$ -axis indicates that the mobilised shaft friction is zero, and point F represents that the limit shaft friction of the pile is mobilised. Therefore, any point $(a/D, (F_v - f_{b0})/f_{b0})$ can be taken as a design-state point of the pile.

The design plane was divided into three zones by two lines, EI and GH. Line EI is a straight line that is parallel to the a/D -axis. The value of point E on the $(F_v - f_{b0})/f_{b0}$ -axis can be calculated by

$$E_{(F_v - f_{b0})/f_{b0}} = \frac{f_{b, \max} - f_{b0}}{f_{b0}} \quad (23)$$

where $f_{b, \max}$ is the maximum tip resistance.

Equation (23) demonstrates that point E represents the situation where the initial mobilised shaft friction of the pile is equal to the redundant bearing capacity of the pile base. Line GH is composed of a series of points, on which the pile would be in the limit state after many cyclic lateral loads.

When the design-state point of the pile was in the zone below line EI, the settlement of the pile would converge; when the design-state point of the pile was in the zone above line GH, the settlement of the pile would diverge; therefore, the two zones were called the convergent zone and divergent zone, respectively. If the design-state point of the pile was in the zone between lines EI and GH, the settlement of the pile was conditionally convergent and dependent on both $(F_v - f_{b0})/f_{b0}$ and a/D . Therefore, this zone was termed the conditionally convergent zone.

It should be noted that current industry standards do not consider the coupling effect of cyclic lateral loads on the vertical performance of piles. The limit state line for the design-state point was FJ (in other words, the design state was acceptable as long as $(F_v - f_{b0})/f_{b0}$ was smaller than the value of F on the $(F_v - f_{b0})/f_{b0}$ -axis). Under this circumstance, if the design-state point is located above line EI, divergent pile settlement might be produced under cyclic lateral loads according to the calculations presented in this study. This could affect both the service performance and the safety of the superstructures, which ought to be considered in the design stage.

Fig. 9. Design plane for a pile under cyclic lateral loads

It should be noted that the theoretical framework presented in this study was established for piles driven in dry sand. For a pile driven in wet sand or clay, the settlement behaviour of the pile can be more complex. For instance, if the pile is driven in wet sand, soil liquefaction should be considered, whereas if the pile is driven in clay, pile-ground gaps can be generated under cyclic lateral loads (Zhang *et al.*, 2011; Liu *et al.*, 2018) and should be considered when predicting the settlement of a cyclic laterally loaded pile. In truth, substantial work is still necessary to incorporate these characteristics into the theoretical model.

CONCLUSIONS

In this study, a theoretical model using the load-transfer method was proposed for the settlement prediction of cyclic laterally loaded piles in dry sand. A simple formula was also established to predict pile settlement quickly in practical engineering. A new concept of 'settlement-controlled design' was proposed to advance the methodology for the design of a pile by considering potential settlement after many cycles of lateral loads. Based on these studies, the following conclusions can be drawn.

- The settlement of a pile under cyclic lateral loads can be attributed to the transmission of the lost mobilised shaft friction to the pile base.
- Theoretical results can provide a reasonable estimate of the pile settlement under cyclic lateral loads. A friction core is revealed in which the mobilised shaft friction is finally maintained at a certain level under cyclic lateral loads. Both the shape and the size of the friction core are closely related to the magnitude of the horizontal displacement.
- The quick prediction formula is a simple and explicit expression with slightly conservative results.
- With regard to the division of the design plane, it was suggested that the design-state points should be limited to the convergent zone to eliminate the risk of non-convergent prediction of pile settlement.

DATA AVAILABILITY STATEMENT

Some or all data used in this study are available from the corresponding author upon reasonable request.

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APPENDIX 1. DETAILS OF THE THEORETICAL MODEL

Load-transfer curve for σ_n

- On the front side, the relationship curve between the normal pressure, $\sigma_{n,f}$, and the normal displacement of the soil, $\omega_{n,f}$, as shown in Fig. 10(a), can be expressed by

$$\sigma_{n,f} = \begin{cases} \sigma_{n,\max} - \frac{k_{n,\text{ini}} (\omega_{n,f} - \omega_{n,f,\max})^2}{2 \omega_{n,f,\max}}, & \text{if } \omega_{n,f} < \omega_{n,f,\max} \\ \sigma_{n,\max}, & \text{otherwise} \end{cases} \quad (24)$$

where $k_{n,\text{ini}}$ is the initial gradient of the curves and can be expressed by (Li *et al.*, 2012)

$$k_{n,\text{ini}} = \frac{8}{\pi(1-\nu)D_p} G_{s,\text{ini}} \quad (25)$$

where D_p is the diameter of the pile (which was 0.006 m in the present study); ν is Poisson's ratio of the soil (which was 0.2); and $G_{s,\text{ini}}$ is the initial shear modulus of the soil and was calculated according to Li *et al.* (2012)

$$G_{s,\text{ini}} = \chi G_0 \quad (26)$$

where G_0 is the shear modulus of the soil; and χ is a coefficient that reflects the effect of the pile installation procedure on the shear modulus of the soil. In this study, the model pile was installed in a wished-in-place manner, so the effect of the pile installation procedure on the soil properties was neglected, and χ was 1. The shear modulus of the soil, G_0 , was obtained according to the Young's modulus, E_s , of the soil, and their relationship was

$$G_0 = \frac{E_s}{2(1+\nu)} \quad (27)$$

Young's modulus, E_s , of the sand could be determined by the equation presented by Janbu (1998)

$$E_s = m \sigma_r \left(\frac{\sigma'}{\sigma_r} \right)^{1-j} \quad (28)$$

where m is the modulus number and was 210 in the present study, according to Fellenius (2020); σ' is the mean effective stress of the soil; j is the stress exponent and was 0.5 in this study, according to Fellenius (2020); and σ_r is the reference stress, which is a constant and was 100 kPa in this study. In equation (24), $\sigma_{n,\max}$ is the maximum of the normal pressure and can be calculated according to Zhang *et al.* (2005)

$$\sigma_{n,\max} = K_p^2 \gamma z \quad (29)$$

where γ is the unit weight of the soil and can be calculated by

$$\gamma = \rho g \quad (30)$$

where ρ is the dry density of the sand and was 1542 kg/m³ in this study. In equation (29), K_p is the passive earth pressure coefficient and can be calculated by

$$K_p = \tan^2 \left(45^\circ + \frac{\phi}{2} \right) \quad (31)$$

where ϕ is the friction angle of the soil and was 31° in this study (Ishihara, 1993). In equation (24), $\omega_{n,f,\max}$ is the displacement required to mobilise the maximum stresses of the soil and can be calculated by

$$\omega_{n,f,\max} = \frac{2\sigma_{n,\max}}{k_{n,\text{ini}}} \quad (32)$$

When the pile moved by da , $\omega_{n,f}$ could be determined based on geometry. According to Chari & Meyerhof (1983) and Prasad & Chari (1999), the pile in this study could be considered a rigid pile. Thus, the displacement of a point on slice abcd, $\omega_{n,f,0}$, is shown in Fig. 11(a) and can be calculated by

$$\omega_{n,f,0} = \left| \frac{D_R - D}{D_R + e_L} \right| da \quad (33)$$

where e_L is the eccentricity of the lateral load and was 50 mm in this study. D_R is the depth of the RC and can be calculated according to Prasad & Chari (1999)

$$D_R = [- (0.567 L_p + 2.7 e_L) + (5.307 L_p^2 + 7.29 e_L^2 + 10.541 e_L L_p)^{0.5}] / 2.1996 \quad (34)$$

where L_p is the embedded length of the pile and was 100 mm in this study. According to equation (34), the value of D_R was 73 mm.

Fig. 10. Load-transfer curves: (a) $\sigma_{n,f}-\omega_{n,f}$; (b) $\sigma_{n,b}-\omega_{n,b}$; (c) $\tau_h-\omega_h$; (d) $\tau_v-\omega_v$; (e) σ_b-s_v

The displacement of the pile on the front side, $\omega_{n,f}$, is shown in Fig. 11(b) and can be calculated by

$$\omega_{n,f} = |\omega_{n,f,0} \cos \theta| \quad (35)$$

- (b) On the back side of the pile, the relationship curve between the normal pressure, $\sigma_{n,b}$, and the normal displacement of the soil, $\omega_{n,b}$, is shown in Fig. 10(b) and can be expressed by

$$\sigma_{n,b} = \begin{cases} \frac{k_{n,ini}}{2} \frac{(\omega_{n,b} - \omega_{n,b,max})^2}{\omega_{n,b,max}}, & \text{if } \omega_n < \omega_{n,b,max} \\ 0, & \text{otherwise} \end{cases} \quad (36)$$

where $\omega_{n,b,max}$ is the displacement when the stress is reduced to 0 and can be expressed by

$$\omega_{n,b,max} = \frac{2\sigma_{n,max}}{k_{n,ini}} \quad (37)$$

$\omega_{n,b}$ is shown in Fig. 11(b) and can be calculated by

$$\omega_{n,b} = |\omega_{n,f,0} \cos \theta| \quad (38)$$

Load-transfer curve for τ_h

The relationship curve between the horizontal shear stress, τ_h , and the normalised shearing displacement of the soil, ω_h/r , is shown in Fig. 10(c) and can be expressed by

$$\tau_h = \begin{cases} \tau_{h,max} - \frac{k_{h,ini}}{2} \frac{(\omega_h - \omega_{h,max})^2}{\omega_{h,max}r}, & \text{if } \omega_h < \omega_{h,max} \\ \tau_{h,max}, & \text{otherwise} \end{cases} \quad (39)$$

where r is the radius of the pile, and $k_{h,ini}$ is the initial gradient of the curves, which can be calculated by Li *et al.* (2012)

$$k_{h,ini} = \frac{G_{s,ini}}{\zeta} \quad (40)$$

where ζ is the dimensionless coefficient and is equal to 4 (Li *et al.*, 2012) in equation (24) (equation (39)), and $\tau_{h,max}$ is the maximum horizontal shear stress and can be calculated by

$$\tau_{h,max} = \sigma_n \tan \delta \quad (41)$$

where σ_n is the normal pressure, σ_n is equal to $\sigma_{n,f}$ on the front side of the pile, σ_n is equal to $\sigma_{n,b}$ on the back side of the pile, and δ is the interface friction angle between the pile and soil and was 0.8ϕ in the present study, following Zhang *et al.* (2005). In equation (24) (equation (39)), $\omega_{h,max}$ is the shearing displacement required to mobilise the maximum horizontal shearing stress and can be calculated by

$$\omega_{h,max} = \frac{2r\tau_{h,max}}{k_{h,ini}} \quad (42)$$

ω_h is shown in Fig. 11(b) and can be calculated by

$$\omega_h = |\omega_{n,f,0} \sin \theta| \quad (43)$$

Load-transfer curve for τ_v

The relationship curve between the vertical shear stress, τ_v , and the normalised shearing displacement of the soil, ω_v/r , is shown in

(a)

(b)

Fig. 11. Derivation of ω_{n_f} , ω_{n_b} and ω_h based on da : (a) relationship between da and $\omega_{n_f,0}$; (b) relationship between $\omega_{n_f,0}$ and ω_{n_f} , ω_{n_b} , ω_h

Fig. 10(d) and can be expressed by

$$\tau_v = \begin{cases} \tau_{v,max} - \frac{k_{v,ini}}{2} \frac{(\omega_v - \omega_{v,max})^2}{\omega_{h,max} r}, & \text{if } \omega_v < \omega_{v,max} \\ \tau_{v,max}, & \text{otherwise} \end{cases} \quad (44)$$

where $k_{v,ini}$ is the initial gradient of the curves and can be calculated by

$$k_{v,ini} = \frac{G_{s,ini}}{\varsigma} \quad (45)$$

In equation (24) (equation (44)), $\tau_{v,max}$ is the maximum vertical shear stress and can be obtained by

$$\tau_{v,max} = \sigma_n \tan \delta \quad (46)$$

$\omega_{v,max}$ is the vertical shear displacement required to mobilise the maximum vertical shear stresses and can be calculated by

$$\omega_{v,max} = \frac{2r\tau_{v,max}}{k_{v,ini}} \quad (47)$$

Because the pile in this study was considered a rigid pile, τ_v was equal to the settlement of the pile for each movement of da .

Load-transfer curve for σ_b

The relationship curve between the base pressure, σ_b , and the settlement of the pile, s_v , is shown in Fig. 10(e) and can be expressed by

$$\sigma_b = \begin{cases} \sigma_{b,max} - \frac{k_{b,ini}}{2} \frac{(s_v - s_{v,max})^2}{s_{v,max}}, & \text{if } s_v < s_{v,max} \\ \sigma_{b,max}, & \text{otherwise} \end{cases} \quad (48)$$

where $k_{b,ini}$ is the initial gradient of the curves and can be expressed according to Li *et al.* (2012)

$$k_{b,ini} = \frac{8}{\pi(1-\nu)D} G_{s,ini} \quad (49)$$

In equation (24) (equation (48)), $s_{v,max}$ is the displacement required to mobilise the maximum stress of the base pressure and can be calculated by Li *et al.* (2012)

$$s_{v,max} = \lambda D \quad (50)$$

where λ is the coefficient that reflects the effect of the pile installation methods on the capacities of the base soil and was 0.2 in this study, according to Li *et al.* (2012). In equation (24) (equation (48)), $\sigma_{b,max}$ is the maximum base pressure and can be calculated by

$$\sigma_{b,max} = \frac{k_{b,ini}s_{v,max}}{2} \quad (51)$$

In this study, all calculations of the pile settlement were implemented using Matlab with self-coded programs.

APPENDIX 2. STEPS FOR SETTLEMENT-CONTROLLED DESIGN

The ‘settlement-controlled design’ steps are shown in Fig. 12. In a traditional pile design, the designers are mainly concerned as to whether the pile capacities (vertical capacity and horizontal capacity) are sufficient – that is, larger than the external loads (vertical load, F_v , and horizontal load, F_h). However, this study found that the settlement of the pile under cyclic lateral loading should also be considered. In the settlement-controlled pile design, judgements can be made first according to the design plane shown in Fig. 9. This helps avoid non-convergent settlements of the pile under cyclic lateral loads. Then, the settlements of the pile can be

Fig. 12. Flow chart of the settlement-controlled pile design

calculated according to the proposed theoretical model. If the value of the predicted settlements is acceptable, the design is completed; otherwise, the design should be further optimised (e.g. soil parameters and pile dimensions can be changed).

NOTATION

a	amplitude of horizontal displacement of cyclic lateral loads
a_i	horizontal displacement of pile at the i th half-cycle movement
D_p	diameter of pile
D_{RC}	depth of rotation centre (RC)
$dF_{f,da}$	loss in mobilised shaft friction in a da movement
$dF_{f,da,b}$	forces transmitted to pile base in a da movement
ds_v	pile settlement in a da movement
E_s	Young's modulus of the soil
e_L	eccentricity of the lateral load
F_h	horizontal load
F_{M0}	mobilised shaft friction of pile before cyclic lateral loading
F_v	vertical load
f_b	force acting on pile base
f_{b0}	initial tip resistance
$f_{b,max}$	maximum tip resistance
G_0	shear modulus of the soil
$G_{s,ini}$	initial shear modulus of the soil
g	acceleration of gravity
i	number of half-cycle movements
j	stress exponent
$K_{b,da}$	contribution of pile base to stiffness of pile against vertical displacement
K_p	passive earth pressure coefficient
$K_{s,da}$	contribution of pile shaft to stiffness of pile against vertical displacement
$k_{b,ini}$	initial gradient of the load-transfer curve for σ_b
$k_{h,ini}$	initial gradient of the load-transfer curve for τ_h
k_{h_QP}	slope of the load-transfer curves corresponding to τ_h in quick prediction formula
$k_{n,ini}$	initial gradient of the load-transfer curve for $\sigma_{n,f}$
k_{n_QP}	slope of the load-transfer curves corresponding to σ_n in quick prediction formula
$k_{v,ini}$	initial gradient of the load-transfer curve for τ_v
k_{v_QP}	slope of the load-transfer curves corresponding to τ_v in quick prediction formula
L_p	embedded length of the pile
m	modulus number
n	cycle number of cyclic lateral loads
r	radius of the pile
S	surface of the pile shaft
S_{j_QP}	settlement of a pile under cyclic lateral loads for quick prediction after j half-cycle movements
s_v	pile settlement
$s_{v,max}$	displacement required to mobilise the maximum stress of the base pressure ($\sigma_{b,max}$)
α	sign function
β	dimensionless coefficient
γ	unit weight of the soil
ΔF_{M_QP}	loss of shaft friction of pile during the lateral loading process in quick prediction formula
δ	friction angle of pile–soil interface
ζ	dimensionless coefficient
η_{QP}	ratio
λ	coefficient that reflects the effect of the pile installation methods on the capacities of the base soil
ν	Poisson's ratio of the sand
ρ	dry density of the sand
σ'	mean effective stress of the soil
σ'_t	reference stress
σ_b	base pressure
$\sigma_{b,max}$	maximum base pressure of the pile
σ_n	normal pressure acting on the pile shaft
$\sigma_{n,b}$	normal pressure acting on the back side of the pile shaft
$\sigma_{n,da}$	normal pressure after pile head moves horizontally at a distance of da

$\sigma_{n,f}$	normal pressure acting on the front side of the pile shaft
$\sigma_{n,max}$	maximum of the normal pressure
τ_h	horizontal shear stress acting on pile shaft
$\tau_{h,max}$	maximum horizontal shear stress
τ_v	vertical shear stress acting on pile shaft (i.e. mobilised shaft friction)
τ_{v,D_1-0}	vertical shear stress acting on slice abcd at depth D_1 in quick prediction formula
$\tau_{v,D_1-\theta}$	vertical shear stress acting on the pile at depth D_1 in quick prediction formula
τ_{v,D_1-ini}	initial shear stress acting on the pile at depth D_1 in quick prediction formula
$\tau_{v,D_{RC}}$	shaft friction at depth D_{RC} in quick prediction formula
$\tau_{v,max}$	maximum vertical shear stress
ϕ	friction angle of the soil
χ	coefficient that reflects the effect of the pile installation procedure on the shear modulus of the soil
ω_h	horizontal shearing displacement of the soil
$\omega_{h,max}$	shearing displacement required to mobilise the maximum horizontal shearing stress ($\tau_{h,max}$)
$\omega_{n,b}$	normal displacement of the soil on the back side
$\omega_{n,b,max}$	displacement when the normal stress ($\sigma_{n,b}$) is reduced to 0
$\omega_{n,f}$	normal displacement of the soil on the front side
$\omega_{n,f,max}$	displacement required to mobilise the maximum stresses ($\sigma_{n,max}$) of the soil
ω_v	vertical shearing displacement of the soil
$\omega_{v,max}$	vertical shear displacement required to mobilise the maximum vertical shear stresses ($\tau_{v,max}$)

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