Non-dimensional solutions for the stabilising piles in landslides in layered cohesive soils considering non-linear soil–pile interactions

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The pile response is of great importance to the effectiveness of a row of piles used as a landslide mitigation method. However, the current classifications of the pile responses and failure modes do not consider the effects of soil displacement and the relative pile rigidity. An attempt is made in this paper to improve this using a non-dimensional model based on simplified ground consisted of two layers of cohesive soils. A hyperbolic p – y model is adopted to consider non-linear soil reactions. The calculations of three case histories show that the proposed model predicts the pile responses well. The dependencies for obtaining a pile response with maximum pile resistance have been identified and the influencing factors have been examined. The efficacy of the proposed approach has been demonstrated by the satisfactory result that it achieves in reproducing the more complex three-dimensional finite-element analyses. The application of the non-dimensional solutions indicates that it can increase the efficiency of the preliminary pile design.

KEYWORDS: clays; finite-element modelling; landslides; piles & piling; soil stabilisation; soil/structure interaction

INTRODUCTION

Mass movements are catastrophic events which threaten both infrastructure and human lives. One of the main mitigation methods against these events is to install rows of piles, transferring the sliding force from the moving mass to the stable ground underneath. Each pile interacts with the adjacent piles, giving rise to an arching effect. The main factor influencing the soil arching effect is the axis-to-axis pile spacing. This factor has been studied by many research groups, using various approaches (Chen & Martin, 2002; Liang & Zeng, 2002; Durrani *et al.*, 2008; Yoon & Ellis, 2009; Ellis *et al.*, 2010; Kourkoulis *et al.*, 2011; Li *et al.*, 2013). From these works, it is commonly concluded that the soil arching between adjacent piles can be maintained when the pile spacing is smaller than a limit value, often referred as about 4 times the pile diameter. With a larger pile spacing, the soil arching effect diminishes and the piles behave almost as single isolated elements. Such a limit value is of significant practical importance in design methods; it can be thought of as the most cost-effective arrangement (Kourkoulis *et al.*, 2011). This study deals with the response calculation of a row of stabilising piles, including the pile deflection, internal forces and the profile of the pile–soil contact forces. As regards the pile spacing, the limit value is assumed

and empirical results of the maximum pile resistance are adopted.

The pile response is of great importance to the effectiveness of the stabilisation method and the associated potential failure mode. By performing limit equilibrium analysis, Viggiani (1981) identified six possible failure mechanisms (mechanisms A, B, C for non-yielding piles and mechanisms B1, BY, B2 for yielding piles) depending on two parameters – namely, the ratio of the embedded length to the pile length in the sliding layer (λ) and the ratio of the soil lateral capacity between the sliding and stable layers (χ). As shown in Fig. 1 ($\chi = 0.5$), mode A refers to the case where the whole pile translates together with the sliding soil, which corresponds to the ‘short pile mode’ by Poulos (1995), meaning that the length of pile in the stable soil is relatively shallow and the yield value of the pile–soil interaction is only attained in the stable soil layer. This failure mode must be avoided because limited contribution to soil stability is expected (Di Laora *et al.*, 2017). Mode C, also the ‘flow mode’ by Poulos (1995), occurs when the sliding layer is relatively shallow and embedment is sufficiently long. The unstable soil becomes plastic and flows around the pile, providing the maximum resistance from the pile for the slope. Mode B describes the cases between the above two extreme cases (also the ‘intermediate mode’ by Poulos (1995)), in which soil failure occurs both above and below the slip surface. As the one most likely to be associated with a plastic hinge, mode B could become mode B1, BY or B2 depending on where the yielding moment of the pile cross-section is first attained.

Figure 1 also depicts that the pile resistance becomes much lower for modes A and B (especially B1 and BY). As a result, slopes reinforced by piles with these pile responses have lower overall stability (Lei & Wu, 2020). With the perspective of resisting soil movement, mode C is preferred, in which the ultimate pile–soil contact pressure in the sliding soil can be applied to the pile as an equivalent loading condition (Ito & Matsui, 1975). In this regard, the framework presented by Viggiani (1981) is convenient in predicting the pile response; however, the inherent drawbacks of the pressure-based methods based on limit equilibrium analyses remain.

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Fig. 1. Illustration of different failure modes identified by Viggiani (1981)

The evolution of the pile response with soil movement and the pile rigidity is not considered (Kanagasabai *et al.*, 2011). As a result, these methods are often not satisfied for free-headed, slender piles (White *et al.*, 2008). To incorporate lateral soil displacements and the pile rigidity into pile response analyses, more sophisticated displacement-based methods are necessary (Poulos, 1973; Byrne *et al.*, 1984; Cai & Ugai, 2003).

A dimensionless pile length βL is often used to determine whether a laterally loaded pile, equivalent to the embedded part of the stabilising pile, behaves rigidly or flexibly (Broms 1964; Poulos 1971; Randolph 1981), where L is the length of the laterally loaded pile, the coefficient $\beta = \sqrt[4]{k_h/(4EI)}$, where k_h is the coefficient of subgrade reaction, E is the pile's Young's modulus and I is the second moment of inertia for the pile's cross-section. Based on a subgrade reaction method with the assumption of linear soil reaction, Cai & Ugai (2003, 2011) have studied the influence of the soil displacement profile on the pile response for flexible and rigid pile segments in the sliding region. In their studies, a dimensionless length $\beta L_1 > 3$ is adopted to identify a flexible pile segment (mode BY), where L_1 is the pile length in the sliding soil. Based on an equivalent load transfer approach, Guo (2013) has developed elastic (sliding layer)–elastic (stable layer) (E–E) solutions, and plastic (sliding layer)–elastic–plastic (stable layer) (P–EP) solutions for the analyses of deep (mode BY) and normal sliding modes (modes B2 and C), respectively. The pile response is calculated by the former solutions when the pile segment in the sliding soil is longer than $1.2L_{c1} + x_{p1}$, where L_{c1} is the critical length of an active long pile in the sliding soil and x_{p1} is the thickness of the plastic zone in the sliding soil. The classifications adopted by Cai & Ugai (2003, 2011) and Guo (2013) provide the scope of application of the simplified calculation methods for long flexible piles, which can also be seen as an exploration of the boundary between modes B2 and BY.

Alternative studies on the responses of stabilising piles employ numerical simulations (Martin & Chen, 2005; Kanagasabai *et al.*, 2011; Kourkoulis *et al.*, 2011), physical models (Guo & Qin, 2010; Lei & Wu, 2020; White *et al.*, 2008; Tang *et al.*, 2014; Li *et al.*, 2016a) and simplified methods (Chow, 1996; Chen & Poulos, 1997), without exploring the boundaries among the failure modes. The stabilisation of a failing slope requires the maximum resistance from the piles, thus identifying the boundary between modes B and C appears to be more useful. This is

achieved by developing a non-dimensional formulation of the subgrade reaction method proposed by Su *et al.* (2018) for the analysis of stabilising piles incorporating a hyperbolic p – y model, where p is the soil–pile contacting pressure and y is the absolute difference between the pile displacement y_p and the soil displacement y_s . Advanced three-dimensional (3D) numerical methods reveal clearly the characteristics of different pile responses (Kanagasabai *et al.*, 2011). The main drawback of such an approach lies in the significant dependency on the model geometry and material properties, which usually demands a large amount of modelling effort for design optimisation. The proposed approach, like other accepted macroscopic methods (Byrne *et al.*, 2020; Z. Salciarini & Tamagnini, 2009; Li *et al.*, 2016b; Page *et al.*, 2018), overcomes the above drawback and is also able to deliver results with comparative accuracy by providing well-calibrated parameters using representative testing and simulation data.

The objectives of this study are: (a) to reveal the evolution of pile response with the sliding soil displacement and the relative pile stiffness, and explore the boundary between mode B and C; (b) to demonstrate the efficacy of the proposed approach by the satisfactory result that it achieves in reproducing the more complex 3D finite-element analyses; and (c) to show the ability of the proposed method to facilitate the design of piles as a mitigation structure.

NON-DIMENSIONAL MODEL FOR STABILISING PILES

Similarly to previous theoretical studies of stabilising piles (Poulos, 1995; Chow, 1996; Cai & Ugai, 2003, 2011), the non-dimensional analysis is based on an idealised ground model. In this section, the idealised ground model for landslides in cohesive soils is introduced. Subsequently, the hyperbolic p – y model and the normalisation method are described. Finally, the p – y model is verified through three reported practical cases.

Idealised ground model for landslides in cohesive soils

For the topic of pile–soil interaction, the accuracy of the analysis depends greatly on the p – y curves, which ideally should be obtained by way of in situ tests. However, measured soil reactions are not always available. In this paper, the authors focus on the case where the landslide occurs in cohesive soils with a constant soil strength profile. Modification should be made when using the proposed approach for the case where the soil strength varies erratically with depth. The geometry of the pile and soil layers is shown in Fig. 2. The soil slides along a slip surface at a depth L_1 ,

Fig. 2. Diagram of the model geometry, the simplified soil movement y_s and the depth profiles of k_h and p_u

dividing the stabilising pile into a passive loading segment (L_1) and an active loading segment (L_2).

In the current analysis two parameters related to the soil reactions are needed: the horizontal subgrade reaction modulus k_h and the ultimate pile–soil contacting pressure p_u . k_h depends not only on the soil elastic modulus E_s , but also on the relative stiffness of the soil–structure interaction system (Zhang & Ahmari, 2013). Many k_h – E_s relations have been proposed by previous studies of laterally loaded piles. As summarised by Zhang & Ahmari (2013), k_h/E_s ranges from 0.48 to 2. The intention in this paper is to study the pile–soil interaction in a non-dimensional scale as a function of the ratio of k_h between the sliding soil and the stable soil. When the value of k_h is needed in the analysis, the aforementioned range of k_h/E_s can be used. The sensitivity of k_h will also be investigated in this paper. E_s is commonly determined from the undrained shear strength, c_u . Poulos (1995) suggested that E_s/c_u lies between 150 and 400. Table 1 lists the values of E_s at different values of c_u given in Zhang & Ahmari (2013). It is postulated that E_s/c_u and the k_h – E_s relation does not vary with depth in each region; therefore, k_h is also assumed constant, as shown in Fig. 2.

The ultimate soil–pile contact pressure p_u in clay is related to the undrained shear strength c_u as follows

$$P_u = N_p c_u \quad (1)$$

where N_p is the limit yield pressure coefficient. Viggiani (1981) suggested that, for the sliding soil above the slip surface, $N_p = [3:4]$, and for the stable soil below the slip surface, $N_p = [8:12]$. The lower value of N_p in the sliding zone is due to its proximity to the ground surface and the weakening of the soil due to the sliding movements (Chow, 1996). However, Poulos (1995) believes that N_p should be similar for both the sliding and stable layers, except the near-surface part. Although the numerical simulation performed in this paper, similarly to previous simulations (Kanagasabai *et al.*, 2011), supports the opinion of Poulos (1995), this topic is not further explored. Apart from that, a group effect such as the arching effect also influences the ultimate soil capacity (Chen & Poulos, 1997; Pan *et al.*, 2002); however, it can be considered in the calibration process. Assuming that N_p does not vary with depth, the distribution of p_u is also constant as plotted in Fig. 2.

Normalisation of the pile flexural equation and the p – y model

The proposed model is a non-dimensional version of the subgrade reaction method (Su *et al.*, 2018). The governing equation for the deflection of a pile facing lateral soil movement is expressed as

$$EI \frac{d^4 y_p}{dz^4} + p = 0 \quad (2)$$

where z is the depth measured from the surface.

The hyperbolic function (Kondner, 1963) is commonly used to describe non-linear soil behaviour. Making use of the

initial horizontal soil subgrade reaction modulus k_h and the ultimate pile–soil contacting pressure p_u , the hyperbolic p – y model takes the following form

$$p = \frac{y}{(1/k_h) + (y/p_u)} \quad (3)$$

According to the normalisation method (Su *et al.*, 2018), the reference displacement y_r , the normalised displacement difference y_n , the normalised soil resistance p_n , the relative stiffness factor T (Reese & Van Impe, 2011) and the normalised depth z_n are defined as follows

$$y_r = \frac{p_u D}{k_h} \quad (4)$$

$$y_n = y_{pn} - y_{sn} = \frac{y_p}{y_r} - \frac{y_s}{y_r} = \frac{y}{y_r} \quad (5)$$

$$p_n = \frac{p}{p_u} \quad (6)$$

$$T = \left(\frac{EI}{k_h} \right)^{1/4} \quad (7)$$

$$z_n = \frac{z}{T} \quad (8)$$

where D is the pile diameter; y_{pn} and y_{sn} are the normalised pile and soil displacements, respectively. Substituting equations (4)–(6) into equation (3) yields

$$p_n = \frac{y_n}{1 + y_n} \quad (9)$$

Equation (9) is plotted in Fig. 3 together with the p – y model for the soft clay suggested by the American Petroleum Institute (API, 2000). Note that the displacement at which the pile–soil contact pressure reaches 50% of the ultimate value (y_{50}) coincides in both models, since $p_n = 0.5$ when $y_n = 1$ (i.e. $y = y_r$). Both p – y curves agree well in the range of y_n between 0 and 5. When y_n is larger than 5, the API model increases rapidly, reaching a maximum, while the hyperbolic model approaches the line $p_n = 1$ slowly. The difference between the two models is less than 5% when y_n is larger than ~ 20 , which is the displacement range considered in this study. Note that it is always possible to use

Fig. 3. Non-dimensional p – y relationships

Table 1. Clay elastic modulus based on its undrained shear strength (Zhang & Ahmari, 2013)

Clay consistency	Undrained shear strength, c_u : kPa	Elastic modulus, E_s : MPa
Very soft clay	2–25	0.5–5
Soft clay	25–50	5–20
Medium	50–100	20–50
Stiff	100–200	50–100

a more complex model in the description of the soil behaviour. For instance, the H-model proposed by Su *et al.* (2018), which needs an additional model parameter, can be used to study the influence of soil non-linearity. For the purpose of this study, the simplicity of the hyperbolic model is appreciated.

When piles are used as a landslide stabilisation method, the soil–pile interaction in the sliding region is of most interest. Therefore, the sliding soil is taken as the reference for the normalisation in equations (4)–(8). For the stable layer, two parameters are introduced

$$R_k = \frac{k_{h1}}{k_{h2}} \quad (10)$$

$$R_p = \frac{p_{u1}}{p_{u2}} \quad (11)$$

where subscripts 1 and 2 denote the parameter for the sliding and stable soil layers, respectively. Note that R_p is equivalent to χ in Viggiani (1981). The normalised p – y model for the pile segment in the stable layer is given by

$$p_n = \frac{y_n}{R_k + R_p y_n} \quad (12)$$

Substituting equations (4)–(8) into equation (2) results in the normalised pile flexural equation

$$\frac{d^4 y_{pn}}{dz_n^4} + p_n = 0 \quad (13)$$

To solve the pile response numerically, equations (9), (12) and (13) can be rewritten in incremental form as

$$\Delta p_n = \frac{\Delta y_n}{(1 + y_n)^2} \quad (14)$$

$$\Delta p_n = \frac{R_k \Delta y_n}{(R_k + R_p y_n)^2} \quad (15)$$

$$\frac{d^4 \Delta y_{pn}}{dz_n^4} + \Delta p_n = 0 \quad (16)$$

A finite-difference code is developed for solving equations (14)–(16). The soil displacement profile in the sliding region can be obtained through filed measurements. When not available, a constant profile or a linear distribution with angle θ_s (see Fig. 2) can be adopted (Cai & Ugai, 2003). The non-dimensional solution can be scaled into a prototype through the equations listed in Table 2.

Table 2. Conversions from the non-dimensional solution to the prototype scale

Parameters	Conversion formulas
Pile displacement	$y_p = y_{pn} y_r$
Soil displacement	$y_s = y_{sn} y_r$
Depth	$z = z_n T$
Pile length	$L = L_n T$
Soil resistance	$p = p_n p_{ult}$
Rotation of pile cross-section	$\theta = \frac{y_r}{T} \theta_n$
Angle of soil displacement profile	$\theta_s = \frac{y_r}{T} \theta_{sn}$
Bending moment	$M = \frac{EI}{T^2} y_r M_n$
Shear force	$F = \frac{EI}{T^3} y_r F_n$

Model verification and pile response characteristics

Three reported case studies are used to validate the use of the hyperbolic p – y relationship (Esu & D'Elia, 1974; Carrubba *et al.*, 1989; Kalteziotis *et al.*, 1993). The parameters of these cases are summarised in Table 3. The main difference from previous analyses lies in the selection of the k_h/E_s value. Ideally, k_h should be determined using the measured p – y curves by way of equation (4), as y_r corresponds to the relative pile–soil displacement at $p = 0.5p_u$. In the case of Esu & D'Elia (1974), k_h/E_s is set to 2 while the value of 1 is adopted by Chow (1996). Note that k_h may vary in different methods. In the approach of Chow (1996), k_h is the gradient of the linear soil reaction before the ultimate value; therefore, the secant modulus of the soil reaction curve (k_{sec}) should be used. If y_{50} (or y_r) is used to govern the soil reaction, k_{sec} is 50% of the initial tangent modulus in the hyperbolic model. $k_h/E_s = 2$ is also given to the case of Carrubba *et al.* (1989). Special attention

Table 3. Parameters of the case histories

Case history	Esu & D'Elia (1974)		Carrubba <i>et al.</i> (1989)		Kalteziotis <i>et al.</i> (1993)	
Soil region	Sliding	Stable	Sliding	Stable	Sliding	Stable
L_1 or L_2 : m	7.5	22.5	9.5	12.5	3.64*	8.36
c_u : kPa	40	40	30	30	100	356
E_s/c_u	200	200	500	500	150	197
k_h/E_s	2†	2†	2‡	2‡	0.22§	1
p_u/c_u	3	8	3.05	5.6	3	8
y_s : mm	110 (uniform)	—	95 (uniform)	—	Measured	—
$L_1 + L_2$: m	30	30	22	22	12	12
D : m	0.79	0.79	1.2	1.2	1	1
EI : MN m ²	360	360	2035.8	2035.8	1540	1540
T : m	2.18	2.18	2.87	2.87	4.65	4.65
L_{n1} or L_{n2}	3.44	10.33	3.31	4.36	0.79	1.79
R_k	1	1	1	1	0.047	0.047
R_p	0.375	0.375	0.545	0.545	0.105	0.015

*Determined by comparing the measured pile and soil displacements.

†The value 1 is adopted in Chow (1996). It is adjusted to 2 for the hyperbolic model.

‡Not given in Chen & Poulos (1997). It is set to 2 for the hyperbolic model.

§The value 1 is adopted by Chow (1996), in which the influence of adjacent piles due to close pile spacing is intrinsically considered in the calculation. It is adjusted to 0.22 for the hyperbolic model.

is paid in the case of Kalteziotis *et al.* (1993), where two rows of piles with a spacing-to-diameter ratio of 2.5 were installed, which, according to the analysis of Chow (1996), largely

reduces the pile deflection and internal forces. Therefore, k_{h1}/E_s is set to 0.22 to consider this effect and match the measured pile displacement. Fig. 4 shows that the computed

Fig. 4. Comparisons of the calculated results and field measurements: (a) Esu & D'Elia (1974); (b) Carrubba *et al.* (1989); (c) Kalteziotis *et al.* (1993)

pile deflection and internal forces match generally well with field measurements and previous analyses. Note that the purpose of the case studies is to verify that the simplified ground model and hyperbolic p - y model can reproduce the previous observations with sufficient precision. However, in this study the authors do not intend to compare the accuracies of various methods, which may need the bending moment and shear force profiles for each magnitude of soil movement (Guo, 2013).

Some major characteristics of the pile responses are also observed from both the computed and measured data. Figs 4(a) and 4(b) present a typical mode B pile response, while Fig. 4(c) shows a typical mode C response. In the mode B case, the relative pile-soil displacement is significantly larger in the bottom section of the sliding soil than in the upper part. Therefore, the ultimate pile-soil contact pressure is first reached in this section. Moreover, the pile head displacement is larger than the soil displacement, resulting in negative soil pressures in the pile top section. Also, a negative bending moment (M_1) exists from the ground surface to a certain depth below, and the pile cross-sectional rotation is smaller at the pile head than at the slip surface. In the mode C case, by contrast, the soil displacement is much larger than the pile displacement in the whole sliding layer. Neither negative bending moment nor negative pile-soil contacting pressure is observed above the slip surface; and the maximum rotation of the pile cross-section above the slip surface is located at the pile top. Note that in the mode B case, the magnitude of the negative bending moment in the sliding region becomes close to the maximum value in the stable region (mode BY) when the pile becomes relatively more flexible. As discussed earlier, the determination of the pile response case is of great importance in the pile design method. The mode C case creates the least damaging effect of soil movement on the pile and provides the largest resistance for slope stabilisation. However, the mode BY case makes the most use of the pile by generating large bending moments in both sliding and stable layers, which is preferred from the perspective of economy.

FAILURE MODES IDENTIFICATION

As pointed out previously, the identification of the failure modes by Viggiani (1981) does not incorporate the effect of soil displacement (y_{sn}) and relative pile-soil stiffness (L_{n1}). In this section, this is improved by using the proposed non-dimensional formulations.

Failure mode evolution with y_{sn} and L_{n1}

The pile response depends primarily on the normalised pile length (Broms, 1964), which corresponds to L_{n1} in this study. The calculation of L_{n1} is similar to those proposed previously by Broms (1964), Poulos (1971) and Randolph (1981). Previous studies characterise the pile response by the use of some critical values of L_{n1} , which usually does not take into account the influence of soil non-linearity. However, the study of Su *et al.* (2018), which incorporates non-linear soil-pile interaction, shows that the critical L_{n1} for a long pile increases significantly with pile head displacement. Therefore, the study of the effect of L_{n1} should be based on the possible free-field soil displacements.

To reveal the evolution of failure modes with y_{sn} and L_{n1} , the influence of the soil properties should be excluded. For this purpose, $R_k = 1$ and $R_p = 3/8$ (as in the case of Esu & D'Elia (1974)) are used for the calculations. Fig. 5(a) displays the t_u - λ relation (where $t_u = F_n/F_{n(max)}$) provided by Viggiani (1981) at $\chi = 0.375$ and the computed results (solid lines) for various y_{sn} , when $L_{n1} = 1$. A full list of formulations of

Viggiani (1981) refers to Di Laora *et al.* (2017). The variation of λ ($= L_{n2}/L_{n1}$) is achieved by fixing L_{n1} while varying L_{n2} . As y_{sn} increases, the solid lines tend to approach the solution of Viggiani (1981). Several lines can be drawn based on this calculation.

- λ_{ult} is the ultimate value of λ , meaning that further increase of L_{n2} only provides a negligible contribution to t_u (i.e. long pile in the stable layer) as all the solid curves become horizontal with large λ . In this study, λ_{ult} is estimated by searching for the minimum L_{n2} , which provides the t_u value that has less than 0.1% difference to the t_u value when $L_{n2} = 10$. Note that $L_{n2} = 10$ has been proved to provide a mode C behaviour for all the cases calculated in this paper.
- λ_{crit} is the critical value of λ where the maximum bending moment in the sliding layer ($M_{n1(max)}$) equals that in the stable layer ($M_{n2(max)}$) – that is, mode BY. This differs from the outcome of Di Laora *et al.* (2017), in which the failure mode BY is identified as a zone.
- $m_u = M_{yn}/L_{n1}^2$: the contour lines (dashed lines) for the formation of plastic hinges in the pile provide a different level of pile section capacity M_{yn} , and therefore various m_u . It is also possible to determine the limit value $m_{u(lim)} = M_{yn(lim)}/L_{n1}^2$, meaning that additional pile section capacity would be the conservative part.

From Fig. 5(a), the evolution of the failure mechanism with soil displacement y_{sn} for different λ ranges can be revealed. When λ is smaller than about 0.5, modes A or B1 are prescribed. The boundary between modes A and B1 is not further explored considering the small pile resistance. A plastic hinge can only be formed in the sliding layer with a small level of m_u (smaller than about 0.07). When λ lies between about 0.5 and 1, with the increase of y_{sn} , the pile response changes from the type of mode B1 to BY and finally becomes the type of B2. Failure mode depends on m_u . When λ is larger than 1 but smaller than about 1.5, pile response of the type of B2 is expected. A plastic hinge can be formed with m_u smaller than about 0.65. With even larger λ , failure mode C will finally take place, providing a pile section capacity of $m_{u(lim)}$ (about 0.7).

Similar observations are made for various L_{n1} ($L_{n1} = 2$ in Fig. 5(b) and $L_{n1} = 3$ in Fig. 5(c)). However, λ_{ult} becomes much smaller and t_u increases less with the soil displacement. Even though the final critical state seems to be approachable for unlimited soil displacement with pile section capacity larger than $m_{u(lim)}$, the consideration of the pile response at an extremely large y_{sn} is meaningless, as the purpose of the pile installation is to prevent the landslide. For a row of infinitely rigid piles, it would imply that the piles are ineffective if the sliding soil has moved in the direction perpendicular to the pile row for a distance greater than half the pile diameter. Half the pile diameter is used here as the limit for y_{sn} . Therefore, the maximum soil displacement in the non-dimensional model is given by

$$y_{sn(0.5D)} = \frac{D}{2y_r} = \frac{k_{h1}}{2p_{u1}} \quad (17)$$

Equation (17) shows that $y_{sn(0.5D)}$ depends also on the soil properties of the sliding region. To make an example, a case of soft clay is considered ($k_h = 12.5$ MPa; $p_u = 112.5$ kPa), which gives $y_{sn(0.5D)} = 55.6$.

The t_u - λ relationships at $y_{sn(0.5D)}$ for various L_{n1} values are plotted in Fig. 5(d). λ_{crit} seems to be slightly influenced by L_{n1} , while large variation of λ_{ult} is observed. The m_u lines on the left side of λ_{crit} are inclined, whereas on the right side they are horizontal, indicating that $m_{u(lim)}$ depends only on L_{n1} for

Fig. 5. The evolution of pile response with L_{n1} and y_{sn} . Variation of t_u - λ with y_{sn} when: (a) $L_{n1} = 1$; (b) $L_{n1} = 2$; (c) $L_{n1} = 3$. (d) Variation of t_u - λ with L_{n1} at $y_{sn(0.5D)} = 55.6$; (e) determination of $L_{n1(crit)}$

a certain soil displacement limit. Reduction of t_u is significant with increasing L_{n1} , which means that the mode C type pile response may not occur for certain L_{n1} , even though the value of λ suggests that. Since mode C is the preferred pile response for slope stabilisation, it would be convenient to define a critical L_{n1} , which is the maximum L_{n1} to provide the mode C type of pile response at $y_{sn(0.5D)}$. As

observed previously, mode C differs from mode B in the features of relative pile-soil displacement at the pile head and the negative bending moment. Fig. 5(e) shows the contour lines of $t_u = F_n/F_{n(max)} = 0.9$, $y_{pn0}/y_{sn0} = 1$ and $M_{1n(max)}/M_{2n(max)} = 0.01$ with varying L_{n1} and y_{sn} . On the right side of the curves, the pile top displacements are larger than the soil displacements, negative bending moments appear in the

sliding layer and less than 90% of the maximum sliding force is mobilised. As y_{sn} increases, the L_{n1} of the curves increase first rapidly, and then the increasing rate becomes much smaller when y_{sn} is larger than about 30. In the following analysis, $F_n/F_{n(max)} = 0.9$ and $y_{sn} = y_{sn(0.5D)}$ are used to estimate $L_{n1(crit)}$. In the current case, $L_{n1(crit)}$ is about 2.7. Provided $L_{n1(crit)}$, the corresponding λ_{ult} can be obtained in Fig. 5(d) (filled circle), which shows a large difference from the results in Viggiani (1981) (filled square). Note that λ_{ult} may be slightly different, depending on the definition (here, less than 0.1% difference in t_u compared to the case when $L_{n2} = 10$).

Effect of R_k and R_p

Previous analyses are based on a given set of R_k and R_p , the values of which vary with the soil properties in both the sliding and stable layers. Generally, k_h and p_u of the stable soil are higher than that of the sliding layer – that is, both R_k and R_p should be smaller than 1. To study the effect of stable soil, the sliding layer is fixed as a soft clay ($y_{sn(0.5D)} = 55.6$). For each combination of R_k and R_p , $L_{n1(crit)}$ is obtained by assuming a long embedment – that is, $\lambda > \lambda_{ult}$, then λ_{ult} is calculated by fixing $L_{n1} = L_{n1(crit)}$ and varying L_{n2} (filled circle in Fig. 5(d)). The variations of $L_{n1(crit)}$ and λ_{ult} with R_k and R_p are, respectively, shown in Figs 6(a) and 6(b),

indicating that R_p has a much greater influence than R_k . The reason for this is that at a soil movement of $0.5D$, the maximum pile–soil contact pressure has been mobilised for the major part of the pile in the sliding layer. As a result, R_p controls the pile response. Generally, $L_{n1(crit)}$ increases and λ_{ult} decreases with reducing R_k and R_p – that is, stiffer, stable soil tends to drag the pile response towards the type of mode C and shorter embedment length is needed. It is also observed that λ_{ult} varies linearly with R_p and the λ_{ult} – R_p line approaches the solution of Viggiani (1981) (filled square in Fig. 5(d)) at an extremely low R_k . A greater effect of R_k on the pile response is expected for the modes A and B.

When R_k and R_p vary due to the change of the sliding soil (i.e. k_{h1} and p_{u1}), the non-dimensional parameters $y_{sn(0.5D)}$, L_{n1} and L_{n2} also vary. Calculations of each combination of R_k and R_p are performed by using a stiff clay ($k_{h2} = 75$ MPa and $p_{u2} = 1.2$ MPa) as the stable layer with $y_{sn(0.5D)}$ calculated by equation (17). The variations of $L_{n1(crit)}$ and λ_{ult} are, respectively, plotted in Figs 6(c) and 6(d), which also suggests the dominant role of R_p . The variation ranges of $L_{n1(crit)}$ and λ_{ult} in Figs 6(c) and 6(d) are larger than those in Figs 6(a) and 6(b), meaning that the sliding layer has a greater impact than the stable layer. Therefore, the p_u profile in the sliding soil may need more attention in the pile design routine. A large difference between λ_{ult} and the solution of Viggiani (1981) is observed and λ_{ult} seems not to be affected by R_k ,

Fig. 6. The effect of R_k and R_p . Variation of (a) $L_{n1(crit)}$ and (b) λ_{ult} with R_k and R_p due to the change of stable soil. Variation of (c) $L_{n1(crit)}$ and (d) λ_{ult} with R_k and R_p due to the change of sliding soil

except for those cases where R_k is much smaller than R_p , which, however, are unlikely to occur. In general, overestimated R_k or R_p results in underestimated $L_{n1(crit)}$ and overestimated λ_{ult} , both resulting in conservative design; that is, a lower sliding force than expected.

Overall, the solution of Viggiani (1981), as expected from its assumptions of rigid pile and limit pile–soil contact pressure, can be seen as an upper bound for the t_u – λ space (Fig. 5(d)). As a result, it is likely to provide a non-conservative design – in other words, overestimated t_u and underestimated λ . The proposed non-dimensional solutions are able to take into account the effect of soil displacement and relative pile rigidity. Once the soil parameters are estimated (R_k and R_p), non-dimensional solutions are calculated and design charts are obtained covering the whole range of y_{sn} , L_{n1} and L_{n2} (i.e. including the whole range of D , EI , L_1 and L_2), which has great potential in the design optimisation process.

DISCUSSION ON THE ACCURACY

The use of the proposed method may throw up some challenges regarding its accuracy, including the soil displacement profile, the c_u profile, the k_h – c_u relationship, the p_u – c_u relationship and so on.

Field measurements show that the movement of the sliding layer is uniform for some landslides, but more frequently it decreases approximately linearly from the slope surface to the sliding plane (Cai & Ugai, 2011). As shown in Fig. 2, $\theta_s = 0$ represents the uniform case, while $\theta_s > 0$ is the case where the soil movement decreases linearly with depth. θ_{sn} is the corresponding parameter of θ_s in the non-dimensional model (see Table 2), which can be readily incorporated in the calculations. Comparisons between the cases with and without θ_{sn} show that its effect on the pile response is significant when y_{sn} is small or $L_{n1} > L_{n1(crit)}$. Ignoring θ_{sn} gives rise to overestimated bending moments in the pile.

For the k_h – c_u and the p_u – c_u relationships, inaccurate selections will induce similar consequences for using different sliding or stable layers, which refer to the observations in the previous section entitled ‘Effect of R_k and R_p ’ and are not repeated here. When assuming constant k_h – c_u and p_u – c_u relationships, the variation of the distribution of c_u affects both profiles of k_h and p_u . As the model normalisation is based on k_h , the non-uniform distribution of k_h can only be considered by using a representative k_h for the whole soil layer. Its influence refers to the effect of R_k . As for the p_u distribution, it may be reasonable to give smaller values from the ground surface to a certain depth (Broms, 1964; Ito & Matsui, 1975; Viggiani, 1981; Kanagasabai *et al.*, 2011), which has a great influence on F_n and $M_{n2(max)}$ when $L_{n1} < L_{n1(crit)}$.

The proposed model is based on the simplified ground model of uniform c_u profile and assuming constant k_h – c_u and p_u – c_u relationships. This ground model is often considered reasonable in overconsolidated clay soil. As concluded previously, R_p and the profile of p_{u1} and y_{sn} may have a great impact on the model accuracy. This can be improved by modifying the ground model, such as using the measured soil displacement profile and calibrating the p_u and k_h profiles through field testing or advanced numerical studies. It will be demonstrated in the next section that, given calibrated parameters, the proposed model can deliver satisfactory results.

NUMERICAL VERIFICATION

In this section, the accuracy of the proposed method is verified against 3D finite-element simulations carried out

using the numerical code Abaqus. First, the modelling details are provided. Second, the soil parameters are calibrated and the p_{u1} profile is obtained. Finally, the simulated results of four test series are used to demonstrate the predictive efficacy of the proposed method.

Numerical model

The geometry of the numerical model is displayed in Fig. 7; it consists of a $L_1 + L_2$ long pile with a diameter of 1 m. The distance from the pile tip to the model bottom is four diameters. The pile axis is five diameters away from all the side boundaries, representing a pile-to-pile spacing of ten diameters perpendicular to the sliding direction (Kourkoulis *et al.*, 2011). The finite-element meshes exploit symmetry, so that only one half of the model needs to be discretised. The pile is assumed to behave as an elastic material while plasticity in the cohesive soils is considered. The constitutive model for cohesive soils proposed by Gerolymos & Gazetas (2006) is adopted, consisting of a non-linear kinematic hardening law in conjunction with a von Mises envelope as the bounding surface and an associated plastic flow law. Generally, this constitutive model predicts well the behaviour of clay deposits in the analysis of soil–structure interactions under both monotonic and cyclic loads (Zafeirakos & Gerolymos, 2013; Gerolymos *et al.*, 2020, 2015). The material properties for both the pile and soils are listed in Table 4. Three additional parameters are defined (Gerolymos & Gazetas, 2006): σ_0 is the stress at zero plastic strain, which is usually expressed as a small fraction of the uniaxial strength (1/20–1/5), and C and γ are hardening parameters that define the maximum transition of the yield surface. C is obtained as equal to the modulus of elasticity (E) by differentiating the back-stress with respect to the plastic strain. γ is obtained by

$$\gamma = \frac{E}{\sqrt{3}c_u - \sigma_0} \quad (18)$$

Interface elements are used to model both the interaction between the pile and the surrounding soils and the slippage between the two soil layers. The friction coefficient 0.3 is used for the former, whereas a frictionless interface is used for the latter. Sliding and detachment of the interfaces are allowed – that is, geometric non-linearities are considered. For all of the side boundaries, the bottom boundary and the symmetry boundary, the movement perpendicular to the boundary is restrained. The soil movement is simulated by applying

Fig. 7. Mode geometry and 3D finite-element mesh

Table 4. Numerical simulation parameters

Material	E : MPa	μ	ρ : kg/m ³	c_u : kPa	σ_0 : kPa	$C = E$: MPa	γ
Pile	25000	0.2	2500	—	—	—	—
Soft clay	12	0.45	2000	30	5.20	12	256.6
Medium clay	32	0.45	2000	80	13.86	32	256.6

horizontal displacement (up to $0.5D$) at both the front and rear boundaries of the top layer.

Parameter calibration

To reproduce the pile response from the finite-element model using the proposed method, basic parameters – namely, k_h and p_u for both soil layers – must be calibrated. For this purpose, a failure mode in which the ultimate lateral capacity is mobilised in both soil layers (i.e. mode B) should be obtained. The case $L_1 = L_2 = 4$ m ($\lambda = 1$) has been selected. As widely reported in both numerical simulations and field observations (Broms, 1964; Ito & Matsui, 1975; Viggiani, 1981; Kanagasabai *et al.*, 2011), p_{u1} near the ground surface (shallower than about $1.5D$) may be significantly lower than the deeper soil. Viggiani (1981) adopted a simple solution by reducing L_1 for the analysis. The proposed method can also be modified to consider this effect by

applying linearly increasing p_{u1} near the ground surface while still using p_{u1} of deep soil for the normalisation. To obtain the p_{u1} profile, another numerical simulation ($L_1 = 4$ m, $L_2 = 8$ m), which delivers the mode C pile response is needed (i.e. p_u is attained only in the sliding layer).

Depth profiles of the pile displacement, bending moment and shear force are readily extracted from the numerical model. The depth profile of pile–soil contact pressure (p) and thus the p – y curves are derived by differentiating the shear force with respect to depth. Fig. 8(a) shows the p profile increasing with soil displacement (y_s) for the case $\lambda = 1$. It is obvious that near the pile rotation points ($z \sim 0.5$ m and ~ 6.5 m), the developed relative lateral pile–soil displacement appears to be too small to mobilise the ultimate lateral capacity (Byrne *et al.*, 2020). Apart from that, the abnormal result is also observed near the slip surface, which is influenced by local vertical stress reduction due to pile–soil gapping (Kanagasabai *et al.*, 2011). Therefore, the p – y curves

Fig. 8. Parameter calibration: (a) variation of p profile with y_s ; (b) fitted p – y curves; (c) selections of parameters representing the soil layers and the p_{u1} profile

used for calibration need to be carefully selected. A two-step calibration is conducted. First, curve fitting is applied for each extracted p - y curve. The initial p_u is set to the maximum p in that p - y curve. Initial k_h can be calculated by equation (4), using y_r (i.e. y_{50}) as the y value where half of the maximum p occurs. The final parameters are selected by varying both p_u and k_h and minimising the least-square error between the numerical data and the fitted hyperbolic p - y curve. Second, parameters at valid depths are selected to generate the parameters for the layer. As shown in Fig. 8(c), excluding the previously mentioned areas, two depth ranges with fully mobilised soil reactions are identified. The results of the case $\lambda = 2$ (p_u attained only above the slip surface) are also plotted, showing good agreement for the identified range above the slip surface. Reduction of p_u is observed due to the influence of soil heaving near the ground surface. Final

p_u and k_h for each layer are obtained by averaging the data in each identified depth range. For the p_u variation near the ground surface, a linear increase with $1/3p_{u1}$ at $z = 0$ appears to fit the data well from the case $\lambda = 2$ (filled squares). Note that there is no pile rotation point above the slip surface for the case $\lambda = 2$; therefore, the ultimate lateral capacity near the surface is obtained using only this calculation. A comparison of the numerical p - y curves and the fitted data in the fully mobilised depth ranges is given in Fig. 8(b).

The calibration results are given in Table 5. The above two numerical calculations are well reproduced. The profiles of pile displacement, bending moment, shear force and soil pressure are compared in Fig. 9(a), while the histories of the pile resistance and the bending moment at the slip surface are checked in Figs 9(b) and 9(c), respectively.

Table 5. Calibration results

No.	L_1 : m	L_2 : m	k_{h1} : MPa	k_{h2} : MPa	p_{u1} : kPa	p_{u2} : kPa	R_k	R_p
1	4	4	16.69	51.54	268.2	580.9	0.32	0.46
2	4	8	16.69	—	268.2	—	0.32	0.46

Fig. 9. Reproducing the calibrated simulations: (a) comparison of the depth profiles of pile displacement, bending moment, shear force and soil pressure when $y_s = 0.5$ m; (b) comparison of the pile resistance; (c) comparison of the pile bending moment at the slip surface

Predictive efficacy

Once the soil parameters have been calibrated, the pile response can be predicted for arbitrary combination of L_{n1} and λ . To demonstrate the predictive efficacy of the proposed model, four series of simulation cases are performed and the configurations are specified in Table 6. The $F_n/F_{n(\max)}-\lambda$, $M_{ns}/L_{n1}^2-\lambda$, $M_{n1(\max)}/L_{n1}^2-\lambda$ and $M_{n2(\max)}/L_{n1}^2-\lambda$ relationships are shown in Fig. 10. In general, the transition from mode B to mode C is well predicted for all the tested L_{n1} values. As for the accuracy, the mean difference between the proposed method and the finite-element method for F and M_s is around 5%, and 9.2% for $M_{2(\max)}$. The maximum difference of around 15% is observed in both M_s and $M_{2(\max)}$. The difference in $M_{1(\max)}$ is not calculated due to its relatively small magnitude. However, it shows better agreement, which

also supports the finding that the mode transition is well estimated. In the view of the current authors, the accuracy can be seen as satisfactory provided that so few parameters are used in the proposed method.

APPLICATION OF THE NON-DIMENSIONAL SOLUTION

This paper has provided some charts that are useful for pile design. However, as emphasised earlier, these results are based on specific R_k , R_p and $y_{sn(0.5B)}$ values. The calculation process can be readily implemented in any coding program. For an existing landslide, k_h and p_u are obtained using in situ or laboratory test results (i.e. given R_k and R_p). Then, design charts like Fig. 5 can be easily generated and used for the

Table 6. Numerical simulation configurations

Case series	L_1 : m	L_2 : m	EI : MN M ²	L_{n1}	$\lambda = L_2/L_1$
1	4	4, 6, 8, 10	1227.2	1.37	1, 1.5, 2, 2.5
2	6	4, 6, 8, 10	1227.2	2.05	
3	6	4, 6, 8, 10	613.6	2.44	0.67, 1, 1.33, 1.67
4	6	4, 6, 8, 10	245.4	3.06

Fig. 10. Reproducing the finite-element method simulations. Variations of (a) $F_n/F_{n(\max)}$, (b) M_{ns}/L_{n1}^2 , (c) $M_{n1(\max)}/L_{n1}^2$ and (d) $M_{n2(\max)}/L_{n1}^2$ with λ

Fig. 11. Application of the non-dimensional solution in the case of Esu & D'Elia (1974): (a) pile response prediction for various pile configurations; (b) sensitivity analysis of soil parameters

response prediction of any pile type (different EI and D , represented by various L_{n1} and y_{sn}). For instance, the computed data for the presentation of Fig. 5 can be applied to the case of Esu & D'Elia (1974). In Fig. 11(a), curves are extracted according to the parameters of the case history. The figure shows that the reinforced concrete pile used to stabilise the slope (hollow circle) lies in the mode B range and cannot provide maximum resistance. Moreover, for the explored conditions, varying D greater than 0.95 m or increasing the pile elastic modulus more than 2.12 times that of the current pile, the pile response moves to the mode C range. The calculation data used for the design charts of Fig. 6 can be used to perform sensitivity analyses of soil parameters, as shown in Fig. 11(b). The prediction of $F_n/F_{n(max)}$ may be unconservative if higher k_h/E_s or lower R_p are estimated. More attention needs to be paid to p_{u1} , which influences the results the most. Such analysis is done directly using the calculation data created for the previous figures. Furthermore, a database can be created in advance for commonly encountered ranges of soil parameters, which can increase the efficiency of the preliminary pile design workflow.

CONCLUSIONS

The response of stabilising piles for landslide prevention is studied by way of a non-dimensional model based on a simplified ground model consisting of two layers of cohesive soils. The soil reaction is described by a hyperbolic p - y model, which has been verified through three case studies. The failure modes identified by Viggiani (1981) have been further studied considering the effect of the soil displacement and the relative pile rigidity. $L_{n1(crit)}$ and λ_{ult} , which determine the mode C pile response, have been identified. The effects of stable and sliding soils have been explored and other influencing factors have been discussed. The efficacy of the proposed approach was demonstrated by the satisfactory result that it achieves in reproducing the more complex 3D finite-element analyses. The non-dimensional model has only four basic parameters related to pile and soil properties, and two parameters related to the soil movement profile. The model provides the opportunity to generate a database in advance, increasing the efficiency of the preliminary pile design workflow.

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NOTATION

C	hardening parameter of the cohesive soil (FL^{-2})
c_u	undrained shear strength of the soil (FL^{-2})
D	pile diameter (L)
E	Young's modulus of the pile (FL^{-2})
E_s	soil elastic modulus (FL^{-2})
F	sliding force applied on the pile (F)
I	second moment of inertia for the pile's cross-section (L^4)
k_h	coefficient of subgrade reaction (FL^{-2})
k_{sec}	secant modulus of the soil reaction (FL^{-2})
L_1	pile length in the sliding layer (L)
L_2	pile length in the stable layer (L)
L_{c1}	critical length of an active long pile in the sliding layer (L)
M	pile bending moment (FL)
M_s	pile bending moment at the slip surface (FL)
M_y	pile section capacity (FL)
m_u	normalised pile section capacity
N_p	limit yield pressure coefficient
p	soil-pile contacting force per unit pile length (FL^{-1})
p_u	ultimate pile-soil contact pressure (FL^{-2})
R_k	ratio of the modulus of the subgrade reaction
R_p	ratio of the ultimate soil pressure on pile
S	pile shear force (F)
T	relative stiffness factor (L)
t_u	pile resistance normalised by its theoretical maximum value
x_{p1}	thickness of the plastic zone in the sliding layer (L)
y	absolute displacement difference between the pile and soil at the same depth (L)
y_{50}	pile displacement corresponding to 50% p_u (L)
y_p	pile displacement (L)
y_r	reference displacement (L)
y_s	soil movement (L)
$y_{sn(0.5D)}$	y_{sn} corresponds to 0.5D soil displacement
z	depth measured from the ground surface (L)
β	relative stiffness factor for pile length normalisation (L^{-1})
γ	hardening parameter of the cohesive soil
$\Delta()$	increment of the parameter
θ	rotation of the pile cross-section
θ_s	angle between the soil displacement profile and the vertical line

λ	ratio of the embedded length to the pile length in the sliding layer
μ	Poisson's ratio
ρ	soil density (FL^{-3})
σ_0	soil stress at zero plastic strain (FL^{-2})
χ	ratio of the soil lateral capacity between the sliding and stable layers
0_0	parameter value at the pile top or ground surface
0_1	parameter for the sliding layer
0_2	parameter for the stable layer
0_{crit}	critical value of the parameter
0_{lim}	limit value of the parameter
0_{max}	maximum value of the parameter
0_n	normalised version of the parameter
0_{ult}	ultimate value of the parameter

REFERENCES

- API (American Petroleum Institute) (2000). *Recommended practice for planning, designing and constructing fixed offshore platforms – working stress design*. Washington, DC, USA: API.
- Broms, B. B. (1964). Lateral resistance of piles in cohesive soils. *J. Geotech. Engng Div.* **90**, No. 2, 27–63.
- Byrne, P. M., Anderson, D. L. & Janzen, W. (1984). Response of piles and casings to horizontal free-field soil displacements. *Can. Geotech. J.* **21**, No. 4, 720–725, <https://doi.org/10.1139/t84-079>.
- Byrne, B. W., Houlsby, G. T., Burd, H. J., Gavin, K. G., Igwe, D. J. P., Jardine, R. J., Martin, C. M., McAdam, R. A., Potts, D. M., Taborda, D. M. G. & Zdravković, L. (2020). PISA Design model for monopiles for offshore wind turbines: application to a stiff glacial clay till. *Geotechnique* **70**, No. 11, 1030–1047, <https://doi.org/10.1680/jgeot.18.P255>.
- Cai, F. & Ugai, K. (2003). Response of flexible piles under laterally linear movement of the sliding layer in landslides. *Can. Geotech. J.* **40**, No. 1, 46–53, <https://doi.org/10.1139/T02-103>.
- Cai, F. & Ugai, K. (2011). A subgrade reaction solution for piles to stabilise landslides. *Geotechnique* **61**, No. 2, 143–151, <https://doi.org/10.1680/geot.9.P026>.
- Carrubba, P., Maugeri, M. & Motta, E. (1989). Esperienze in vera grandezza sul comportamento di pali per la stabilizzazione di un pendio. In *Atti del XVII convegno nazionale di geotecnica*, vol. 1, pp. 81–90. Rome, Italy: Associazione Geotecnica Italiana (in Italian).
- Chen, C. Y. & Martin, G. R. (2002). Soil–structure interaction for landslide stabilizing piles. *Comput. Geotech.* **29**, No. 5, 363–386, [https://doi.org/10.1016/S0266-352X\(01\)00035-0](https://doi.org/10.1016/S0266-352X(01)00035-0).
- Chen, L. T. & Poulos, H. G. (1997). Piles subjected to lateral soil movements. *J. Geotech. Geoenviron. Engng* **123**, No. 9, 802–811, [https://doi.org/10.1061/\(ASCE\)1090-0241\(1997\)123:9\(802\)](https://doi.org/10.1061/(ASCE)1090-0241(1997)123:9(802)).
- Chow, Y. K. (1996). Analysis of piles used for slope stabilization. *Int. J. Numer. Analyt. Methods Geomech.* **20**, No. 9, 635–646.
- Di Laora, R., Maiorano, R. M. S. & Aversa, S. (2017). Ultimate lateral load of slope-stabilising piles. *Geotechnique Lett.* **7**, No. 3, 237–244, <https://doi.org/10.1680/jgele.17.00038>.
- Durrani, I. K., Ellis, E. A. & Reddish, D. J. (2008). Numerical modelling of lateral pile-soil interaction for a row of piles in a frictional soil. In *Advances in transportation geotechnics* (eds E. Ellis, H. S. Yu, G. McDowell, A. Dawson and N. Thom), pp. 291–298, <https://doi.org/10.1201/9780203885949>. Boca Raton, FL, USA: CRC Press/Balkema.
- Ellis, E. A., Durrani, I. K. & Reddish, D. J. (2010). Numerical modelling of discrete pile rows for slope stability and generic guidance for design. *Geotechnique* **60**, No. 3, 185–195, <https://doi.org/10.1680/geot.7.00090>.
- Esu, F. & D'Elia, B. (1974). Interazione terreno-struttura in un palo sollecitato da una frana tipo colata. *Riv. Ital. Geotech.* **8**, No. 1, 27–38 (in Italian).
- Gerolymos, N. & Gazetas, G. (2006). Static and dynamic response of massive caisson foundations with soil and interface nonlinearities – validation and results. *Soil Dyn. Earthq. Engng* **26**, No. 5, 377–394, <https://doi.org/10.1016/j.soildyn.2005.12.001>.
- Gerolymos, N., Zafeirakos, A. & Karapiperis, K. (2015). Generalized failure envelope for caisson foundations in cohesive soil: static and dynamic loading. *Soil Dyn. Earthq. Engng* **78**, 154–174, <https://doi.org/10.1016/j.soildyn.2015.07.012>.
- Gerolymos, N., Giannakos, S. & Drosos, V. (2020). Generalised failure envelope for laterally loaded piles: analytical formulation, numerical verification and experimental validation. *Geotechnique* **70**, No. 3, 248–267, <https://doi.org/10.1680/jgeot.18.P051>.
- Guo, W. D. (2013). Pu-based solutions for slope stabilizing piles. *Int. J. Geomech.* **13**, No. 3, 292–310, [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0000201](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000201).
- Guo, W. D. & Qin, H. Y. (2010). Thrust and bending moment of rigid piles subjected to moving soil. *Can. Geotech. J.* **47**, No. 2, 180–196, <https://doi.org/10.1139/T09-092>.
- Ito, T. & Matsui, T. (1975). Methods to estimate lateral force acting on stabilizing piles. *Soils Found.* **15**, No. 4, 43–59, https://doi.org/10.3208/sandf1972.15.4_43.
- Kalteziotis, N., Zervogiannis, H., Frank, R., Seve, G. & Berche, J. C. (1993). Experimental study of landslide stabilization by large diameter piles. In *Geotechnical engineering of hard soils-soft rocks: proceedings of an international symposium* (ed. A. Anagnostopoulos), pp. 1115–1124. Rotterdam, the Netherlands: Balkema.
- Kanagasabai, S., Smethurst, J. A. & Powrie, W. (2011). Three-dimensional numerical modelling of discrete piles used to stabilize landslides. *Can. Geotech. J.* **48**, No. 9, 1393–1411, <https://doi.org/10.1139/t11-046>.
- Kondner, R. L. (1963). Hyperbolic stress–strain response: cohesive soils. *J. Soil Mech. Found. Div.* **89**, No. 1, 115–143.
- Kourkoulis, R., Gelagoti, F., Anastopoulos, I. & Gazetas, G. (2011). Slope stabilizing piles and pile-groups: parametric study and design insights. *J. Geotech. Geoenviron. Engng* **137**, No. 7, 663–677, [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0000479](https://doi.org/10.1061/(ASCE)GT.1943-5606.0000479).
- Lei, G. & Wu, W. (2020). Centrifuge study on the effect of pile bending stiffness on the slope stabilised by piles. *Int. J. Phys. Model. Geotech.* **20**, No. 4, 212–223, <https://doi.org/10.1680/jphmg.18.00087>.
- Li, C., Tang, H., Hu, X. & Wang, L. (2013). Numerical modelling study of the load sharing law of anti-sliding piles based on the soil arching effect for Erliban landslide, China. *KSCE J. Civ. Engng* **17**, No. 6, 1251–1262, <https://doi.org/10.1007/s12205-013-0074-x>.
- Li, C., Wu, J., Tang, H., Hu, X., Liu, X., Wang, C., Liu, T. & Zhang, Y. (2016a). Model testing of the response of stabilizing piles in landslides with upper hard and lower weak bedrock. *Engng Geol.* **204**, 65–76, <https://doi.org/10.1016/j.enggeo.2016.02.002>.
- Li, Z., Kotronis, P., Escoffier, S. & Tamagnini, C. (2016b). A hypoplastic macroelement for single vertical piles in sand subject to three-dimensional loading conditions. *Acta Geotech.* **11**, No. 2, 373–390, <https://doi.org/10.1007/s11440-015-0415-7>.
- Liang, R. & Zeng, S. (2002). Numerical study of soil arching mechanism in drilled shafts for slope stabilization. *Soils Found.* **42**, No. 2, 83–92, https://doi.org/10.3208/sandf.42.2_83.
- Martin, G. R. & Chen, C. Y. (2005). Response of piles due to lateral slope movement. *Comput. Struct.* **83**, No. 8–9, 588–598, <https://doi.org/10.1016/j.compstruc.2004.11.006>.
- Page, A. M., Grimstad, G., Eiksund, G. R. & Jostad, H. P. (2018). A macro-element pile foundation model for integrated analyses of monopile-based offshore wind turbines. *Ocean Engng* **167**, 23–35, <https://doi.org/10.1016/j.oceaneng.2018.08.019>.
- Pan, J. L., Goh, A. T. C., Wong, K. S. & Teh, C. I. (2002). Ultimate soil pressures for piles subjected to lateral soil movements. *J. Geotech. Geoenviron. Engng* **128**, No. 6, 530–535, [https://doi.org/10.1061/\(ASCE\)1090-0241\(2002\)128:6\(530\)](https://doi.org/10.1061/(ASCE)1090-0241(2002)128:6(530)).
- Poulos, H. G. (1971). Behavior of laterally loaded piles: I-single piles. *J. Geotech. Engng Div.* **97**, No. 5, 711–731.
- Poulos, H. G. (1973). Analysis of piles in soil undergoing lateral movement. *J. Soil Mech. Found. Div.* **99**, No. 5, 391–406.
- Poulos, H. G. (1995). Design of reinforcing piles to increase slope stability. *Can. Geotech. J.* **32**, No. 5, 808–818, <https://doi.org/10.1139/t95-078>.
- Randolph, M. F. (1981). The response of flexible piles to lateral loading. *Geotechnique* **31**, No. 2, 247–259, <https://doi.org/10.1680/geot.1981.31.2.247>.
- Reese, L. C. & Van Impe, W. F. (2011). *Single piles and pile groups under lateral loading*, 2nd edn. Leiden, the Netherlands: CRC Press/Balkema.

- Salciarini, D. & Tamagnini, C. (2009). A hypoplastic macroelement model for shallow foundations under monotonic and cyclic loads. *Acta Geotech.* **4**, No. 3, 163–176, <https://doi.org/10.1007/s11440-009-0087-2>.
- Su, D., Huang, J. J. & Yan, W. M. (2018). Parametric investigation on the responses of laterally loaded piles in overconsolidated clay using nondimensional solutions addressing nonlinear soil–pile interaction. *Comput. Geotech.* **96**, 203–214, <https://doi.org/10.1016/j.compgeo.2017.11.004>.
- Tang, H., Hu, X., Xu, C., Li, C., Yong, R. & Wang, L. (2014). A novel approach for determining landslide pushing force based on landslide–pile interactions. *Engng Geol.* **182**, 15–24, <https://doi.org/10.1016/j.enggeo.2014.07.024>.
- Viggiani, C. (1981). Ultimate lateral load on piles used to stabilize landslides. In *Proceedings of 10th international conference on soil mechanics and foundation engineering* (ed. Publications Committee of X ICSMFE), pp. 555–560. Rotterdam, the Netherlands: A.A. Balkema.
- White, D. J., Thompson, M. J., Suleiman, M. T. & Schaefer, V. R. (2008). Behavior of slender piles subject to free-field lateral soil movement. *J. Geotech. Geoenviron. Engng* **134**, No. 4, 428–436.
- Yoon, B. S. & Ellis, E. A. (2009). Centrifuge modelling of slope stabilisation using a discrete pile row. *Geomech. Geoengng* **4**, No. 2, 103–108, <https://doi.org/10.1080/17486020902856991>.
- Zafeirakos, A. & Gerolymos, N. (2013). On the seismic response of under-designed caisson foundations. *Bull. Earthq. Engng* **11**, No. 5, 1337–1372, <https://doi.org/10.1007/s10518-013-9465-0>.
- Zhang, L. & Ahmari, S. (2013). Nonlinear analysis of laterally loaded rigid piles in cohesive soil. *Int. J. Numer. Analyt. Methods Geomech.* **37**, No. 2, 201–220, <https://doi.org/10.1002/nag.1094>.