DISCUSSION

A novel elasto-viscoplastic formulation for compression behaviour of clays

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Contribution by Evert J. den Haan

The model presented by Yuan & Whittle (2018) is impressive. It is indeed novel and does provide for a wide range of viscous phenomena, while isotache models do grapple with the mismatch between initial field rate of strain and the usually much higher value deduced from oedometer tests. The method given for the rational estimate of initial creep rate could, however, admittedly in an irrational manner, also be applied in isotache modelling – for example, to adjust (increase) measured preconsolidation pressure. Both models require such an estimate, and to embrace the presented model, other advantages would have to be proven. In this regard, the rate sensitivity parameter, β , and the additional features presented by Yuan (2016), such as non-linear small-strain behaviour and variation of the creep parameter, certainly warrant evaluation.

The rational estimate of initial creep rate uses a value for T_{99} of 3. This is incorrect. $T=3$ pairs with an average degree of consolidation of 99.95% not 99%. $T=1.78$ pairs with 99%.

The model can range between hypothesis A and B behaviour by varying β from 0 to ρ_a/ρ_c . The authors present indications that β may be strain rate dependent, and the low strain rates in thicker laboratory samples (and even more so in field circumstances), favour low β and hypothesis A behaviour. It may be little known, but isotache models can also produce a transition from hypothesis B to hypothesis A behaviour for increasing layer thickness. This is best exemplified by Yoshikuni *et al.* (1995; Figs 16 and 17 in that paper). Seven layer thicknesses varying from 2 cm to 40 m are analysed. In all cases the same initial rate of creep strain is taken, and initial states all plot on the same isotache, thicker layers plotting lower because of the higher self-weight. It is the self-weight (higher stiffness, lower permeability) that causes the shift from B type to A type behaviour. (The isotaches in Yoshikuni *et al.* (1995) may be shown to be given by $e = e_0 + C_a \log(\dot{\epsilon}/\dot{\epsilon}_0) - C_\beta \log(\sigma'_v/\sigma'_{v0})$, proving that it is indeed an isotache model.)

Figure 13 shows similar results using an isotache model based on natural strain. The parameters are loosely inspired by the test results for San Francisco Bay mud (SFBM) reported in Fig. 5, although permeability is taken a factor 10 higher, as laboratory deduced permeability is often quite low. Here too the transition from B type to A type behaviour

occurs as thickness increases. Note, however, that end of primary (EOP) (99% degree of consolidation) strain actually decreases at larger thickness, and this may be characteristic of this approach, so that unique EOP strain independent of thickness is probably not to be expected. (The isotaches here are given by

$$\ln v = \ln v_1 - c \ln(\tau/\tau_1) - b \ln(\sigma'_v/\sigma'_{v1})$$

where v is specific volume ($1+e$), and τ is intrinsic time in days and equals the reciprocal of creep rate of strain times c . v_1 is specific volume on the reference isotache; $\tau_1 = 1$ day at $\sigma'_{v1} = 1$ kPa.)

Does the presented model with $\beta = \rho_a/\rho_c$ also produce this transition, and how then does $\beta = 0$ perform?

Figure 8(c) shows ‘a unique loge–log σ'_v curve’ for $H=2$ and 20 cm with $\beta=0$. A completely unique curve is, however, not a fundamental property of the model, as shown by Fig. 9(b), where the curves ‘concentrate in a narrow band’.

The model linearises the limiting compression curve (LCC) by using the logarithm of voids ratio. It can be recast in terms of natural strain or logarithm of specific volume v , and Fig. 14 shows that this linearises LCC in a constant rate of strain (CRS) test on a highly compressible Dutch peat. (During the unloading branch, absolute rate of strain is used in equation (7) to prevent collapse of the calculation. Unlike the numerical predictions of the CRS tests in Fig. 5, straightforward integration of the model equations is applied without accounting for hydraulic flow. The one-dimensional simulation of the upper curve in Fig. 5 can be closely replicated in this manner.)

In the group of four CRS curves in Fig. 3(a), $R_{a0}=0$ may have been taken. Then $\dot{\epsilon}^{vp}=0$, and $O(\dot{\epsilon})=\dot{\epsilon}$ is invoked. For this case at least, almost the same result, not noticeably different, is found by requiring the same initial $\dot{R}_{a0}$ for $R_{a0}=0$ with $O(\dot{\epsilon})=\dot{\epsilon}$ and $R_{a0}\neq 0$ without $O(\dot{\epsilon})$. A quadratic equation in R_{a0} as a function of $\dot{\epsilon}$ results in

$$R_{a0}^2 - f(\dot{\epsilon})R_{a0} + (\sigma'_{p0}/m\sigma'_{v0})\dot{\epsilon}f(\dot{\epsilon}) = 0$$

in which $m = (\rho_c - \rho_a)/\rho_a\rho_r n$.

The lesser roots of R_{a0} replicate Fig. 3(a) very well (curves 1–4 in Fig. 15). For the lowest curve, $R_{a0} = 1.34 \times 10^{-4}$ %/h is determined, and it increases more or less proportionally with applied strain rate for the higher curves. The initial state therefore effectively varies for the curves shown. The greater root essentially produces $R_{a0}=f(\dot{\epsilon})$, which then remains constant throughout the test.

The LCCs of these four tests, extrapolated downwards to pass through the initial state, give a strain rate of 4.62×10^{-11} %/h. The model could be expected to produce a straight line at slope ρ_c with this rate. The greater root for this strain rate (1.66×10^{-10} %/h), being so close to $f(\dot{\epsilon})$, does of course (curve 5') but the lesser root (1.24×10^{-13} %/h) initially overshoots the LCC (curve 5). The $O(\dot{\epsilon})=\dot{\epsilon}$ solution does also.

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Fig. 13. Isotache simulations for compression of layers varying in thickness from 0.02 to 20 m: (a) midplane voids ratio–midplane effective stress; (b) natural strain–elapsed time. All cases have equal initial rate of strain. Initial surface load is 25 kPa. Applied load is 100 kPa

A reasonable common value of R_{a0} for the four curves would be that producing the straight line through the origin. This value, $1.66 \times 10^{-10} \text{ %/h}$, produces (strong) initial overshoot of the LCC for all four curves (curves 1'–4').

Overshoot of the LCC can be produced not only from an initial state as in Fig. 3(a), but also during resumption of CRS loading after a relaxation period or a creep period. It is generally enhanced by very low values of R_{a0} and higher rate of loading, so longer relaxation or longer creep will produce more overshoot.

Yuan (2016; Fig. 5.6) also finds the overshoot of the LCC, during reloading following unloading, using a non-linear small-strain formulation. It is attributed to material viscosity, which is contrasted to earlier explanations in terms of structure effects. Note the (slight) tendency for overshooting in Fig. 14 after the relaxation and unloading phases. The

measured behaviour lacks this, so here at least it is an undesirable trait of the presented model.

The Batiscan example in Fig. 4(a) is from a CRS test interrupted by two relaxation phases. The relaxation phase in Fig. 4(a) (the first) is treated as an isolated segment, and the compression parameters ρ_a and ρ_c are not related to the CRS compression curve. Ideally, a model would seamlessly fit all CRS and relaxation stages. This is not the case. Is this because the clay is brittle (Feng, 1991: p. 97), for which the model is not designed?

The immediate elastic strain at the draining boundary of a soil layer after load application produces an infinite spike in elastic strain rate, and presumably also in $\dot{R}_{a0}$. What is the effect on R_a at the commencement of creep on the boundary? This has a bearing on the numerical solution of the consolidation response using, for example, finite-difference methods, as in Figs 7–9 and 12. Perhaps the authors could

Fig. 14. Prediction of CRS test on Bloemendalerpolder peat with adapted model using natural strain (test 4D, depth 5.3 m, 52°19'15"-745°N, 5°01'54.239"E, water content 1258%)

Fig. 15. Recalculation of CRS tests at different strain rates shown in Fig. 3(a)

provide a worked example for the solution of the upper boundary conditions for one of these cases.

Authors' reply

The authors very much appreciate the discussor's efforts in reproducing their formulation and examining their work. They also appreciate the detailed and helpful diagnosis/review, and hope that the following responses will help to clarify the points raised.

- (a) The discussor stated that both isotache model and the authors' proposed model require the estimate for the initial creep rate in predicting field-scale consolidation.

The authors agree that the estimate for initial creep rate is important for both models to predict field-scale consolidation. The typical calibration for isotache models tends to overlook the estimate for the initial creep rate, as the creep rate is in default determined by the assumed pre-consolidation pressure. Adjusting the preconsolidation pressure in an isotache model can

improve the estimate of the initial creep rate in the field-scale prediction. But as the discussor suggested, it is 'less rational' and difficult to justify the alternation in stress history parameters from the calibration based on laboratory test results. In addition, changing the preconsolidation pressure in a three-dimensional isotache framework alters the initial size of the yield surface and can affect the prediction of shear behaviour too. The authors' proposed model explicitly requires the calibration for the initial creep rate (so it would not be overlooked) and estimates it in terms of the assumed initial internal strain rate (with less dependency on the preconsolidation pressure).

- (b) The discussor is correct that a consolidation time factor, $T = 3$, corresponds to an average consolidation degree, $U = 99.95\%$ (instead of 99% as stated in the paper under discussion)

$$T_{99.95} = 1.781 - 0.933 \log_{10} (100 - 99.95) \approx 3$$

$$T_{99} = 1.781 - 0.933 \log_{10} (100 - 99) \approx 1.78$$

This discrepancy does not alter the intention of the proposed method, which uses a consolidation time

Fig. 16. Illustration of the effects of ρ_a/ρ_c on the predictions of reloading behaviour and irrecoverable deformation

Fig. 17. Flow chart for integration at the top drainage boundary under incremental load condition

$T=3$ to give an order of magnitude estimate of the initial creep rate (beyond 99% consolidation).

- (c) The discussor suggested that the isotache model, such as the one presented by Yoshikuni *et al.* (1995), can also predict a transition from hypothesis B to hypothesis A behaviour for increasing layer thicknesses.

The formulation presented by Yoshikuni *et al.* (1995) produces typical isotache predictions. As shown in the

discussion (Figs 13(a) and 13(b)), the 2 cm and 10 cm thick samples have almost identical initial void ratio (specific volume, $v = 3.5$) and effective stress states, and exhibit behaviour consistent with hypothesis B, with a larger strain at EOP conditions for the thicker sample (Fig. 13(b)).

However, the authors disagree that isotache models (such as the Yoshikuni *et al.* model) can predict hypothesis A type behaviour for thick clay layers (10 m, 20 m and 40 m in Fig. 13(b)). As shown in Fig. 13(a), these thicker samples start the compression from different initial void ratio, e_0 and effective stress, σ'_{v0} (due to self-weight effects noted by Yoshikuni *et al.*). The EOP results in Fig. 13(b) only show apparent hypothesis A behaviour, because the compression curves are plotted in strain against log (time) space, rather than void ratio–log(time) space.

The use of strain plotted against log(time) curves for samples of different thickness that are consolidated from different initial void ratio and effective stress states is not an appropriate way to classify/distinguish between hypothesis A or B behaviour. The authors have illustrated similar misleading predictions from isotache models in a prior paper (where they compared the soft soil creep and Imai models; Yuan & Whittle (2013)). For evaluating field-scale consolidation, it is typical to have lab specimens reproduce the in situ state of void ratio and effective stress prior prediction.

The authors' proposed model can predict a full range of behaviour between hypothesis A and B, assuming the same initial void ratio, stress and stress history variable for the thin and thick specimens, as illustrated in Fig. 8.

- (d) The discussor questioned why Fig. 9(b) does not show the unique $\log e - \log \sigma'_v$ curve, as shown in Fig. 8(c). There is an important difference between the assumptions for the initial R_{a0} used in the predictions in Figs 8(c) and 9(b): Fig. 8(c) shows a unique $\log e - \log \sigma'_v$ curve because the predictions for 2 cm and 20 cm thick samples assume different initial internal strain rates, R_{a0} (estimated based on thickness); these are noted in the legend in Fig. 8(b). There are small differences in the stress–strain ($\log e - \log \sigma'_v$, Fig. 9(b)) curves for Osaka Bay mud because the predictions for different thickness assume the same R_{a0} . This assumption represents the initial consolidation state for the interconnected oedometer test. As described in the paper by Watabe *et al.* (2008), the specimens in each consolidation cell were initially consolidated separately from the initial stress level, before they were connected in series (to emulate different thicknesses) and loaded to the target stress level. Therefore, it is reasonable to assume the same initial creep rates (R_{a0}) prior to the consolidation phase under the target load. It is noted that this assumption has limited impact, as the curves quickly converge to a unique $\log e - \log \sigma'_v$ behaviour, suggesting hypothesis A behaviour.

- (e) The discussor suggested that the linearised LCC curve for CRS tests can be reproduced with an implementation of the proposed one-dimensional model in an elemental level simulation without coupling of hydraulic flow (Fig. 15).

The authors agree with the discussor's approach. Actually, the authors also developed a three-dimensional elasto-viscoplastic formulation, MIT-SR (generalised from the one-dimensional model in *Géotechnique*), and verified its prediction for a variety of triaxial shear behaviour in elemental level simulations (Yuan & Whittle, 2021a, 2021b). The

Fig. 18. Evolution of strain rate components at the top drainage boundary at the beginning of consolidation

authors are pleased to see that the proposed model has been used in describing a highly compressible Dutch peat with initial water content as high as 1258% (initial void ratio, $e_0 \approx 33$). Also, the discussor's plot clearly demonstrates the 'steady state' of R_a , linearised over the LCC range.

- (f) When simulating CRS tests in Fig. 3(a), the authors assumed a zero internal strain rate, $R_{a0} = 0$ for convenience, and adopted a contingency term $O(\dot{\epsilon}) = \dot{\epsilon}$ in the transient coefficient, m_t , (equation (9)) to allow the R_a (and $\dot{\epsilon}^{vp}$) to increase as compression continues

$$m_t = \left(\frac{\rho_c}{\rho_a} - 1 \right) \frac{\dot{\epsilon}^{vp}}{\rho_t n} + O(\dot{\epsilon}) \quad (9)$$

The discussor offered an alternative way to initialise the CRS simulation, which directly prescribes a non-zero value for R_{a0} obtained from a function of the applied constant strain rate (this eliminates the term $O(\dot{\epsilon})$ in the definition of m_t). The discussor suggested that the alternative approach replicates well the predictions in Fig. 3(a). However, it should be noted that the internal strain rate R_{a0} is intended to represent the effects of strain rate history. The approach proposed by the discussor with a prescribed initial value based on the subsequent applied strain rate differs from the authors' original assumption.

- (g) The discussor observed the overshooting behaviour in reproducing the LCC curves for CRS tests, as shown in Fig. 15.

The overshooting issues can likely be attributed to the inadequacy of the transient coefficient m_t (equation (9)). As illustrated in Fig. 2 in the original paper, a smaller value of m_t retards the evolution of the internal strain rate, R_a , in approaching the steady-state conditions under constant rate of strain compression conditions. This can result in a relatively high elastic strain rate upon initial imposition of a high external strain rate, and cause an overshoot in the effective stress.

When predicting the curves 1' to 4' in Fig. 15, the discussor neglected the term $O(\dot{\epsilon})$ from equation (9),

which led to a smaller transient coefficient m_t , and hence obtained an overshooting response. The discussor also showed a slight overshooting tendency in the CRS predictions for the Bloemendalerpolder peat in Fig. 14. This highly compressible clay has a very high ratio, $\rho_a/\rho_c \approx 0.087$ (as opposed to 0.02–0.06 for typical clays), which also leads to a relatively small m_t and triggers the overshoot. Similar behaviour has been illustrated by Yuan (2016), Fig. 16, showing undesirable overshooting at high ρ_a/ρ_c ratios. It should be noted that the results in Fig. 16 were predicted with an extended one-dimensional formulation (Yuan (2016), chapter 5, which is also included in the generalised three-dimensional model; Yuan & Whittle (2021a)), which includes a non-linear small-strain stiffness formulation to capture hysteretic behaviour under unloading/reloading condition, controlled by an additional set of parameters C_b , D and r .

- (h) The discussor questioned why the authors did not present all the CRS compression and relaxation stages in fitting the relaxation data for Batiscan clay, as shown in Fig. 4(a).

The relaxation data for Batiscan clay and Väsby clay shown in Fig. 4(a) were added during the paper review phase to support the prediction of a linearised decrease in effective stress with time. Fig. 4(a) was added to illustrate schematically the relaxation behaviour, rather than to fit any specific clay (i.e. modelling the CRS behaviour for Batiscan clay was not the authors' original intention). The authors agree that it would not be easy to capture the highly non-linear compression properties of Batiscan clay, even in a $\log \sigma' - \log e$ space.

There is an erratum in the index data reported for Väsby clay in Fig. 4. The correct values are $w_L = 122\%$, $I_p = 81\%$, based on the data in Table 3.1 in Feng (1991). The authors appreciate this correction.

- (i) The discussor questioned the integration of the proposed formulation at the top drainage boundary when simulating consolidation under an instant load. At the beginning of consolidation, an immediate loading on the top drainage boundary can theoretically produce an infinite gradient of excess pore pressure and

thus an infinite total strain rate. When solving the problem using a finite-difference scheme, this immediate loading condition can be approximated as a constant strain rate loading on the top boundary over a very small time increment (e.g. $\Delta t < 0.01$ s), where the required total strain rate is determined through an iterative approach. The iterations search to find the total strain rate, where the change in effective stress, $\Delta\sigma'$, is equal to the applied total stress, $\Delta\sigma$, at the top boundary at the end of the small time increment, as illustrated in the flow chart in Fig. 17.

As indicated in Fig. 17, the consolidation time increment can be further divided into multiple sub-steps, and the proposed constitutive equations integrated in each sub-step. The activated internal strain rate R_a is then updated using equation (6), through the course of sub-step integration, which is a function of the total strain rate and the transient coefficient, m_t . The authors incorporated the automatic sub-stepping algorithm proposed by Sloan (1987) for the time integration of the elasto-viscoplastic (EVP) constitutive model. A similar sub-stepping algorithm was also used for integrating the general three-dimensional EVP model (Yuan & Whittle, 2021a).

Figure 18 illustrates the computed strain rate components (total, elastic, viscoplastic rate) at the top boundary for a 2 cm thick specimen of Osaka Bay mud (Figs 9(c) and 9(d)) for the first few consolidation steps. It can be seen that very high total and elastic strain rates occur at the end of the first consolidation step ($t \approx 0.007$ s). The internal strain rate has been stimulated throughout the sub-step integration, leading to a noticeable viscoplastic strain rate. Afterwards ($t > 0.007$ s), the elastic strain rate vanishes (viscoplastic strain rate = total strain rate), which leads to a creep process under constant stress condition at the top drainage boundary. Thus, the proposed method reasonably describes consolidation behaviour at the top boundary.

However, since the finite-difference simulation discretises the soil into a number of thin sub-layers, the above calculation mainly represents the behaviour of a thin layer at the top boundary. When a finer discretisation is used (e.g. with 30 sub-layers), the calculated behaviour at the top boundary does not

significantly impact the average compression behaviour across the thickness of the samples (Figs 7–9 and 12).

NOTATION

c	reciprocal of creep rate of strain times
m_t	transient coefficient
T	consolidation time
U	degree of consolidation
v	specific volume ($1 + e$)
v_1	specific volume on the reference isotache
τ	intrinsic time (days), equals the reciprocal of creep rate of strain times, c

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