DISCUSSION

Results of a critical state line testing round robin programme

DAVID REID*, ANDY FOURIE†, JUAN LUIS AYALA‡, SIMON DICKINSON§,
 FELIPE OCHOA-CORNEJO||, RICCARDO FANNI¶, JUAN GARFIAS**,
 ANTÓNIO VIANA DA FONSECA††, MASON GHAFGHAZI‡‡, CARLOS OVALLE§§,
 MICHAEL RIEMER|||, AMIN RISMANCHIAN¶¶, ROBERTO OLIVERA***, GONZALO SUAZO††† and
 LUIS ALBERTO TORRES-CRUZ‡‡‡

Contribution by Luis Alberto Torres-Cruz

The authors made a most valuable contribution to the technical literature on the experimental aspects of critical state line (CSL) determination (Reid *et al.*, 2021). The discussor wishes to address two points. The first point is to request further information regarding the methods used to perform the cross-sectional area correction required for the calculation of the deviator stress, q . In the discussor's experience, q is generally calculated by assuming that the specimen deforms as a right circular cylinder (RCC). However, this assumption is often not met, even when using lubricated and enlarged platens. Since the paper provides an indication of the state of practice across geotechnical laboratories around the world, it would be interesting to know the extent to which the participating laboratories adopted the RCC assumption, or the alternative methods they may have used to perform the area correction.

The second point is to elaborate on the uncertainty of Γ and λ_e , a topic that is seldom discussed in the technical literature. The logarithmic fit equation can be expressed as

$$e = \Gamma_{p^*} - \lambda_e \times \ln\left(\frac{p'}{p^*}\right) \quad (1)$$

where Γ_{p^*} is the void ratio at $p' = p^*$; λ_e is the slope; and p^* is an arbitrary stress that ensures dimensional consistency by cancelling the units in the argument of the logarithm.

The authors effectively adopted $p^* = 1$ kPa, which is perhaps the most common choice in the literature. The choice of p^* is inconsequential to the regressed CSL as it does not affect the underlying values of e and p' . However, p^* does affect the uncertainty of Γ_{p^*} because this uncertainty decreases when p^* is close to the stress levels of the underlying (e, p') points of the CSL. This leads to Γ_{100} generally having a lower uncertainty than Γ_1 since, as in the case of the authors' data, CSLs typically have underlying points closer to $p' = 100$ kPa than to $p' = 1$ kPa. Beyond statistical considerations, the practical implication is that Γ_{100} correlates more strongly than Γ_1 to soil index properties (Torres-Cruz, 2019). Accordingly, the current discussion uses Γ_{100} ($= \Gamma_1 - \lambda_e \times \ln(100)$) instead of the more widely adopted Γ_1 . It is worth noting that a base-10 logarithm is also commonly used in equation (1) and that this produces a different slope $\lambda_{10} = 2.303\lambda_e$.

The results of the round robin programme allow an estimate of the uncertainty of Γ_{100} and λ_e by calculating the standard deviation of the multiple non-excluded measurements of these parameters listed in Table 4. However, geotechnical practitioners cannot count on Γ_{100} and λ_e estimates from multiple laboratories and must instead rely on one set of (e, p') points from a single source. In this case, the standard error of Γ_{100} and λ_e , which depends partly on the scatter of the data and the number of experimental (e, p') points that define the CSL, provides an estimate of their standard deviation. For instance, a recent analysis computed the standard error of Γ_{100} and λ_{10} of 91 CSLs with known underlying (e, p') points (Torres-Cruz & Santamarina, 2020). To enable comparisons, the standard errors of both parameters were normalised by the size of the range of values observed in non-plastic soils. Based on an analysis of 160 CSLs, Γ_{100} appears to vary from 0.2 to 1.2, yielding a range of 1.0; and λ_e from 0 to 0.13, yielding a range of 0.13. The normalised standard error (NSE) of λ_e ($\text{NSE-}\lambda_e$), and of Γ_{100} ($\text{NSE-}\Gamma_{100}$) was also calculated herein for the 13 round robin CSLs whose (e, p') data points are available in the online supplementary material provided by the authors. Fig. 17 shows that the data points of the 91 CSLs and of the round robin CSLs exhibit the same trend: $\text{NSE-}\lambda_e$ is always greater than $\text{NSE-}\Gamma_{100}$, with the ratio of $\text{NSE-}\lambda_e$ to $\text{NSE-}\Gamma_{100}$ mostly varying from 3 to 12. That is, estimates of Γ_{100} are generally more reliable than estimates of λ_e . More specifically, $\text{NSE-}\Gamma_{100}$ varies predominantly from 0.2 to 2% with maximum values of $\sim 5\%$, while $\text{NSE-}\lambda_e$ varies predominantly from 1 to 10% with maximum values of $\sim 17\%$. It should be noted that Fig. 17 is independent of the logarithmic base used in equation (1) – that is, $\text{NSE-}\lambda_e = \text{NSE-}\lambda_{10}$.

These results point to the importance of inspecting the underlying (e, p') points that define a CSL to, at least, qualitatively assess the expected precision of Γ_{100} and λ_e . Ideally, the standard errors of Γ_{100} and λ_e should be formally calculated to enable the assessment of how their uncertainty propagates into other calculations. However, it is important

* School of Civil and Environmental Engineering, The University of Western Australia, Crawley, WA, Australia (Orcid:0000-0002-1867-1676).

† School of Civil and Environmental Engineering, The University of Western Australia, Crawley, WA, Australia (Orcid:0000-0001-7250-2104).

‡ School of Civil and Environmental Engineering, The University of Western Australia, Crawley, WA, Australia (Orcid:0000-0001-9297-319X).

§ BGC Engineering Inc., Fredericton, NB, Canada.

|| GeoEngineering Faculty, Universidad de Chile, Santiago, Chile.

¶ Golder, Perth, WA, Australia.

** Klohn Crippen Berger, Vancouver, BC, Canada.

†† Construct-Geo, Faculty of Engineering of University of Porto, Porto, Portugal (Orcid:0000-0002-9896-1410).

‡‡ Department of Civil & Mineral Engineering, University of Toronto, Toronto, ON, Canada (Orcid:0000-0002-4890-8291).

§§ IRME UQAT-Polytechnique, Department of Civil, Geological, and Mining Engineering, Polytechnique Montréal, Montreal, Canada (Orcid:0000-0002-9648-5262).

||| Department of Civil and Environmental Engineering, UC Berkeley, Berkeley, CA, USA.

¶¶ Fugro, Wallingford, UK (Orcid:0000-0001-5565-769X).

*** Golder, Vancouver, BC, Canada.

††† Department of Civil Engineering, Universidad Tecnica Federico Santa Maria, Valparaíso, Chile.

‡‡‡ University of the Witwatersrand, Johannesburg, South Africa.

Fig. 17. Normalised standard errors of Γ_{100} and λ_e . UWA, The University of Western Australia; BGC, BGC Engineering Inc.; UC, Universidad de Chile; KCB, Kohn Krippen Berger (KCB); UPorto, Universidade do Porto; UofT, University of Toronto; PUC, Pontificia Universidad Católica de Chile; UTFSM, Universidad Técnica Federico Santa María

to note that the standard errors reflect only the precision of the CSL parameters, not their accuracy. The latter can only be ensured by adopting good laboratory practices as emphasised by the work of the authors.

Authors' reply

The authors thank the discussor for his interest in their work. The authors are pleased that the data produced in their work could be included in the database collected by the discussor to provide further insight into the trends observed for normalised standard errors for CSL parameters.

With respect to the area correction applied in the program, all participants independently elected to use the RCC approach in their work. This was generally justified on the basis of the use of oversized lubricated end platens by many of the participants. However, this represents an interesting area for further discussion and study, as even when using oversized lubricated end platens it is not uncommon for specimens still to exhibit a slight bulging pattern at large strains that is not entirely consistent with the RCC assumption. This may be consistent with the observations of Tatsuoka & Haibara (1985) and Ueng *et al.* (1988) that lubricated platens with silicone grease still have an interface friction angle in the order of 1–2°. The visual observation of bulging, even when using oversized lubricated end platens, probably warrants further investigation numerically. For

example, Sheng *et al.* (1997) saw significant differences when comparing an idealised 'perfect' end platen to a frictional one with a 20° interface friction angle. Further analysis of this type using more realistic interface friction angles for oversized lubricated end platens may provide guidance on the effect such small friction may have on divergence of specimens from ideal conditions and how this may affect parameters inferred from global test measurements.

NOTATION

e	void ratio
$\text{NSE-}\Gamma_{100}$	normalised standard error of Γ_{100}
$\text{NSE-}\lambda_e$	normalised standard error of λ_e
p'	mean effective stress
p^*	arbitrary stress value in the same units as p'
q	deviator stress
Γ	void ratio of the CSL at $p' = 1$ kPa, as defined by the authors
Γ_{p^*}	void ratio of the CSL at $p' = p^*$
λ_e	slope of the logarithmic fit critical state line (CSL) equation when using a natural logarithm
λ_{10}	slope of the logarithmic fit CSL equation when using a base-10 logarithm

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