DISCUSSION

Taylor's slope stability chart for combined effects of horizontal and vertical seismic coefficients

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Contribution by R. Ganesh

The recent paper by Sahoo & Shukla (2019) presents a simple and alternative approach for assessing the seismic stability of homogeneous cohesive-frictional soil slopes with the help of Taylor's slope stability chart (Taylor, 1937, 1948). The main contribution of the work reported in the paper under discussion appears to be the incorporation of the influence of vertical seismic forces in the reduction factor approach formulated earlier by Majumdar (1971). The authors' attempt in proposing such an easy-to-use approach is welcomed with appreciation by the discussor. However, some aspects of the authors' approach for determining the factor of safety (F) of slopes need to be discussed for better understanding by the geotechnical community. The present discussion paper considers two important aspects. One relates to the interpretation of the authors' results and the other is regarding the simplifications followed in their approach. The discussor would also like to contribute with respect to the correct interpretation of the authors' results for the determination of the safety factor of slopes. This has been demonstrated by taking one of the working examples reported by the authors in their paper.

In the original paper, the reduction friction factor m was obtained by considering an equivalent unit weight γ_E , aiming to use stability numbers in Taylor's slope stability chart. With the definition of equivalent unit weight γ_E , the authors have actually converted the given pseudostatic seismic problem into a static one with no external forces except the one given by the equivalent vertical body forces. Thus, the original unit weight γ used in the definition of the stability number in the Taylor's slope stability chart must be replaced by an equivalent unit weight γ_E in the determination of safety factors. According to the reduction factor approach originally proposed by Majumdar (1971), the unit weight of soil γ in the stability number (see Fig. 1 of the original article) is the equivalent unit weight, γ_E .

Based on the above discussion, the correct application of the authors' approach has been demonstrated herein by considering one of the illustrative examples reported by the authors in their paper as follows. Consider a 10 m high soil slope with an inclination of 60° to the horizontal (Fig. 9). The soil has the following properties: total unit weight, $\gamma = 16 \text{ kN/m}^3$; cohesion, $c = 20 \text{ kPa}$; angle of internal friction, $\phi = 25^\circ$.

Determine the factor of safety $k_h = 0.1$ taking into account the effect of vertical seismic acceleration.

Assuming that $k_v = 0.5k_h$, the solution is as follows. From Fig. 7(e), $m \approx 0.788$ for both the vertically downward and upward directions of k_v . Therefore, using equation (11), the value of ϕ_m can be determined as $\phi_m \approx 20^\circ$.

From Taylor's chart (Fig. 1), for slope angle, $i = 60^\circ$ and $\phi_m \approx 20^\circ$. The stability factor can be expressed by replacing the original unit weight γ with an equivalent unit weight γ_E as given below

$$\frac{c}{F\gamma_E H} = 0.114$$

From equation (7)

$$F = \frac{c}{(0.114)\gamma[1 \pm k_v + k_h(l/d)]H}$$

From Fig. 3

$$l/d \approx 0.85$$

$$F = \frac{20}{(0.114)(16)(1 \pm 0.05 + 0.1 \times 0.85)(10)}$$

Considering a vertically upward direction of k_v

$$F = \frac{20}{(0.148)(16)(1 - 0.05 + 0.1 \times 1.3)(10)} = 1.059$$

Considering a vertically downward direction of k_v

$$F = \frac{20}{(0.148)(16)(1 + 0.05 + 0.1 \times 1.3)(10)} = 0.966$$

The safety factors of slopes actually vary with respect to the assumption of the direction of seismic forces. The critical direction of vertical acceleration in the present analysed case is found to be acting vertically downwards, as would be expected. If the value of factor of safety of slopes is interpreted correctly, as illustrated in this discussion paper, the simple slope considered in this example with the given soil properties and seismic loadings is actually 'unstable'. However, for the same problem, the authors have reported that the slope will be in an 'apparently stable' condition owing to the incorrect interpretation of their results.

It is further noticed that the reduction friction factor m in the original paper has been determined without considering the effect of soil cohesion in the analysis. It appears that the original problem has been overlooked by the authors because the reduction friction factor m only modifies the contribution of soil friction angle towards the stability of slopes for the given pseudostatic seismic loadings. If the original problem was to be considered as it is (without omitting the contribution of cohesion in the analysis), one would actually end up with two reduction factors – one for reducing the contribution of friction angle and another for reducing the contribution of cohesion towards the stability of slopes.

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These two reduction factors must remain the same for the given problem owing to the consideration of a single factor of safety with respect to the shear strength of the soil. This unique value of the reduction factor for reducing the contribution of the friction angle and cohesion of the soil can be easily obtained by adopting an iterative approach or the graphical approach as reported by Taylor (1937, 1948). Such an approach would be expected to give a meaningful interpretation of the results from the Taylor's slope stability chart. Despite this shortcoming, the approach of the authors would be easily expected to give safe estimates of the safety factors of slopes, depending upon the magnitude of the cohesion of soil. If the cohesion of the soil is lower, the factor of safety estimated using the authors' approach is expected to be nearly equivalent to the correct value. However, if the contribution of cohesion towards the stability of slopes becomes higher, as would be expected for larger cohesion of the soil, the values of F obtained using the authors' approach can be expected to be significantly different from the correct values. Nevertheless, the approach devised by the authors could be useful as an alternative approach for the preliminary stability assessment of simple slopes taking into account the influence of pseudostatic seismic forces.

The authors of the paper under discussion are invited to shed some light on the various issues raised here pertaining to their work and to clarify the understanding of the interpretation of the results they have obtained for practical applications.

Authors' reply

The authors thank the discussor for expressing this opinion on the interpretation of results presented as illustrative examples using the total unit weight, γ , of soil, and on the assumption of no change in the value of cohesion, c , during the earthquake, but there is nothing incorrect about these two aspects, as pointed out in the discussion.

It is important to note that, during the combined horizontal and vertical seismic loadings, the total unit weight, γ , of the soil is modified to γ_E , as presented in equation (7). As any change in the value of γ results in a corresponding change in the angle of internal friction, ϕ , of the soil (Lambe & Whitman, 1979; Duncan *et al.*, 2014), the analysis has been presented with the derivation of an expression for the modified angle of internal friction, ϕ_m , of the soil in order to take into account the combined effects of horizontal and vertical seismic loads. In equation (11), it may be noticed that ϕ_m is a function of ϕ , γ , γ_E and the geometrical parameters related to the slope. As the seismic loads have been properly considered in the estimation of ϕ_m , the Taylor's slope stability chart for the c - ϕ soil slope, as presented in Fig. 1, may be used for the stability number, reading ϕ as ϕ_m . The consideration of γ_E in determining the factor of safety F of the slope using Fig. 1, as presented by the discussor, includes the effect of seismic loads twice, resulting in an unrealistic value of F . The calculation carried out by the discussor for the example with horizontal seismic coefficient, $k_h = 0.1$, by considering γ_E , gives $F = 1.059$ for the upward vertical seismic coefficient $k_v(\uparrow) = 0.05$ and $F = 0.966$ for the downward vertical seismic coefficient $k_v(\downarrow) = 0.05$, whereas with consideration of γ , the calculation as presented in the authors' original paper gives $F = 1.09$. Thus the factor of safety of the slope with $k_v(\uparrow)$ is greater than the factor of safety with $k_v(\downarrow)$, which is not a correct observation, as experienced and reported by many researchers in the past (Richards & Shi, 1994; Kramer, 1996; FHWA, 1997; Ling & Leshchinsky, 1997; Ling *et al.*, 1997; Ling & Leshchinsky, 1998; Ling, 2001; Huang, 2005; Shukha & Baker, 2008; Das & Ramana, 2011; Iskander *et al.*, 2013; Zhao *et al.*, 2017).

It appears that the discussor has followed the statement made by Majumdar (1971) about the use of γ_E , but no illustrative example was given to justify it. As explained here, with proper justification, the total unit weight γ (not γ_E) should be used in the expression for Taylor's stability number while calculating the factor of safety, F , of the slope. This approach has also been presented by Das & Ramana (2011) while describing the work of Majumdar (1971), who explained how to use the Taylor's chart (Fig. 1) for determining the factor of safety of the slope under a horizontal seismic loading condition only.

In the authors' paper, cohesion of the slope soil has been assumed to be constant with depth and its value remains unchanged during the earthquake period (see assumption (d) on p. 345) for the total stress analysis as considered in the paper. This assumption of unchanged cohesion during the earthquake is based on the past observations and reports by several researchers (Richards & Shi, 1994; Das & Ramana, 2011; Agusti & Sitar, 2013; Duncan *et al.*, 2014). It should be noted that the factor of safety F in the expression of stability number as presented in Fig. 1 and also the calculated value in the illustrative example in the authors' paper refer to the factor of safety with respect cohesion $F = F_c (= c/c_d, c_d$ being the developed/mobilised cohesion) based on full mobilisation of the modified friction angle ϕ_m . As the discussor suggests, the traditional graphical approach, originally explained by Taylor (1937, 1948), and also reported in the literature later (Lambe & Whitman, 1969; Das & Ramana, 2011; Steward *et al.*, 2011), may be used by readers while calculating the value of F based on the shear strength of the soil (equation (1), p. 345), and the same has been explained below for the second illustrative example of the authors' paper.

Referring to the c - ϕ soil slope in Fig. 9 (height, $H = 10$ m; slope angle, $i = 60^\circ$; total unit weight, $\gamma = 16$ kN/m³; cohesion, $c = 20$ kPa and angle of internal friction, $\phi = 25^\circ$), from Fig. 7(e), for $k_h = 0.5$, $k_v = 0.5k_h = 0.25 \downarrow$, $m \approx 0.281$.

From equation (11)

$$m = \frac{\tan \phi_m}{\tan \phi} = 0.281$$

or

$$\begin{aligned}\phi_m &= \tan^{-1}[0.281(\tan \phi)] \\ &= \tan^{-1}[0.281(\tan 25^\circ)] \\ &= \tan^{-1}(0.131)\end{aligned}$$

or

$$\phi_m \approx 7.5^\circ$$

(for downward direction of vertical seismic loading)

Similarly, from Fig. 7(e), for $k_h = 0.5$, $k_v = 0.5k_h = 0.25 \uparrow$, $m \approx 0$.

From equation (11)

$$m = \frac{\tan \phi_m}{\tan \phi} = 0$$

or

$$\begin{aligned}\phi_m &= \tan^{-1}[0(\tan \phi)] \\ &= \tan^{-1}[0(\tan 25^\circ)] \\ &= \tan^{-1}(0)\end{aligned}$$

or

$$\phi_m \approx 0^\circ$$

(for upward direction of vertical seismic loading)

Table 1. Computation of factor of safety with respect to friction and cohesion ($\phi_m \approx 7.5^\circ$)

Assumed developed/mobilised friction angle, ϕ_d	$\tan \phi_d$	$F_\phi = \frac{\tan \phi_m}{\tan \phi_d}$	$N_s = \frac{c}{F_c \gamma H} = \frac{c_d}{\gamma H}$ (from Fig. 1)	c_d	$F_c = \frac{c}{c_d}$
(1)	(2)	(3)	(4)	(5)	(6)
1.5°	0.0262	5.028	0.1803	28.848	0.693
2.5°	0.0436	3.015	0.1745	27.920	0.716
3.5°	0.0611	2.152	0.1687	26.992	0.741
4.5°	0.0787	1.673	0.1629	26.064	0.767
5.5°	0.0963	1.367	0.1571	25.136	0.796
6.5°	0.1139	1.155	0.1513	24.208	0.826
7.5°	0.1316	1.000	0.1480	23.680	0.844

After the computation of ϕ_m , for determining the factor of safety F of the slope with respect to strength under combined horizontal and vertical seismic loadings, the step-by-step procedure as explained below with numerical values in Table 1 may be followed.

Step 1. Assume several values of developed/mobilised friction angle, $\phi_d (\leq \phi_m)$ (see column 1 in Table 1) of the soil.

Step 2. Calculate $\tan \phi_d$ (see column 2).

Step 3. Determine the factor of safety of the slope with respect to friction, F_ϕ , for each assumed value of ϕ_d (see column 3), as defined below

$$F_\phi = \frac{\tan \phi_m}{\tan \phi_d} \quad (26)$$

Step 4. For each assumed value of ϕ_d with slope angle i , from Fig. 1, obtain the value of stability number ($N_s = [(c/F_c \gamma H) = (c_d/\gamma H)]$) (see column 4)

Step 5. Determine $c_d (= N_s \gamma H)$ corresponding to each value of ϕ_d for the given value of γ and H of the slope (see column 5).

Step 6. Compute the factor of safety of the slope with respect to cohesion, $F_c = c/c_d$ (see column 6).

Step 7. Plot the graph of F_c against F_ϕ , on the same scale, using the values given in Table 1, and draw a line bisecting the angle between the axes, as shown in Fig. 12. The dashed part of the curve indicates that the developed/mobilised angle of internal friction, ϕ_d , cannot exceed the modified angle of internal friction ϕ_m .

Step 8. In Fig. 12, determine the factor of safety of the slope where the bisecting line intersects the curve, giving $F_c = F_\phi = 0.884$.

Step 9. As $F_c = F_\phi$ will give the same factor of safety with respect to strength F (Taylor, 1937, 1948; Steward *et al.*, 2011), $F = F_\phi = F_c = 0.884$ for $k_h = 0.5$ and $k_v = 0.25 (\downarrow)$ if the general practice of estimating factor of safety with respect to strength F is followed. As ϕ_d cannot exceed ϕ_m in the field situation, $F = F_c = 0.844$ is the more realistic value. Hence, for the slope considered, the factor of safety with respect to strength, F , is controlled by the cohesion of the soil.

It may be noted that the factor of safety with respect to cohesion with full mobilisation of the modified angle of internal friction, $F_c = 0.844$ (see section (b) of the illustrative example, p. 351, and also the last row of Table 1) is also obtained here by following the traditional graphical approach, mainly because of the low value of modified angle of internal friction, $\phi_m = 7.5^\circ$. For $k_h = 0.5$, $k_v = 0.5k_h = 0.25 \uparrow$, the modified friction angle under combined horizontal and vertical seismic coefficients is obtained as $\phi_m \approx 0^\circ$; hence, the factor of safety of the slope with respect to strength, F , is controlled by cohesion of the soil only; thus $F = F_c = 0.694$ as obtained in the authors' paper (see section (b) of the illustrative example, p. 351). In view of the facts presented here, it may be concluded that, for a low value of modified angle of internal friction, say $\phi_m \leq 10^\circ$, practising

Fig. 12. Variation of F_c against F_ϕ

engineers may avoid the iterative/graphical approach while using the Taylor's slope stability chart for designing the c - ϕ soil slope under combined horizontal and vertical seismic loads, for the sake of simplicity, without significant error.

NOTATION

c_d	developed/mobilised cohesion (kPa)
F_c	factor of safety with respect to cohesion
F_ϕ	factor of safety with respect to friction
ϕ_d	developed/mobilised angle of internal friction due to generalised earthquake (degrees)

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