DISCUSSION

A simplified working stress design method for reinforced soil walls

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Contribution by D. S. Xu

The discussor thanks the authors for their effort in developing a simplified working stress design method for calculating the maximum reinforcement loads ($T_{\max}$) in reinforced soil walls (Ehrlich & Mirmoradi, 2016). The authors tried to simplify the EM method (Ehrlich & Mitchell, 1994), which was further developed by Liu & Won (2014), Liu (2015) and Liu (2016a, 2016b) to take account of the non-constant Poisson ratio of soil and soil dilatancy. However, the proposed simplified working stress design method has some points that require further discussion or explanation, as presented in the following sections.

Derivation of K_c

In the discussed paper, the derivation of K_c is the most important parameter to determine the maximum tensile forces mobilised in the reinforcement, $T_{\max}$; however, the derivation of K_c has some points that seem to be inappropriate.

- (a) The expression K_c in equation (27) has a typographical error. It should be

$$K_c = \frac{K_0}{1 + (1/S_i^*)} \quad \text{where} \quad S_i^* = \frac{A_r E_r (1 - \nu^2)}{A_s E_s} \quad (41)$$

- (b) The expression of the stress level (SL) given in the discussed paper contradicts the other equations (i.e. equations (27) and (29)) given by the authors.
(c) It seems that equation (29) in the discussed paper cannot be obtained based on the authors' derivations presented in the Appendix. According to the authors' derivation, the following expression is obtained

$$K_c = \frac{K_0}{1 + \{A_s E_s / [(1 - \nu^2) A_r E_r]\}} \quad (42)$$

where $K_0 = \frac{\nu}{1 - \nu}$, $K_c = \frac{\sigma'_x}{\sigma'_z}$

Incorporating the expression of E_s , $E_s = k P_a (\sigma'_x / P_a)^n \text{SL}$ and $\text{SL} = 8 K_c \sin(\phi'/2)$, equation (42) can be rearranged as

$$K_c = \frac{K_0}{1 + \{[A_s k P_a (\sigma'_x / P_a)^n \times 8 K_c \sin(\phi'/2)] / [(1 - \nu^2) A_r E_r]\}} = \frac{K_0}{1 + \{[(\sigma'_x / P_a)^n \times 8 K_c \sin(\phi'/2)] / [S_i (1 - \nu^2)]\}} \quad (43)$$

where $S_i = \frac{A_r E_r}{A_s k P_a}$

Thus, K_c is expressed as

$$K_c = \frac{K_0}{1 + \{[(\sigma'_x / P_a)^n \times 8 K_c \sin(\phi'/2)] / [S_i (1 - \nu^2)]\}} \quad (44)$$

Figure 11 shows the comparisons of the derived K_c (i.e. equation (44)) with the original expression of K_c (i.e. equation (29) in the discussed paper) and the modified K_c (i.e. equation (31) in the discussed paper). The values of the derived K_c exhibit a large difference compared with the expression of K_c given by the authors. Fig. 12 shows how K_c varies with the stiffness index S_i . The results show that K_c decreased with S_i . However, when the S_i is close to 0, this means that the backfill soil is not reinforced by the reinforcement, and thus the horizontal force is not in equilibrium and the backfill will collapse. The result of the derived K_c also proves this phenomenon, as it shows $K_c \rightarrow 0$ when $S_i \rightarrow 0$ (i.e. black line in Fig. 12).

Figure 13 shows the normalised reinforcement loads predicted with the above K_c expression and comparisons with the authors' proposed simplified method and Ehrlich & Mitchell (1994). As shown in the figure, the derived normalised reinforcement loads based on the authors' simplified method diverge widely from the results of Ehrlich & Mitchell (1994).

The ultimate soil friction angle

The authors gave two expressions of ϕ'_{ult} in the discussed paper (i.e. equations (18) and (40) in the Appendix). It can be understood that the ultimate soil friction angle of equation (40) was derived based on the hyperbolic stress-strain model. However, the authors gave another expression of the ultimate soil friction angle (i.e. equation (18)) in the discussed paper. It seems the values of ϕ'_{ult} in these two expressions are quite different when $R_f \neq 1$, as shown in Fig. 14. As indicated in the figure, the values of ϕ'_{ult} are different for the different equations.

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Equivalent at-rest K by considering facing inclination, K_{ow}

The authors extended the proposed simplified method to calculate the $T_{\max}$ by considering the effect of facing inclinations. The expressions of K_c for vertical and

Fig. 11. Comparison of K_c plotted against the effective friction angle ($v=0.2$, $S_i=0.71$)

Fig. 12. Comparison of K_c plotted against S_i ($v=0.2$, $\phi'=37^\circ$)

Fig. 13. Comparison of normalised reinforcement load for a 5 m high wall determined by the new derived formula in this discussion (solid lines), the formula provided in the discussed paper (dashed lines) and Ehrlich & Mitchell (1994) (dash-dotted lines)

non-vertical walls are

$$K_c = \frac{[1 + 8S_i^* \sin(\phi'_{ult}/2)]K_0}{2 + 8S_i^* \sin(\phi'_{ult}/2) - K_0} \quad \text{for } \omega < 6^\circ \quad (45)$$

Fig. 14. Comparisons of ϕ'_{ult} for $w=10^\circ$, $S_i=2$, $R_i=0.7$

Fig. 15. Comparisons of the normalised K_{ow} and K_a for different facing inclinations

$$K_c = \frac{[1 + 8S_i^* \sin(\phi'_{ult}/2)]K_{ow}}{2 + 8S_i^* \sin(\phi'_{ult}/2) - K_{ow}} \quad \text{for } \omega > 6^\circ \quad (46)$$

For the inclined facing conditions, the horizontal equilibrium is different from the vertical wall. The K_r equation (i.e. equation (14)) presented in the discussed paper is different from that given in the Appendix (i.e. equation (35)).

Figure 15 shows the differences between K_{ow} , K_a and K_0 for different wall inclinations. Comparisons are presented for three soil friction angles (i.e. 30° , 40° , 50°). The authors mentioned that the K_a was obtained based on the Coulomb earth pressure theory. Unfortunately, the K_a in the discussed paper did not take into account the friction between the backfill and the wall, whereas this friction is considered in the Coulomb theory and the American Association of State Highway and Transportation Officials (Aashto) method. Results of comparisons show that the values of the equivalent at-rest earth pressure coefficient K_{ow} are between the values of K_a and K_0 .

AUTHORS' REPLY

The authors thank the discussor for his comments; the points he raises will be useful to readers. The points are mainly related to the derivation of the three parameters in the discussed paper: K during compaction, K_c ; ultimate soil

friction angle, ϕ'_{ult} ; and equivalent at-rest K considering facing inclination, K_{ow} . Clarifications for these points are provided as follows.

K_c evaluation

As correctly mentioned by the discussor, equation (27) in the discussed paper has a typographical error. This equation should be expressed as

$$K_c = \frac{K_0}{1 + (1/S_i)} \quad (47)$$

where

$$S_i = \frac{A_r E_r (1 - \nu^2)}{A_s E_s}$$

S_i is the relative soil–reinforcement stiffness index considering linear elastic behaviour for soil and reinforcement. To take into consideration the non-linearity of the soil behaviour, E_s can be expressed as

$$\frac{E_s}{1 - \nu^2} = k P_a \left(\frac{\sigma'_x}{P_a} \right)^n SL \quad (48)$$

The stress level, SL, decreases with soil strain and thus with K_c . Assuming $SL = K_c/8 \sin(\phi'/2)$, which was back-calculated to fit better with the original method, the parameter S_i can be rearranged as follows (in the discussed paper, the SL equation has a typographical error)

$$S_i = \frac{8 \sin(\phi'/2) S_i^*}{K_c} \quad (49)$$

Thus

$$K_c = \frac{K_0}{1 + [K_c/8 \sin(\phi'/2) S_i^*]} \quad (50)$$

Substitution of $K_c = K_0 / \{1 + [1/8 \sin(\phi'/2) S_i^*]\}$ into the equation above gives

$$K_c = \frac{K_0}{1 + [(K_0 / \{1 + [1/8 \sin(\phi'/2) S_i^*]\}) / 8 \sin(\phi'/2) S_i^*]} \quad (51)$$

Rearrangement of equation (51) gives equation (29) as presented in the discussed paper. As mentioned in the original paper under discussion, an adaptation is required for equation (29); for $S_i = 0$, a value of $K_c = K_a$ is required. Therefore, equation (29) is adapted as equation (31).

It should be noted that in the actual field conditions the wall may be far from failure and, in this condition, the determined T_{max} is higher than the corresponding Rankine K_a condition (see Fig. 16). In a specific layer, hypothetically, under zero horizontal strain, ϵ , the soil would be in the stress state corresponding to at-rest conditions and there would be no tension load in the reinforcements. With increasing horizontal strain, the horizontal soil stresses, σ_s , decrease, approaching the active condition. Simultaneously, the reinforcement loads, T_{max} , increase until equilibrium of the reinforced soil mass is reached. This equilibrium can be achieved with relatively small strains, and T_{max} under those conditions would be closer to the value corresponding to the at-rest condition, K_0 , when the reinforcement is stiffer. When the reinforcement has lower stiffness, the strain required to reach equilibrium is higher and T_{max} approaches the value corresponding to the active condition, K_a . This behaviour is controlled by the soil–reinforcement relative stiffness, S_i , as defined by Ehrlich & Mitchell (1994). Note that this simple

Fig. 16. Stress–strain compatibility between soil and reinforcement

model may represent the stress–strain behaviour of a reinforcement layer in a wall without the toe restraint condition. Nevertheless, if the combined effect of facing stiffness and toe restraint cannot be neglected, the reinforcement load may significantly reduce (Ehrlich & Mirmoradi, 2013; Mirmoradi & Ehrlich, 2015, 2016a, 2017; Mirmoradi *et al.*, 2016).

ϕ'_{ult} evaluation

As mentioned in the discussed paper, the ultimate soil friction angle of equation (40) was derived based on the hyperbolic stress–strain model. Then, based on this equation, equation (18) was derived to fit better the values calculated by the original EM method. As shown in Fig. 3 and Fig. 7 of the discussed paper, the results determined using the new proposed method agree well with those calculated by the original EM method and the numerical analysis considering different R_f values.

K_{ow} evaluation

The parameter K_{ow} defined in the discussed paper is the equivalent at-rest earth-pressure coefficient that takes into consideration the effect of the facing inclination on the calculation (equation (20)). As stated in the discussed paper, equation (20) is a rearrangement of equation (32) as used by the Aashto simplified method. Note that for the calculation of T_{max} using the original EM method, the effective vertical stress is calculated using Meyerhof's (1955) method (equation (2)). As mentioned in the discussed paper, equation (2) was developed for a vertical wall. In the new proposed method based on the results of numerical studies, this equation is recommended to be used for $\omega < 6^\circ$ (ω is the facing inclination to the vertical). For $\omega > 6^\circ$, the effective vertical stress can be calculated as $\sigma'_z = \gamma' z$.

In summary, the discussed paper proposed a simple working stress design procedure for the calculation of T_{max} that can explicitly take into account the effect of compaction-induced stress, reinforcement and soil stiffness properties, and facing inclination. A good agreement was observed between the new proposed method and data from physical and numerical model studies considering different wall heights, compaction efforts, facing inclinations, reinforcement stiffnesses and soil parameters. This method does not consider the effect of the soil dilatancy in its calculation. It should be noted that the dilatancy of soil may not significantly affect the reinforcement load in reinforced soil walls under working stress conditions (Mirmoradi & Ehrlich, 2016b).

Regarding the comment raised by the discussion contributor about the magnitude of K_c for $S_i = 0$, it should be noted that this hypothetical condition would lead to $K_c = K_a$, which is the limit equilibrium condition for a reinforced soil mass, as correctly considered by the proposed method.

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