Ultimate lateral load of slope-stabilising piles

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The paper deals with the design and analysis of slope-stabilising piles and adopts limit equilibrium concepts to derive pile contribution to stability. Analytical expressions for pile ultimate load are derived in terms of the force and the moment to be used in routine slope-stability analyses to take into account pile contribution. Free and head-restrained isolated piles, with infinite and finite section capacity, in drained and undrained conditions are considered.

KEYWORDS: limit equilibrium methods; piles & piling; slopes

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NOTATION

A, B, C	failure mechanisms for infinite pile sectional strength
B1, BY, B2	failure mechanisms involving plastic hinging
d	pile diameter
k_{d1}, k_{d2}	bearing capacity factors in drained conditions for sliding and firm soil
k_{p1}, k_{p2}	passive earth pressure coefficients for sliding and firm soil
k_{u1}, k_{u2}	bearing capacity factors in undrained conditions for sliding and firm soil
L	pile length
l_1	length of the portion of the pile embedded in the sliding soil
l_2	length of the portion of the pile embedded in the firm soil
M_y	pile sectional yielding moment
m_d	dimensionless pile yielding moment in drained conditions
$m_{d,lim}$	limit dimensionless pile yielding moment for free-head piles in drained conditions
$m_{d,lim,r}$	limit dimensionless pile yielding moment for fixed-head piles in drained conditions
m_u	dimensionless pile yielding moment in undrained conditions
$m_{u,lim}$	limit dimensionless pile yielding moment for free-head piles in undrained conditions
$m_{u,lim,r}$	limit dimensionless pile yielding moment for fixed-head piles in undrained conditions
m_u^*	dimensionless pile yielding moment separating mechanisms B1 and BY in undrained conditions
s_{u1}	undrained shear strength of the sliding soil
s_{u2}	undrained shear strength of the firm soil
T_i, M_i	stabilising force and moment with respect to slip surface for mechanism i
t_d	dimensionless stabilising force in drained conditions
t_{di}	dimensionless stabilising force in drained conditions for mechanism i
t_u	dimensionless stabilising force in undrained conditions
t_{ui}	dimensionless stabilising force in undrained conditions for mechanism i
t_u^*	dimensionless stabilising force separating mechanisms B1 and BY in undrained conditions
α	slope inclination
γ	soil unit weight

λ	dimensionless pile embedment ($= l_2/l_1$)
λ_{lim}	limit dimensionless pile embedment for free-head piles
$\lambda_{lim,r}$	limit dimensionless pile embedment for fixed-head piles
λ^*	dimensionless pile embedment separating mechanisms B1 and BY in undrained conditions
χ_u, χ_d	strength ratios between sliding and firm soil in undrained and drained conditions

INTRODUCTION

The use of piles for stabilising unstable slopes or for increasing the safety conditions of a precarious slope is a popular technique reported as successful in a number of works since the 1970s (Ito & Matsui, 1975; Fukuoka, 1977; Sommer, 1977). Despite the reliability, effectiveness and success of such a technique, the design of slope-stabilising piles still presents a number of obscure points, and a unique, widely accepted procedure for routine design has not been developed. A common approach is that proposed by Viggiani (1981) which involves three steps: (a) evaluation of the shear force required to increase the stability of the precarious slope up to a desired amount; (b) evaluation of the shear force contribution that piles can offer to transfer forces from the unstable to the stable ground; (c) from comparison of calculations in (a) and (b), establish the type, number and the location of piles. In the framework of Modern Geotechnical Codes, this method is suitable for the ultimate limit state conditions, whereas the serviceability conditions, intended to evaluate the admissibility of displacements, must be necessarily tackled by more complex numerical analyses involving pile–soil interaction. Step (a) may be achieved by conventional slope-stability analyses on the basis of a limit equilibrium approach and thus computing the required additional resisting force needed to increase the safety factor up to the target value. Many contributions dealt with the evaluation of the stabilising contribution offered by isolated or rows of piles (step b). Analytical solutions were offered by Ito & Matsui (1975), De Beer & Carpentier (1977) and Viggiani (1981). The latter provided solutions for a single free-head pile embedded in fine-grained soil in undrained conditions (total stress analysis) in the framework of the limit equilibrium method. With a similar approach, Muraro *et al.* (2014) have obtained the solution for the case of a rigid (not yielding) single free-head pile in drained conditions. Ito & Matsui (1975) studied the pile reaction forces when the soil is forced to squeeze between two infinitely long piles, providing two expressions obtained by using either the theory of plastic deformation or the theory of plastic flow. De Beer & Carpentier (1977) offered improved equations, by overcoming some limitations due to the

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strong hypotheses involved in the derivation of the previous methods. A number of contributions dealt with pile–soil interaction, by means of Winkler approach (e.g. Baguelin *et al.*, 1977; Fukuoka, 1977; Guo, 2006) or continuum models (e.g. Poulos, 1973, 1995, 1999; Hull, 1993). Finally, in some studies numerical analyses by way of the finite-difference or finite-element method are performed (Ito *et al.*, 1979; Oakland & Chameau, 1984; Poulos & Chen, 1997; Kourkoulis *et al.*, 2011, 2012). Regarding step (c), different works dealt with the optimal position of piles along the slope (Lee *et al.*, 1995; Hassiotis *et al.*, 1997; Cai & Ugai, 2000; Ausilio *et al.*, 2001; Pan *et al.*, 2012). Regarding pile spacing, Smethurst & Powrie

(2007) and Kourkoulis *et al.* (2011) have highlighted how a spacing of four pile diameters represents a general trend in engineering practice as it optimises the benefit/cost ratio.

A relevant limitation of the traditional approach is that the calculation of the piles' contribution to stability refers to the critical slip surface (i.e. the one possessing the least safety factors) for the unreinforced slope. However, the presence of the piles changes the stability conditions and the critical slip surface may even not intersect piles for very close transversal spacing among the reinforcement elements. Assuming that the safety factor of the reinforced slope is the improved safety factor of the slip surface which was the critical one in the absence of reinforcement is a very coarse approximation that may lead to a severely unconservative design.

To overcome this limitation, one could take into account the pile contribution directly in the stability analysis, for every slip surface under consideration (Fig. 1), by means of a force and a moment representing the ultimate pile–soil contact pressures. Such an approach is feasible only if ready-to-use analytical expressions are available for ultimate pile load. The scope of this work is therefore to propose a comprehensive set of analytical solutions for single piles, both free and fixed against rotation at the top, for drained and undrained conditions, as reported in the following. Group/arching effects are thereby not covered in this work.

Fig. 1. Problem under consideration: assessment of pile contribution in the realm of limit equilibrium stability analyses of reinforced slopes

ULTIMATE PILE LOAD IN UNDRAINED CONDITIONS

Viggiani (1981) has derived analytical expressions for the ultimate load of a free-head single rigid-plastic pile of diameter d

Fig. 2. Failure mechanisms and associated distribution of soil reaction, shear force and bending moment in undrained conditions and total stress analysis (modified from Viggiani, 1981)

and length L , crossing an unstable horizontal layer of thickness l_1 subjected to uniform horizontal displacement, and penetrating the underlying stable soil for a length l_2 , both modelled as rigid-plastic purely cohesive materials. Thereby, such a formulation is suitable for treating undrained conditions through a total stress analysis in fine-grained soils where the upper and lower layers have undrained shear strengths equal to s_{u1} and s_{u2} , respectively. Viggiani (1981) followed a limit equilibrium approach, similar to that proposed by Broms (1964a) for piles loaded by transverse loads applied at their upper end, and identified six possible failure mechanisms, corresponding to a stiff pile having infinite (mechanisms A, B, C) and finite (mechanisms B1, BY, B2) yielding moment M_y of the cross-section (Fig. 2). More specifically, mode A occurs when the embedment in the stable layer is small, so that the sliding soil carries the pile and the latter fully mobilises soil strength in the stable layer. Limited contribution to stability is obtained by the insertion of piles. Mode C, also referred as 'flow mode', occurs for a large embedment and the unstable soil fails around the pile. Maximum contribution is expected from the pile to stabilise the slope. In between the above

mechanisms, an intermediate mode B is found, where the soil strength is fully mobilised everywhere. When bending moments reach the yielding moment in a certain pile section, plastic hinging occurs and modes B1, BY and B2 are possible.

It is convenient to define the dimensionless quantities

$$\chi_u = \frac{k_{u1}s_{u1}}{k_{u2}s_{u2}} \quad (1a)$$

$$\lambda = \frac{l_2}{l_1} \quad (1b)$$

$$m_u = \frac{M_y}{k_{u1}s_{u1}dl_1^2} \quad (1c)$$

$$t_u = \frac{T}{k_{u1}s_{u1}dl_1} \quad (1d)$$

where k_{u1} and k_{u2} are the bearing capacity factors (taken as 4 and 8, respectively, by Viggiani, 1981); T is the shear force

Table 1. Shear force and bending moment offered by the pile for the six mechanisms (modified from Viggiani, 1981)

Mode A	$t_{uA} = \frac{T_A}{k_{u1}s_{u1}dl_1} = \frac{\lambda}{\chi_u} \quad (2a)$ $\frac{M_A}{k_{u1}s_{u1}dl_1^2} = \frac{\lambda}{2} \frac{T_A}{k_{u1}s_{u1}dl_1} = -\frac{\lambda}{2} t_{uA} \quad (2b)$
Mode B*	$t_{uB} = \frac{T_B}{k_{u1}s_{u1}dl_1} = \sqrt{\frac{(1+\lambda)^2}{(1+\chi_u)^2} + \frac{(\chi_u + \lambda^2)}{\chi_u(1+\chi_u)}} - \frac{(1+\lambda)}{(1+\chi_u)} \quad (3a)$ $\frac{M_1}{k_{u1}s_{u1}dl_1^2} = \frac{(1-t_{uB})^2}{4} \quad (3b)$ $\frac{M_2}{k_{u1}s_{u1}dl_1^2} = \frac{(\lambda - \chi_u t_{uB})^2}{4\chi_u} \quad (3c)$ $\frac{M_B}{k_{u1}s_{u1}dl_1^2} = \frac{(\lambda - \chi_u) \left(\lambda + \chi_u(3 + \chi_u + 3\lambda) - 2\sqrt{\chi_u(\chi_u^2 + \lambda^2 + 2\chi_u(1 + \lambda + \lambda^2))} \right)}{4\chi_u(1 + \chi_u)^2} \quad (3d)$
Mode C	$t_{uC} = \frac{T_C}{k_{u1}s_{u1}dl_1} = 1 \quad (4a)$ $\frac{M_C}{k_{u1}s_{u1}dl_1^2} = \frac{1}{2} \quad (4b)$
Mode B1†	$t_{uB1} = \frac{T_{B1}}{k_{u1}s_{u1}dl_1} = \frac{\lambda}{\chi_u + 2} \left[\sqrt{\frac{2\chi_u + 2}{\chi_u} + 4\frac{\chi_u + 2}{\lambda^2} m_u} - 1 \right] \quad (5a)$ $\frac{M'_2}{k_{u1}s_{u1}dl_1^2} = \frac{1}{4\chi_u} (\lambda - \chi_u t_{uB1})^2 \quad (5b)$ $\frac{M_{B1}}{k_{u1}s_{u1}dl_1^2} = \frac{(2 + 3\chi_u)\lambda^2 - 2\chi_u^2(2 + \chi_u)m_u - 2\sqrt{2}\lambda\sqrt{\chi_u(1 + \chi_u)\lambda^2 + 2\chi_u m_u(2 + \chi_u)}}{2\chi_u(2 + \chi_u)^2} \quad (5c)$
Mode BY	$t_{uBY} = \frac{T_{BY}}{k_{u1}s_{u1}dl_1} = 2\sqrt{\frac{m_u}{(1 + \chi_u)}} \quad (6a)$ $\frac{M_{BY}}{k_{u1}s_{u1}dl_1^2} = m_u \frac{1 - \chi_u}{1 + \chi_u} \quad (6b)$
Mode B2	$t_{uB2} = \frac{T_{B2}}{k_{u1}s_{u1}dl_1} = \frac{\sqrt{1 + (2\chi_u + 1)(1 + 4m_u)} - 1}{2\chi_u + 1} \quad (7a)$ $\frac{M''_1}{k_{u1}s_{u1}dl_1^2} = \frac{1}{4} (1 - t_{uB2})^2 \quad (7b)$ $\frac{M_{B2}}{k_{u1}s_{u1}dl_1^2} = \frac{2m_u - \chi_u(3 + 2\chi_u - 4m_u) + 2\sqrt{2}\chi_u\sqrt{1 + \chi_u + 2m_u + 4\chi_u m_u}}{2(1 + 2\chi_u)^2} \quad (7c)$

*In the original work there is an error in the expression equivalent to equation (3c): the parameter χ (herein denoted as χ_u) multiplying the dimensionless shear force t_{uB} is omitted. The same correction has been reported by Chmoulian (2004)

†In the original work, there is a typographical error in the expression equivalent to equation (5a): parameter λ instead of χ is reported in the denominator in the first term under the square root. The same correction has been reported by Chmoulian (2004)

and M_y is the yielding moment. Dimensionless values of the reaction force and moment, with respect to the slip surface, offered by the pile for the six mechanisms may be obtained from equilibrium equations and are reported in Table 1. As an element of novelty over previous studies, the value of moment has been reported.

For a given soil and pile configuration, the minimum shear force at the level of the interface between the two layers among these mechanisms represents the contribution offered by the pile to stabilise the upper soil layer in Fig. 1.

It is evident that such a method is directly applicable when a clear slip surface may be identified as the separation between two layers with sharply differing strengths. However, the analytical solutions provided by the author may be used in routine stability analyses for curved slip surfaces (where the particular condition of no strength contrast between unstable and firm soil occurs) along with bending moments at the level of the slip surface derived in this work, reported in Table 1. Note that this moment is negative and of small magnitude for mode A while it is positive for mode C. In the latter case, the resultant force is located above the interface and thereby represents an unfavourable contribution.

With the aim of further exploring the analytical results and their physical meaning, a schematic representation of all the different mechanisms in the plane $t_u:\lambda$ is offered in Fig. 3. For any value of m_u , the trend of dimensionless shear force t_u by varying λ has three distinct zones. First, for small embedment ratios, a linear trend is observed (mechanism A), followed by a parabolic trend (mechanism B1 or B, depending on m_u). For larger λ the value of t_u is constant (mechanisms BY, B2 or C). Mechanisms B1 and BY are separated by a straight line having the equation

$$t_u = \frac{\lambda}{\chi_u} \frac{1}{1 + \sqrt{\chi_u + 1}/\chi_u} = \lambda \left(\sqrt{\frac{\chi_u + 1}{\chi_u}} - 1 \right) \quad (8)$$

for $\lambda < \lambda^*$

the value of λ^* being

$$\lambda^* = \frac{(\sqrt{\chi_u + 1} - 1)}{\chi_u} \quad (9)$$

For larger λ , the limit value t_u^* separates mechanisms BY and B2 and assumes the value

$$t_u^* = t_u(\lambda^*) = \left(\frac{\sqrt{\chi_u + 1} - 1}{\chi_u} \right) \left(\sqrt{\frac{\chi_u + 1}{\chi_u}} + 1 \right) \quad (10)$$

for $\lambda > \lambda^*$

Failure according to mechanism BY or B2 depends on whether the dimensionless yielding moment m_u is lower or larger than a critical value m_u^* , whose analytical expression is

$$m_u^* = \frac{1 + \chi_u}{2\chi_u^2} \left[\frac{\chi_u + 2}{2} - \sqrt{1 + \chi_u} \right] \quad (11)$$

There is a threshold value of λ beyond which the dimensionless shear force on the sliding plane is constant and equal to the maximum value $t_u = 1$. This is particularly interesting from a design viewpoint, because it indicates the minimum value of embedment length in the firm soil, for a fixed thickness of the unstable layer, to achieve the maximum shear force for a given section capacity. This limit value, λ_{lim} , is obtained by intersecting mechanisms B and C as

$$\lambda_{lim} = \chi_u + \sqrt{2\chi_u^2 + 2\chi_u} \quad (12)$$

Fig. 3. Schematic representation of dimensionless shear force for a free-head pile in undrained conditions and associated failure mechanisms

The maximum shear force is obtained for mechanism C and for a particular condition occurring in mechanism B2 – that is, when the plastic hinge is located at the sliding plane ($f_2 = 0$ and $g_1 = 0$ in Fig. 2) with a value of dimensionless section capacity

$$m_{u,lim} = \frac{\chi_u + 1}{2} \quad (13)$$

which thereby indicates the value of the yielding moment above which no benefit is obtained in terms of pile performance. For different values of strength ratios and bending sectional capacity, the normalised pile shear force at the interface is represented in Fig. 4.

With the aim of providing a more comprehensive set of ready-to-use expressions for pile contribution to slope stability, the following sections report an extension of Viggiani's (1981) solutions for head-restrained piles, in undrained and drained conditions.

Piles with head restrained against rotation

In engineering practice, it is not rare the use of two or more parallel rows of slope-stabilising piles connected at the top by stiff structural elements. In these conditions, modelling piles' heads as fixed against rotation may be a more suitable choice.

By inspecting Fig. 2, it is straightforward to notice that among the six mechanisms detected for free-head piles, B and B2 imply rotation at pile top and thus only mechanisms A and C (for infinitely resistant piles) and mechanisms B1 and BY (involving section yielding) are admissible for the kinematics of the problem under investigation. It can be proven, yet is not done here in the interest of space, that any mechanisms other than the ones considered previously are not admissible.

A schematic representation of the different mechanisms in the plane $t_u-\lambda$ is offered in Fig. 5. The beneficial effect of the head restraint is evident for large values of sectional capacity: a pile that would fail with mechanism B under free-head conditions, due to the head fixity fails with mechanism A or C, whereas piles associated to mechanism B2 turn to BY, increasing their ultimate load. On the contrary, for low values of sectional yielding capacity, no benefit is observed in terms of magnitude of the stabilising force as even for free-head piles a plastic hinge below the pile head would control pile failure.

Mechanisms B1 and BY are separated again from the line described by equation (8), up to a limit value of λ equal to

Fig. 4. Dimensionless shear force for free-head piles in undrained conditions for different values of dimensionless yielding moment and strength ratio of sliding and firm soil: (a) $\chi_u = 0.25$; (b) $\chi_u = 0.5$

$$\lambda_{lim,r} = \frac{1}{\sqrt{(\chi_u + 1/\chi_u) - 1}} = \chi_u \left(\sqrt{\frac{\chi_u + 1}{\chi_u}} + 1 \right) \quad (14)$$

For $\lambda > \lambda_{lim,r}$ the maximum shear force is obtained for any value of $m > m_{u,lim,r}$, the latter being equal to

$$m_{u,lim,r} = \frac{\chi_u + 1}{4} \quad (15)$$

It can be noticed that this value is half of the one corresponding to the free-head case. Values of yielding moment larger than $m_{u,lim,r}$ can be helpful in the mechanism B1, where the shear force increases for values of m up to the corresponding limit values for free-head piles, for which mechanism B1 occurs in the conditions described by the point of intersection of A and C. Quantitative results for different values of strength ratios and dimensionless yielding moment are represented in Fig. 6. It is worth noting that for mechanism C the head moment is indeterminate and thereby the interface moment is unknown. However, in a design problem the optimum condition is represented by the point where mechanisms B1, BY and C coincide, where the moment at head is equal to the yielding moment for $m_u = m_{u,lim}$.

ULTIMATE PILE LOAD IN DRAINED CONDITIONS

In this section, the ultimate pile load is derived for drained conditions. Calculations are carried out in the special case of dry cohesionless soil. Nevertheless, when the groundwater level is coincident with the free surface, all the results are still valid if the buoyant unit weight is utilised instead of the dry unit weight. It would be straightforward to extend the derivations to the case of generalised drained conditions (i.e. the soil possesses positive values of both effective

Fig. 5. Schematic representation of dimensionless shear force for a fixed-head pile in undrained conditions and associated failure mechanisms

Fig. 6. Dimensionless shear force for fixed-head piles in undrained conditions for different values of dimensionless yielding moment and strength ratio of sliding and firm soil: (a) $\chi_u = 0.25$; (b) $\chi_u = 0.5$

cohesion and friction angle) and groundwater level at an arbitrary location, yet this is not done here due to space limitations.

Figure 7 is a sketch of the failure mechanisms with associated patterns of soil reaction and internal forces along the pile. From equilibrium considerations, as already shown by Muraro *et al.* (2014) dimensionless shear force and bending moment at the interface for mechanisms A and C are obtained as

Mode A

$$t_{dA} = \frac{T_A}{k_{d1} \gamma d l_1^2} = \frac{\lambda}{\chi_d} \left(\frac{\lambda}{2} + 1 \right) \quad (16a)$$

$$\frac{M_A}{k_{d1}\gamma d l_1^3} = -\frac{\lambda^2}{\chi_d} \left(\frac{1}{2} + \frac{1}{3}\lambda \right) \quad (16b)$$

Mode C

$$t_{dC} = \frac{T_C}{k_{d1}\gamma d l_1^2} = \frac{1}{2} \quad (17a)$$

$$\frac{M_C}{k_{d1}\gamma d l_1^3} = \frac{1}{6} \quad (17b)$$

where γ is the unit weight of the soil, taken for simplicity as identical for the two layers, $k_{d1} = 3k_{p1}$ (following Broms, 1964b) and $\chi_d = k_{p1}/k_{p2}$.

To take into account the inclination of slope α , Rankine's passive earth pressure coefficients k_{p1} and k_{p2} must be evaluated through the classical expressions for infinite slope.

For mechanism B, the expressions for dimensionless shear force cannot be derived in closed form. Thus, the problem is tackled numerically, solving for each set of λ and χ_d values the two equilibrium equations iteratively. The solutions were obtained by means of Matlab software for both cases of infinite and finite values of cross-sectional dimensionless yielding moment (modes B1, BY and B2), defined as

$$m_d = \frac{M_y}{k_{d1}\gamma d l_1^3} \quad (18)$$

Note that parameters m_d^* , $m_{d,lim}$ and $m_{d,lim,r}$ have the same meaning as m_u^* , $m_{u,lim}$ and $m_{u,lim,r}$ yet they are indicated with different symbols due to the different normalisations. The ultimate shear force and moment offered by the pile is represented in Fig. 8 for different values of strength ratios and dimensionless yielding moment. For restrained-head piles, the same trends are observed as for the undrained case and the quantitative results are depicted in Fig. 9.

Limit values of embedment ratio and dimensionless bending moment can be obtained by interpolating the numerical results. Regression formulae for critical embedment ratio and dimensionless bending for both free- and fixed-head piles in drained conditions, depicted in Fig. 10, are

$$\lambda_{lim} = 1.192 \cdot \chi_d^{0.61} \quad (19a)$$

$$m_{d,lim} = 0.108 \cdot \chi_d + 0.17 \quad (19b)$$

$$\lambda_{lim,r} = 0.977 \cdot \chi_d^{0.64} \quad (19c)$$

$$m_{d,lim,r} = 0.053 \cdot \chi_d + 0.086 \quad (19d)$$

It is noted that, as in the undrained case, the dimensionless limit moment for a restrained pile is one-half of that associated to the free-head pile.

Fig. 7. Failure mechanisms and associated distribution of soil reaction, shear force and bending moment in drained conditions

Fig. 8. Dimensionless shear force and moment for free-head piles in drained conditions for different values of dimensionless yielding moment and strength ratio of sliding and firm soil: (a) $\chi_d = 0.5$; (b) $\chi_d = 1$; (c) $\chi_d = 0.5$; (d) $\chi_d = 1$

Fig. 9. Dimensionless shear force for fixed-head piles in drained conditions for different values of dimensionless yielding moment and strength ratio of sliding and firm soil: (a) $\chi_d = 0.5$; (b) $\chi_d = 1$

Fig. 10. Regression formulae for critical embedment ratio and dimensionless bending for both free- and fixed-head piles in drained conditions

CONCLUSIONS

In this paper, several formulae have been provided for the ultimate load of slope-stabilising piles, to be used in routine limit equilibrium slope-stability analyses. Pile contribution may be expressed through a force and a moment, derived in this work for free- and fixed-head piles, in drained and undrained conditions, for infinite and finite pile section flexural capacity. Analytical solutions have been derived for all failure mechanisms in the undrained case (total stress analysis). For the drained case, analytical solutions are provided whenever their derivation is possible, while closed-form approximate expressions, which fit well the numerical data, are reported for the other mechanisms.

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