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Simplified approach to the design of segmental tunnel linings

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A new simplified approach to the hyperstatic reaction method (HRM) for the analysis of segmental tunnel linings is introduced. The influence of segmental joints on the lining behaviour is taken into account by considering opening/rotation of the joints under external loads, which cause a reduction in the contact surface area between two segments at the joint location and certain surrounding zones. Consequently, a decrease in the inertial modulus and cross-sectional area at these sections of the lining ring is expected. A specific implementation was developed using a finite-element method framework in which an iterative procedure was applied with the height of the joint section as the unknown parameter. The results obtained using the new simplified HRM (SHRM) were compared with those obtained using the existing complex HRM, which takes joints directly into account, and a three-dimensional numerical model. The results obtained show that the SHRM can be used to estimate the behaviour of a segmental tunnel lining effectively.

Notation

A_s	area of transversal section
B	width of a lining ring (m)
b_c	width of concrete section (m)
b_j	width of joint section (m)
b_l	width of lining ring (m)
E_c	Young's modulus at concrete section
E_s	elastic modulus
E_j^i	Young's modulus at joint section
$F_1, F_2, F_3, \dots, F_n$	sub-vectors composed of the three external forces applied to each node in the global Cartesian reference system
J_{concrete}	inertial modulus of concrete section (m ⁴)
J_{joint}	inertial modulus of joint section (m ⁴)
J_s	inertial modulus of cross-section (m ⁴)
$\mathbf{K}$	global stiffness matrix
$\mathbf{k}_i$	local stiffness matrix in the global Cartesian reference system
$k_{i,a}, k_{i,b}, k_{i,c}, k_{i,d}$	sub-matrices of k_i , each of which has 3×3 dimensions
k_n, k_s	stiffness of normal springs and shear springs, respectively
L_i	distance between terminal nodes
M	bending moment of cross-section of lining ring
M_c	bending moment induced in concrete part
M_j	bending moment induced in joint part
N	total normal force acting at a cross-section (MN/m)
N_j	normal force acting on joint part (MN/m)

$\mathbf{q}$	vector of nodal displacements
$q_1, q_2, q_3, \dots, q_n$	sub-vectors composed of the three displacements of each node in the global Cartesian reference system
$\mathbf{q}_v, \mathbf{q}_h$	applied active loads
R	tunnel radius
S_j	height of contact section between two segments at the joint location (m)
S_l	height of concrete section (m)
S_{total}	average height of cross-section (m)
$\mathbf{z}_i$	local stiffness matrix of i th element
ω	angle measured counterclockwise from tunnel bottom

1. Introduction

Due to the presence of joints, the behaviour of segmental tunnel linings is different from that of continuous linings (Arnau and Molins, 2012; Do *et al.*, 2013). In the literature, the effects of segmental joints on tunnel lining behaviour have usually been considered indirectly or directly, using analytical, empirical and numerical methods. In indirect methods, the tunnel structure is perceived as a rigid lining ring (Duddeck and Erdmann, 1982; Einstein and Schwartz, 1979; Muir Wood, 1975; Oreste, 2007; Takano, 2000; Zhang *et al.*, 2014). The effect of joints is usually taken into account through a reduced rigidity of the tunnel structure. As far as direct methods are concerned, segmental joints are added directly to the tunnel lining structure and joints are explicitly considered in the analysis (Arnau and Molins, 2012; Blom, 2002; Ding *et al.*, 2004; Do *et al.*, 2013; Lee *et al.*, 2002; Naggar and Hinchberger,

Figure 1. Schematic illustration of support structures adopted in the HRM (from Do *et al.*, 2014a)

2008). A summary of the design methods used for segmental tunnel linings is given by Do *et al.* (2014a, 2014b).

Do *et al.* (2014a) developed a new numerical approach to the hyperstatic reaction method (HRM) (Figure 1), referred to as the complex HRM in this paper. In that work, segmental joints were added directly in the HRM. The effect of joints was considered using a fixity ratio that is estimated on the basis of the rotational stiffness. This method allows the three-dimensional (3D) effects of segment joints in successive rings on tunnel lining behaviour to be taken into account. In addition, it also allows an arbitrary distribution of segment joints in the lining rings to be simulated. A comparison of the numerical results obtained with the HRM and experimental data obtained from a shield-driven tunnel project showed that the HRM could be used to estimate the behaviour of a segmental tunnel lining effectively within a short calculation time.

A new simplified model of the HRM, hereafter called the SHRM, which allows the influence of joints on the behaviour of segmental tunnel linings to be analysed in another way is presented in this paper. Unlike the existing (complex) HRM proposed by Do *et al.* (2014a), in the SHRM, the influence of segmental joints is taken into account by considering the reduction of the contact surface and therefore the inertial modulus at lining sections that are influenced by joints. The applicability of the SHRM was analysed through comparisons with both the existing complex HRM and 3D numerical analysis.

The SHRM can be applied using a common HRM calculation method in order to consider the presence of joints in a segmental lining. In this way, without having to use specific and complex computing systems, anyone is able to analyse the mechanical behaviour of segmental linings and proceed to their dimensioning in a simple and effective way. This innovation allows quick analysis of the mechanical behaviour of segmental linings, taking into account all the relevant aspects, including the 3D effect of the presence of adjacent rings with a different joint arrangement.

2. Mathematical formulation of the HRM

The numerical HRM for continuous linings was developed by Oreste (2007) and the method was extended for the analysis of segmental tunnel linings by Do *et al.* (2014b). Therefore, only a short overview is given here.

In the HRM, the support structure is represented by mono-dimensional elements that are able to develop bending moments, axial forces and shear forces (Oreste, 2007). A beam element i is defined by the inertial modulus (J_s) and area (A_s) of the transversal section, the elastic modulus (E_s) of the constituent material and the length (L_i) (distance between terminal nodes). The interaction between the tunnel structure and surrounding ground is taken into consideration in two ways: through normal and tangential springs connected to the nodes of the structure and through applied active loads (q_v and q_h) (Figure 1).

The local stiffness matrix $\mathbf{z}_i$ of the i th element, which is evaluated according to a local reference system, is transferred to the local stiffness matrix $\mathbf{k}_i$ in the global Cartesian reference system before being assembled in the global stiffness matrix, $\mathbf{K}$, using the principal illustrated by Huebner *et al.* (2001).

Due to the presence of normal and tangential node springs (see Figure 1) along the tunnel structure, modifications of the corresponding elements along the diagonal of the global stiffness matrix $\mathbf{K}$ in terms of the stiffness of the normal springs (k_n) and shear springs (k_s) of each node were made (Do *et al.*, 2014b).

The unknown parameters that need to be estimated are the displacement components of the nodes of the discretised structure. The vector $\mathbf{q}$ of the nodal displacement components

$$1. \quad \begin{matrix} \xrightarrow{\text{Global stiffness matrix } \mathbf{K} \text{ of the segmental lining}} \\ \begin{pmatrix} (k_{n,d} + k_{1,a}) & k_{1,b} & 0 & 0 & 0 & k_{n,c} \\ k_{1,c} & (k_{1,d} + k_{2,a}) & k_{2,b} & 0 & 0 & 0 \\ 0 & k_{2,c} & (k_{2,d} + k_{3,a}) & k_{3,b} & 0 & 0 \\ 0 & 0 & k_{3,c} & (k_{3,d} + k_{4,a}) & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & k_{n-1,b} \\ k_{n,b} & 0 & 0 & 0 & k_{n-1,c} & (k_{n-1,d} + k_{n,a}) \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \dots \\ \dots \\ q_n \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ \dots \\ \dots \\ F_n \end{pmatrix} \end{matrix}$$

obtained from Equation 1 allows the stresses at the nodes of the structure to be determined (Huebner *et al.*, 2001).

3. The new SHRM

A segmental tunnel lining is a 3D structure in which longitudinal joints in successive lining rings are usually staggered along the longitudinal tunnel direction (Figure 2). In order to take into consideration the 3D effect of a segmental tunnel lining, a section, including two halves of two successive lining rings, is considered in the SHRM (Figure 2).

While a fixity factor (Monforton and Wu, 1963) was added to the stiffness matrix of each finite element in order to reflect the semi-rigid joint connections between segments in the research conducted by Do *et al.* (2014a), the fundamental aspect of the proposed SHRM is the assumption that the presence of longitudinal joints in lining rings causes a reduction in the cross-sectional area and the inertial modulus of the lining at this section. In other words, the influence of joints can be taken into consideration on the basis of the contact condition of the joint surface between two segments as follows.

- For a closed surface joint, the joint works as a concrete section without influence on the behaviour of the tunnel lining.
- For an opened surface joint, the joint causes a reduction in the cross-section and inertial modulus of the lining and therefore has a significant influence on the behaviour of the tunnel lining.

It is assumed that each opened joint causes an effect zone as indicated in Figures 2(a) and 2(b).

As shown in Figure 2, the presence of a longitudinal joint weakens the lining at a particular point and this effect is perceived also in neighbouring areas, whereas a dispersion effect of the discontinuity is considered with straight lines inclined at 45° with respect to tunnel perimeter. In practice, it is assumed that the effect of weakening of the longitudinal joint reduces with distance from the longitudinal joint due to the cooperation of the closest rings where a filled concrete section is present. The dispersion according to 45° inclined lines is generally considered in stress transfer problems inside concrete structures.

Figure 2. Segmental lining scheme in the SHRM: (a) true segmental tunnel lining; (b) simplified model in the SHRM; (c) cross-section A-A

The inertial modulus of a section (J_s) affected by a joint is the sum of the inertial moduli of the concrete part (J_{concrete}) and the joint part (J_{joint}) in the section, given by

$$2. \quad J_s = J_{\text{concrete}} + J_{\text{joint}} = \frac{b_c S_1^3}{12} + \frac{b_j S_j^3}{12} = \frac{b_1 S_{\text{total}}^3}{12}$$

The transversal section area (A_s) at a lining section, which is affected by a joint, is estimated to be

$$3. \quad A_s = b_c S_1 + b_j S_j$$

The bending moment of each cross-section of a lining ring, M , is equal to the sum of the bending moments induced in the concrete part (M_c) and in the joint part (M_j)

$$4. \quad M = M_c + M_j$$

From Equations 2 and 4, M_c and M_j can be estimated as follows (considering the compatibility of the section rotations in relation to the inertial moment of each part, due to the fact that the rotation of a section is considered proportional to its bending stiffness EJ)

$$5. \quad M_c = M \frac{b_c S_1^3 / 12}{J_s} = M \frac{(b_c S_1^3 / 12)}{(b_c S_1^3 / 12) + (b_j S_j^3 / 12)}$$

$$6. \quad M_j = M \frac{b_j S_j^3 / 12}{J_s} = M \frac{b_j S_j^3 / 12}{(b_c S_1^3 / 12) + (b_j S_j^3 / 12)}$$

On the other hand, M_j can also be estimated from the normal forces (N_j) as (see Figure 2)

$$7. \quad M_j = N_j \left(\frac{S_1}{2} - \frac{S_j}{3} \right)$$

where

$$8. \quad N_j = N \frac{b_j S_j}{b_c S_1 + b_j S_j}$$

Combining Equations 6 and 7 yields

$$9. \quad M \frac{b_j S_j^3 / 12}{(b_c S_1^3 / 12) + (b_j S_j^3 / 12)} = N \frac{b_j S_j}{b_c S_1 + b_j S_j} \left(\frac{S_1}{2} - \frac{S_j}{3} \right)$$

Knowing the values of M and N at each section, which are determined from the previous step, the height of the joint section S_j can be estimated as the integer root of Equation 10.

$$10. \quad \frac{N b_j}{3} S_j^4 + \left(M b_j - \frac{N S_1 b_j}{2} \right) S_j^3 + M b_c S_1 S_j^2 + \frac{N S_1^3 b_c}{3} S_j - \frac{N S_1^4 b_c}{2} = 0$$

Due to the fact that S_j at each joint is unknown at the beginning, an iterative procedure is applied with the assumption that S_j is firstly equal to the thickness of the lining section S_1 . With the height of the joint section S_j obtained from Equation 10, after each calculation step, if $M_j / (b_j S_j^2 / 6) > N_j / b_j S_j$ (which corresponds to the case in which the joint is partly opened), the above procedure is applied to determine the height of the joint part S_j . On the other hand, $S_j = S_1 = S_{\text{total}}$. These S_j values at joints in the lining are then used to estimate the inertial modulus (J_s) and area (A_s) of the transversal section at each node along the tunnel perimeter, which are used in the next step of the iterative procedure. The iterative procedure is continued until convergence of the joint height S_j is obtained.

An advantage of the new SHRM over the HRM developed by Do *et al.* (2014a) is that it allows consideration of a reduced joint area (see Figure 3). It was impossible to consider this joint type directly in the HRM proposed previously (Do *et al.*, 2014a).

4. Case study: shield tunnelling for the Bologna–Florence railway line project

Parameters from the Bologna–Florence railway line, part of the Italian high-speed rail network, were used to assess the performance of the new SHRM. In this study it is assumed that the behaviour of the soil and the tunnel structure is linear-elastic. The properties of the soil and tunnel lining are given in Table 1.

In order to assess the performance of the SHRM, a simplified 3D numerical model was developed using the generalised finite-difference program Flac3D (Itasca, 2009). A 3D simulation of a tunnel supported by means of a segmental lining

Figure 3. Decreased joint type

Table 1. Section parameters at lining ring 582 (Croce, 2011)

Parameter	Value
Soil properties	
Unit weight, γ : kN/m ³	17
Young's modulus, E : MPa	150
Poisson's ratio, ν	0.3
Internal friction angle, ϕ : degrees	37
Lateral earth pressure factor, K_0	0.5
Overburden, H : m	20
Properties of the tunnel lining	
Young's modulus, E_s : MPa	35 000
Poisson's ratio, ν_s	0.15
Lining thickness, t : m	0.4
External diameter, D : m	9.4

was simulated in the simplified 3D numerical model. In addition, the new SHRM was also compared with the existing complex HRM developed by the authors (Do *et al.*, 2014a). Detailed descriptions and evaluations of the performance of both the simplified 3D model using Flac3D and the existing complex HRM have been presented by Do *et al.* (2014a) and are therefore not repeated here.

Figure 4 shows the bending moment and normal forces induced in the tunnel linings. Without considering the spikes in the bending moment at the segmental joint locations, Figure 4(a) indicates that the bending moment results obtained by the new SHRM were in good agreement with those of the complex HRM and were more less the same as those obtained using the Flac3D model. As far as normal forces induced in the tunnel structure are concerned, Figure 4(b) shows good agreement between the results obtained with the complex HRM and the new SHRM. However, there was a significant difference between the shapes of the normal forces diagrams along the tunnel periphery obtained by the HRMs and Flac3D. This difference could be due to the fact that the methods do not consider soil behaviour in the same way: a continuous medium is assumed in Flac3D while, in the HRMs, soil action is considered by independent springs in each part of the beam elements. This indicates that several interactions cannot be taken into account by the subgrade reaction method, such as the load transfers resulting from arching effects. It should be noted that, compared with the Flac3D model, both the HRM and the SHRM slightly underestimated the normal forces. Caution is therefore necessary when using the HRM or SHRM for the design of tunnel linings. A decrease in normal forces generally causes a decrease of the allowable maximum bending moment induced in the tunnel lining. This combination of structural forces leads to a safer tunnel lining from the design point of view.

Due to movement of the tunnel lining towards the surrounding ground at the springline with angles ω of 90° and 270° in this study, reaction forces from the surrounding soil mass acting on the tunnel lining through springs placed at nodes along the

Figure 4. Comparison of (a) bending moments and (b) normal forces in the tunnel lining obtained using the SHRM, the complex HRM and Flac3D

tunnel section in the HRMs will develop. The compressive normal forces produced in the tunnel lining at these sections will therefore be transferred to the sections at which normal forces are currently smaller (at the crown and invert of the tunnel with angles ω of 90° and 270° in this study). Consequently, normal forces at these sections, which were originally small, will increase, as shown by the HRMs (Do *et al.*, 2018).

The following comments can be made on the basis of the above analysis.

- The structural force results obtained using the SHRM were globally in good agreement with the results of the complex HRM and the numerical Flac3D model. However, this was not the case locally near the joint location.
- The influence of joints in segmental linings can be taken into consideration through the reduction of the transversal section and therefore through the inertial modulus of the lining. This simplified way of taking joints into account gives the same results as a more complex method (the case of a true joint).

5. Conclusions

A new simplified approach to the HRM applied to a segmental tunnel lining has been presented and the adopted mathematical procedure detailed. The influence of segmental joints was considered directly using a reduction in the transversal section and therefore the inertial modulus of sections along lining rings, which are influenced by the presence of segmental joints. A specific implementation was developed using a finite-element method framework. The SHRM allows the arbitrary distribution of segmental joints along a tunnel boundary to be taken into consideration.

A simplified 3D numerical model developed using Flac3D software and the existing complex HRM were used to assess the performance of the new SHRM. The numerical results presented in this paper show that the SHRM can be used to investigate the behaviour of a segmental tunnel lining instead of 3D numerical modelling.

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