A computer design aid for prestressed concrete box beams

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Proc. Instn Civ. Engrs Structs & Bldgs, 1993, **99**, May, 237–238

Paper 9725

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In the quest for lighter, more efficient solutions, wall thicknesses in box girder bridges have been reduced steadily over the years to the point where torsional and distortional warping and shear lag effects cannot be ignored.

Because of this need to consider thin-walled effects there has been a move towards the use of full three-dimensional finite-element (FE) analysis early in the design process. This is a pity since spine beam bridges of medium to long span are essentially linear structures and should, if possible, be analysed as such.

34. For this reason, the Authors are to be commended for presenting a simple analytical technique for box girders in which all the important thin-walled actions are included in a single beam element. Such a method is useful for statutory checking and may be sufficiently accurate for final design. However, it is perhaps in the preliminary design stage where the method will be of greatest value, since the ease with which the geometry and prestressing may be modified will enable the designer to try alternatives and to be much more innovative in identifying the optimum design solution.

35. My own work in this area has followed almost exactly the same path as that presented by the Authors, with the development of a similar thin-walled beam element. However, rather than approximating thin-walled beam behaviour with an isoparametric beam element for inclusion within an FE model, an equivalent beam element has been developed which conforms exactly with the assumptions made by Vlasov in the development of his thin-walled beam theory.^{13,14} The principal advantage of the equivalent beam element is that it is identical to the solid beam element used in conventional beam and grillage analyses, but with additional degrees of freedom to allow for the thin-walled beam actions. This enables designers to use existing computer programs with which they are familiar without recourse to FE packages. Such an approach has been adopted in the analysis of a range of closed thin-walled structures such as box girder bridges, shear cores and grillages,¹⁵ and of open thin-walled sections such as demountable military bridge decks.¹⁶

36. In § 20, the Authors mention the pre-processor program which calculates the 15 section properties required for each beam element. These properties are easily determined

for straight beams¹⁷ but may vary significantly when beam curvature is included.¹⁸ These curvature effects have little influence on the parameters which determine longitudinal performance but have a marked effect on the distribution of stresses around the cross-section. I should be grateful for the Authors' comments on curvature and clarification as to whether or not curvature effects are included in the calculation of section properties in their analysis.

37. With regard to flexural shear lag, I agree with the Authors that this effect may be relatively insignificant near the intermediate supports. This is attributable to the beneficial effects of prestressing counteracting a large proportion of the bending moment due to self weight and externally applied load. As the axial stresses generated at the intermediate supports due to torsional and distortional warping cannot generally be reduced by prestressing, they may assume greater relative significance. It would appear that the Authors have accounted only for flexural shear lag in their model and not for the other components of shear lag associated with the torsional and distortional axial stress distributions shown in Figs 7(c) and 7(d) for single cell, and Figs 8(c) and 8(d) for twin cell sections. Would it be possible to modify the model to accommodate these extra shear lag components, and would the Authors expect their effect to be significant?

38. Finally, could the Authors confirm that the transverse distribution of shear stress due to torsional warping restraint in single and twin cell sections is as shown in Figs 19(a) and 19(b) and not as shown in Figs 7(g) and 8(g) in their Paper.

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Professor Waldron's researches are valuable in developing an alternative analytical method; his earlier publication⁵ described an elegant flexibility approach for a beam matrix which was subsequently inverted. It was not clear to us how this method could be extended to include transverse distortion and longitudinal variation in section thickness, or how the load vector could incorporate HB vehicles and the effect of prestress tendons which follow the curvature of the beam and the vertical profile. We have noted the inclusion of distortion in the forthcoming publication.¹⁴

40. Our method and Professor Waldron's deal with curvature differently; the advantage

Paper published: Proc. Instn Civ. Engrs Structs & Bldgs, 1992, **94**, Feb., 61–72

Fig. 19. Shear stress distribution due to torsional warping restraint around (a) single cell, and (b) twin cell, box girder bridge

of the isoparametric finite-element formulation of the stiffness matrix is that curvature effects are brought into the calculation by the shape functions representing the curvature of the beam.

41. Professor Waldron raises three points in § 37. The first concerns the description of shear lag in § 18. Shear lag can lead to appreciable peaks in the longitudinal stresses in the flanges in the vicinity of the webs when live and dead loading are present, but the nature of the profile required for continuous prestressing leads to small shear lag effect—unfortunately,

this can lead to a need for additional prestress tendons at the supports simply to overcome the local peaks in stresses. The second point is largely a matter of definition, and we would wish to draw attention to Figs 7(h) and 8(h), and also to Maisel's figures A1/9(b), A1/18(b), and A3/36(b) of reference 4. The third point concerns shear lag in the longitudinal stresses caused by torsion and distortion effects; these may be readily introduced into CWBOX, but so far the significance of the effect has not justified this. Nevertheless, Professor Waldron is right to alert readers to this form of shear lag.

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