Discussion.

The President. The PRESIDENT, in moving the vote of thanks to the Author, observed that the Paper referred to various experimental methods which had been investigated in recent years for the purpose of determining stresses in bridges and framed structures, and more particularly structures where the number of members was such as to render calculations indeterminate. The Author indicated that one method of mathematical attack was by the principle of virtual velocities. Those who had investigated the subject mathematically would no doubt comment on that and other methods of a highly theoretical character. The Author also gave an interesting account of the experimental methods on which he had been engaged, and he had made a valuable contribution, both practical and theoretical, to a subject which was engaging a great deal of attention at the present time.

The Author. The AUTHOR, in showing some lantern-slides, observed that a piece of apparatus which was not described in the Paper might be of interest. This was a lateral extensometer, used for measuring changes of thickness of models. The standard type had a horse-shoe shaped body, one end of which carried a needle moved by a micrometer screw. The other end carried the extensometer with another needle, in the same line, operating a bell-crank lever, which latter turned a mirror, and the deformation of the specimen was measured by the movement of a spot of light on a screen. The front part of the extensometer was carried by a pair of spring plates, and a tail rod was provided at the back fitting into a ball-and-socket joint so that the instrument could be aligned with the bar on which the measurements were made. The whole of the measuring-extensometer swung about the centre of the needles. The mounting was similar to, but rather more elaborate than an engineer's compound slide-rest, with micrometer attachments for moving the extensometer into any position desired. The entire apparatus was carried on a vertical pillar, with arrangements which enabled the instrument to be swivelled round to any azimuth and set in any position within wide limits.¹

¹ E. G. Coker, "Lateral Extensometers." *Engineering*, vol. 129 (1930), p. 465.

Mr. H. J. FEREDAY observed that, as the Author had made particular reference to what might be called the secondary stresses in bridge structures, he wished to make certain comments which might be of interest, in particular to those dealing with bridgework. He complimented the Author on the bibliographical notes accompanying his Paper. The Author might perhaps have mentioned that the fundamental law governing all the stresses in question had been formulated in the time of Newton by Hooke. Young's modulus followed some time afterwards. Following those men came Euler, Lagrange, Clerk Maxwell, Castigliano and others, and with them must be included the late Mr. F. E. Robertson, M. Inst. C.E., and the late Mr C. F. Findlay, M. Inst. C. E., who were both Chief Engineers of the East Indian Railway; the former dealt with the stresses in statically indeterminate structures by the law of virtual velocities in an article in *Indian Engineering* in 1892; and the latter dealt with the elasticity of a floor-system in a Paper¹ discussed in 1900. He mentioned the last two names because as far back as 37 years ago in the one case and 29 years in the other those engineers were dealing specifically with secondary stresses, and since that time—he himself had worked under Mr. Robertson—those stresses had been dealt with in bridgework for India wherever it was found necessary to do so. The apparatus shown in *Fig. 4* was extremely interesting; the control of a small linear motion by two flat diaphragms was used in other mechanisms, but not nearly so much as it ought to be. With regard to the method of measuring the load by the deformation of a ring, not many months ago he had come across a similar apparatus resembling the link of a stud-link chain. The stud was sawn through, so that the stud-link became a long link and was thus used on a crane-chain for measuring the load. Putting on the load brought the two faces of the saw-cut in the stud together, and the load was read on a dial-gauge operated by the relative movement of the two parts of the stud. The width of the saw-cut was such that, when the faces touched, the maximum crane-load was reached, and that was indicated by the pointer's ceasing to register; the long link being then a stud-link, its strength was increased at least 25 per cent. so that although the crane-chain might be overstressed, the weighing-apparatus would not be harmed. A question that concerned the bridge-builder was that of making a model which represented the bridge he desired to design or had designed, to prove whether his calculations were correct. He had been engaged on the design of bridgework for a long time, but had

¹ Minutes of Proceedings Inst. C.E., vol. cxli (1900), p. 17.

Mr. Fereday. only once had occasion to make a model, when he made a scale model in steel, three-eighths full size, of a bent column, the stresses in which were very difficult to calculate. It was tested under the conditions to which the member would be exposed in the structure. The results obtained by calculations and by the test did not differ very much, but the results of the test had been taken in determining the rest of the calculations. Secondary stresses, of course, were extremely laborious to evaluate for bridgework. They sometimes had to be found, but they were only a fraction of the main stresses, and there was a margin of safety between the permissible strength and the elastic limit of the material ; and in many cases it was known from experience that they were negligible. The secondary stresses in ordinary trusses were not in fact nearly so difficult and complex as the stresses in the floor-system of the bridge. In India a large number of iron bridges had been strengthened by the addition of steel structures alongside of, and fastened to, the old girders. All the calculation was done by treating the bridges as elastic structures ; only in that way could one be satisfied that the division between the old and the new work was correct. A very difficult problem encountered 25 years ago was the origin of the need that had been felt for a stress-recorder. Many of the so-called secondary stresses in main girders and arch-ribs could be eliminated by special design, accurate manufacture, and a certain order of erection, and this elimination was achieved in many of the bridges sent to India ; many of the bridge-builders in this country were second to none in the accuracy of their bridgework. When such bridges were erected, the secondary stresses were a maximum under no load, decreased under live load, and practically vanished under the maximum load for which the bridge was designed. The secondary stresses in the floor-system were dealt with in a somewhat similar manner. He was not in a position to deal with the physical parts of the Paper ; but he wanted to show that bridge-builders were dealing with the problems described. If they had to read all the literature published on the ultra-scientific aspects of the subject they would have no time to design bridges. They did, however, read much of the work published, picking out the essential for use and discarding the superfluous.

Professor
Guest.

Professor JAMES J. GUEST thought that the Paper might have far-reaching effects. There were, however, one or two places where the meaning was not quite clear. Referring to the stress-determination by means of lateral measurements of the strain across the thickness of the piece, the Author said (p. 36) " The method has an advantage over the strain-ellipse method, in that the total

strain is the sum of all the strains along a line in the member perpendicular to its two parallel faces, and to that extent is more truly representative of the stress-distribution at a point on the surface than are the measurements of the deformation of a surface circle surrounding it." Just before that passage, however, the Author referred to various experimenters who had attacked problems which had not been solved by theory—such as the case of the dam—by making a model of some material, describing circles on it, and then loading it. Measurements of the distortion of the circle gave the principal strains and hence determined the two principal stresses; but if one quantity alone was measured, two independent quantities could not be determined thereby. The Author measured only one thing—which he said was the most important—but several experimenters had measured two, from which all could be determined. He did not think the thickness strain was the most important measurement. It gave the sum of the two principal stresses, but that sum was not the vital stress factor for the material. He had once had put to him—as had probably many others—the problem of the strength of a crank-shaft. Crank-shafts failed in the angle between the web and the pin. The corner, of course, was radiused, and the question was, how much radius was desirable and what was the effect of varying that radius. For the experimental part of the investigation he made an india-rubber model and drew circles on it and strained it. In that way he obtained the two principal stresses at the surface. It was a tedious method, but it had a wider application than the advocated method, because it dealt with materials shaped in three dimensions, while the method of the Paper was confined to two-dimensional work. The method he had used, therefore, was not to be despised, since on occasion it helped an investigator out of a difficulty. A point arose with reference to those stress lines shown in *Fig. 3* that were very close to the corner. He thought that when a corner occurred like that in a material which was thick compared with the radius of the corner there was a wide difference in the stress through the thickness of the material—the variation was 40 per cent. in a case he had calculated—so that the optical method would give only an average value, and not the actual maximum stress in the material. The variation was affected by the value of Poisson's ratio, and he did not know the value of that ratio or of Young's modulus for cellulose. India-rubber had a Poisson's ratio of about 0.45, which made a considerable difference if a model in three dimensions was used for a test, because the ratio for steel was only 0.30 or 0.31. In the two-dimensional cases of the Author's method one was not troubled

Professor
Guest.

Professor
Guest.

about Young's modulus. It would, however, be interesting to know the values for cellulose, as it would enable the nicety of the exceedingly close experimental work described to be better estimated. In the method illustrated in *Fig. 7* the stresses were measured round the perimeter. Taking, for example, the central vertical member, on the one side there was a tension and on the other side a compression. At first sight it would appear that there were only the bending-moment and the axial thrust or pull in the member, and the Author remarked (p. 46) that "observation of the colour-bands in this bridge shows that in the free parts of the members between the joints the stress system is mainly direct stress accompanied by bending." But the bending-moments at the ends of that vertical member were of the same sign, and therefore there must be a shear in order to balance their sum. The same remark, he thought, applied to most of the Figures given. *Fig. 9* showed such a distribution, and the shear-stress curve AEB was shown. It seemed to exist in all cases. He would ask whether the curve AEB giving the distribution had been directly calculated from some of the Author's experimental results? The magnitude of the bending-moment shown in the diagrams generally surprised him, and the secondary stresses would be very much more than one would expect, especially as the axial load in some cases was quite light. The usual way of calculating the secondary stresses was to find the primary stresses first, on the assumption that the structure was pin-jointed. Then the distortion of the figure was calculated, the joints were assumed to be rigid, and the moments were found from the angular fitting of the distorted frame. That method was only permissible if the secondary stresses were very much lower than the primary stresses. Calculated secondary stresses sometimes came to 20 per cent. of the main stresses, or even 40 per cent., and therefore he was doubtful whether it was then justifiable to calculate the secondary stresses in that manner. The Author was throwing very much more doubt on that process. Based on that, presumably, the Author made the suggestion, which he took to be the essence of the Paper, that one should substitute for the present method of calculating those stresses the method given in paragraph (4) p. 50. Paragraphs (1), (2), (3), and (4) were the chief part of the Paper, for there the Author suggested that the method of calculation should be changed. He did not quite understand paragraphs (1) and (2). He supposed that those paragraphs referred to the actual structure, and that certain places for the measuring-instrument had to be marked on the members before assembly, so that the instrument could be put back when the

structure was being erected, and then, as the dead load began to come on, the effect of that loading, and later the effect of the live load, could be ascertained. Presumably the idea put forward in paragraph (4) was that a number of typical structures should be analysed, and figures found similar to those the Author had shown in *Figs. 7 and 8*. From those the bending-moments, the shears, and the axial loads in a structure loaded in a particular way would be determined, and then a similar structure could be designed. The method would necessitate an immense amount of work; and a very large number of typical structures would have to be analysed. The bending-moments, shears, and axial loads having been obtained from a corresponding model, the design of the members would be proceeded with. Then arose a point which he would like the Author to make clear. When one of those models which consisted of a plain, uniformly-thick piece of material was subjected to experiment, certain results were obtained. If the width of the members was reduced as compared with the thickness, other results would be obtained. In calculating the secondary stresses in a structure, having determined all the primary stresses, one was met with the following problem. Taking, say, a triangle of bars, the changes of length of the sides and the changes of angle which had to be allowed for would be calculated. In assuming its final position, a member would bend because the angle it made with adjacent members was fixed, and it had to obey a certain rule about the curvature. The fibre-stress and the width must be proportional, and that tied the designer down to making the thickness and width proportional, or it meant the determination and use of a large series of experimental results. Thus arose another point of difficulty. In design, economy of material and construction had to be considered, which led to certain dispositions of material in different sections. Thus the upper members of a bridge might have such a section as ABCD (*Fig. 10*) and to it might be fixed by rivets and gusset-plates a pair of flat sections, as at E and F, constituting a tension member. When compared with the parts of the model, the depths, areas and second moments of such varied sections had to correspond if the stresses and deformations were to correspond. Design to secure this correspondence and so take advantage of the method seemed to present difficulties, and a much fuller explanation as to the mode of adjusting the sections would be desirable, which he asked the Author to add.

Professor
Guest.

Fig. 10.

Professor
Batho.

Professor C. BATHO observed that the Author had described some very interesting applications of his beautiful photo-elastic method to the determination of the stresses in monolithic structures, or rather in sheets of material of various shapes, loaded two-dimensionally. He had also given another method, that of measuring the changes of thickness of plates, for co-ordinating the results of such experiments with experiments on steel members. Professor Batho thought a question to be considered was how far the results obtained by those methods could be applied to actual structures. The problem could be divided into two parts, the first being the determination of the forces and moments in the members of the structure, and the second the distribution of stress in the members, as determined by their end connections and make-up. For the consideration of the first problem, namely the determination of the forces and moments in the members, regarded simply as members having a certain moment of inertia, etc., he thought the Author's method was excellent; but a rather high degree of experimental technique was required, and the experiments were somewhat lengthy. He had found that Professor Beggs's method, which was mentioned in the Paper, was much quicker. In that method it was only necessary to measure the small deflections produced in different parts of the model by slight given movements in other places. The method, however, required more knowledge of the theory of structures, in arranging the experiments and interpreting the results, than did the photo-elastic method, since it was based entirely on the application of Maxwell's theorem of reciprocal deflections. He would like, in passing, to comment on the remarks made by the Author on the calculation of stresses. He had seldom found that the method of least work was the most useful for calculating either frameworks without diagonals or braced frameworks. In the case of frameworks without diagonals his experience was that the slope-deflection method, as used very largely in America, and notably by Messrs. W. M. Wilson and G. A. Maney in their work on wind stresses in buildings,¹ was capable of giving an accurate analysis of a complicated structure in a fairly simple manner. For structures carrying live loads, he had found that methods based on the theory of virtual work and on Maxwell's theorems, such as had been developed for many years in German text-books on structures, and more recently in American text-books, were the best. A whole body of theory had been developed, giving elegant solutions

¹ University of Illinois Bulletin No. 80.

for the most complicated indeterminate structures. He was rather astonished that those methods had not been used to a greater extent in this country. He thought they were gradually coming into use here, but they had been in general use on the Continent and in America for many years. Of course, all those methods involved, in dealing with a complicated structure, the solution of a number of simultaneous equations, or complex graphical constructions; and although he thought that it was often easier to use a calculating-machine or work at a drawing-board than to experiment with models, there were, undoubtedly, many problems for which either the Author's or Professor Beggs's method would be extremely useful. Coming to the second problem—the actual distribution of the stresses in the members themselves—the Author had remarked that stress-distributions could not usually be calculated accurately near to the end connections. He would like to go further than that and say that in very many cases they could not be calculated accurately in the main bodies of the members without relying upon the results of experimental work on actual structures or members. An example might make that point clearer. The Author had referred to a Paper by Professor Batho in which he described a number of experiments on single- and double-angle members in tension, having various types of end connections. The experiments on single-angle members attached to a gusset-plate by one leg only showed that the line of pull was practically at the junction between the angle and the plate, and along the line of the rivets, provided the plate was flexible. There was, of course, a very uneven distribution of the stress over the cross section. That distribution, however, as shown by extensometer readings, followed a regular law. It was in accordance with the usually accepted theory of eccentric loading, and, once the line of pull was known, the maximum stress in the angle could be calculated. In a 3-inch by 3-inch by $\frac{1}{4}$ -inch angle it would amount to about $2\frac{1}{2}$ times the mean stress. Experiments with broader gusset-plates held rigidly showed that the maximum stress was much reduced by the stiffness of the gusset-plate. The angle was restrained from bending by the gusset-plate, especially in the plane of the gusset-plate, and that modified the distribution of the stress very considerably. In members consisting of two angles back to back, with a gusset-plate between—a very common form of construction—the restraint was usually so great in both directions that the stress was almost uniformly distributed. That simple example showed that, even if there was a given pull on a member, the distribution of the stress—not only near the end connections but also in the main body of the member—might depend

Professor
Batho.

Professor
Batho.

entirely on the end connections. Experiments had been made on built-up columns, and they all showed the same kind of action. In a built-up member each element of the member was trying to go its own way, but was restrained from doing so by its connection to other members, or by the connections within the member itself, and was thus often subjected not only to direct load but also to bending and even torsion. He considered that a knowledge of the distribution of stress in steel structures could be obtained only by experiments such as he had described. The photo-elastic method could not be used, because the problems were three-dimensional and also because the connections could not easily be reproduced in xylonite. Some years ago an investigation of the distribution of strains in a truss was carried out under his direction at Birmingham, somewhat similar to the investigation of Dr. Wyss to which the Author had referred. The results of that investigation had not been published. They were so complicated—there were so many factors influencing the strains in the complete truss—that it was impossible to analyse them fully until the problem had been approached step by step in the way he had described. It was necessary to begin with the simple elements, and, introducing one restraint after another, learn how to take into account the effect of each restraint. He considered such experiments important. The need for more economical use of material in structures, especially in steel-framed buildings, was clearly recognized, and could probably be met by raising the working-stresses and adopting more scientific methods of design. Before this could be done many experimental investigations were needed on actual connections, structural elements, and structures. Although important information on special problems could be obtained by the photo-elastic method, it appeared to be limited at present entirely to the solution of two-dimensional problems on single sheets of material. The Author's lateral extensometer might be of great use for some of the investigations he had considered. He was not sure, however, whether the form the Author had shown was capable of being used in the varied positions and confined spaces where it was sometimes necessary to use an extensometer in such investigations. The Author had also mentioned an interference extensometer. That would be an extremely delicate instrument; and his experience with interference methods had been that the slightest vibrations affected the measurements. He would be glad if the Author would describe an interferometer type of extensometer suitable for work on steel structures and free from that defect.

Mr. J. S. WILSON thought the Author was a little hard on the **Mr. Wilson.** method of determining stresses by the measurement of circles drawn on surfaces. Professor Guest had already dealt with that point. The Author suggested that his method did measure the stress at a point infinitesimally small, but he could not imagine that even the Author's beam of light came through such a very small area of the piece of celluloid: it must have an appreciable size. Therefore the effect on the light would be caused by the average of the stresses taken over the area and in the thickness of the celluloid. For these reasons he thought that the measurement of circles drawn on the surfaces was equally accurate. He felt that the amount of work involved in those methods of analysing stresses was so vast that they were really applicable only in the case of some definite and mysterious problem. Twenty years ago he and Mr. Gore tackled a definite problem which was causing a good deal of worry, and they spent a long time over it. The work seemed interminable, but in the end they did establish something regarding the stresses in a dam. From that point of view the work was worth while, but in the case of ordinary everyday structures he did not think there was time for it, and average or approximate calculations were on the whole all that was necessary. The special methods described gave the intensities of stresses at different points, and he would urge the professorial members to see whether they could not go one step further. The stress might be there, and it was useful to know that it was there, but what the engineer wanted to know was how much harm it did. Many bridges had been put up in which the stress-concentrations and secondary stresses were very high according to the methods in question, but they had not damaged the structures at all, or affected their stability or durability. If in a case where there were high stresses at particular points a specimen were subjected to a fatigue test, the engineer would obtain exactly what he wanted to know. From all one could gather from existing bridges and structures, those high stresses at particular points did not affect them at all. Together with Professor B. P. Haigh,¹ M. Inst. C.E., he made tests some time ago to try to get information on the question. They drilled a very small hole through a fatigue test-piece. According to mathematical analysis the stress near that hole was concentrated to the extent of nearly three times the average stress across the test-piece. They tested the piece, together with another test-piece exactly the same except that it was without the hole, and no appreciable difference between the

¹ *Engineering*, vol. 116 (1923), p. 411.

Mr. Wilson. two results was found after several hundred thousand repetitions. In that case, therefore, in spite of the high stress round the hole the endurance of the test-piece was not reduced. That exemplified what experienced practising engineers felt with regard to large structures. They knew that in every riveted structure there were stress-concentrations round the rivets which might be double the average stress, but did not produce the damaging effect which these experimentally-determined high concentrations might lead one to expect. There was a point he did not quite understand in connection with *Fig. 7*. In the lower part of the main rib near the left-hand hinge there was apparently very little general compression, although the greater part of the 30-lb. load must be carried by a general compression in that inclined member. There was a certain intensity of tension on one edge and of compression on the other edge, and the two seemed to be nearly equal. It seemed, therefore, that the secondary stresses in the example chosen might be twenty or more times the main stress. From that point of view it was rather difficult to understand the diagram. By what he had said he did not wish to detract from the value of the experiments the Author had conducted: he felt they were extremely valuable.

The Author. The AUTHOR, in reply, expressed his grateful thanks for the appreciative discussion and the references which had been made to the work described in the Paper. He would like to mention that, although all the experimental work had been carried out under his personal supervision, yet nearly all the detailed observation had been made by the students named in the Paper. The use of the stud link described by Mr. Fereday was a very interesting way of measuring a heavy load with ease and safety, and its principle was similar to that of the spring balances used by the Author. The main difficulty in constructing weighing-apparatus of that type was the accurate registration of the load; the smallness of the change of form of the link used necessitated an extremely sensitive mechanism for recording. The use of flat diaphragms in the machine described with reference to *Fig. 4* did away with deformation altogether, except incidentally for determining an equilibrium position. He was not aware of a similar device for that purpose. The discussion indicated a considerable variety of opinion regarding the magnitude of the so-called "secondary" stresses. The name implied smallness, and various theories of stress-distribution in framed structures appeared to be based on the fundamental assumption that they were small, as pointed out by Professor Guest. So far as the framed structure itself was concerned, there was no advantage, and there might be danger, in making this somewhat arbitrary division

between "primary" and "secondary" stresses, as in any case the structure had to bear both, and experiments like those of Professors Batho and Wyss and others indicated that the secondary stress-intensity was much larger than was usually supposed. Experiments made photo-elastically and in an entirely different way, as described in the Paper, lent support to that view. The experience of designers of bridges in which mild steel was used led them, apparently, to assume, with considerable confidence, that the unknown stresses, often incalculable ones, were of no great importance, for when plasticity developed it tended to smooth out very high intensities. That was probably true of mild steel, but such material was now being superseded in large bridges by high-tensile alloy steels, which did not possess that "averaging" property nearly so perfectly; while in another material, concrete, now largely used, plastic yielding was negligible. Hence it appeared likely that as time went on the problem of determining the actual total stress-intensity, no matter how obtained, would become more and more important, and the arbitrary division of stress-intensity into two parts would tend to disappear. Owing, possibly, to the somewhat condensed form in which the Paper was written, some doubt as to the meaning of various sections had been expressed, and especially so with regard to strain-measurements on the surface. With regard to the statement made as to those kinds of measurement at the head of the final summary (p. 49) it might be useful to add that the necessity for taking the smallest possible linear dimension could be shown in many ways. When, for example, the maximum stress in an eye-bar was required, the length used in the main body might be considerable but the ends were almost invariably the most highly stressed, and the extreme variation of distribution in them made it imperative to approximate to point measurements there, in order to obtain accurate values of the intensity anywhere in the head. Another matter in that connection which also appeared to require some further explanation was that, among other things, the Paper advocated a particular method of stress-measurement which gave the average stress across the thickness of a plate, as distinguished from an average along a line on the surface. The reason for that preference was that the principal stresses $\hat{r}r = P$, $\hat{\theta}\theta = Q$ parallel to the central plane of the plate probably did not differ very much unless the plate was very thick. That was in fact the basis of the mathematical theory of generalized plane stress when coupled with the further considerations that the third principal stress $R = \hat{z}z$ was zero at the surface, as also were the stresses $\hat{x}z$ and $\hat{y}r$; and

The Author. hence all those latter stresses were unlikely to become large in the interior of the plate. On the other hand, Professor Guest believed, from his calculations, that the variation in P and Q might be as much as 40 per cent. at a corner of a plate where the radius was small compared with the thickness of the plate; but he had given no information about the basis on which he had obtained those results. The Author's photo-elastic observations of the change in the breadth of the isoclinic bands in plates of the same shape and different thickness appeared to indicate only a small variation. A further misapprehension concerned the scope of the method. That probably arose from the insistence in the Paper on one point, namely, the identity of the $(P + Q)$ measurements in a metal and in an elastic transparent substance, from which it followed that lateral measurements at the free boundaries gave the whole stress there. The Appendix, however, made it quite clear that four photo-elastic measurements could be and were usually made at a point, namely, the two principal stresses and their directions. The substantial identity of $(P + Q)$ for opaque bodies and for transparent bodies hence indicated the scope for the former type of body, while the identity established led to the conclusion that the very difficult surface-strain measurements on steel might be replaced by photo-elastic experiment. Some minor points related to the elastic constants of the transparent material used. Young's modulus E was about 330,000 in pound and inch units, and Poisson's ratio was 2.5; but in plane stress or generalized plane stress, those constants did not appear in the fundamental equation of stress, so that, provided the material used was elastic, in the sense that it obeyed the generalized Hooke's law, the values of the constants were immaterial, unless certain multiple boundary conditions existed, which did not, however, arise anywhere here. In connection with Professor Guest's remarks, some photo-elastic measurements at the angles of plane sections of crank-shafts were made by the Author during the war.¹ Generalized plane stress conditions were taken, as only trifling cases of three-dimensional stresses had, so far, yielded to photo-elastic treatment. The severity of the stress at an angle was shown and measured for several cases. Some doubt also appeared to exist whether the measurements described could be regarded as point measurements. Such was easily seen to be the

¹ E. G. Coker, "The Stress in the Fillets of a Crank Shaft," in "Report on Materials of Construction used in Aircraft and Aircraft Engines," by C. F. Jenkin, p. 15, Aeronautical Research Committee, H.M. Stationery Office, 1920.

case when it was considered that an area 1 inch square on a model The Author. could, if required, be projected on to a screen to give a square with sides of about 60 inches, that was, an areal magnification of 3,600. If therefore at a place on the model a small circular area of 0.001 square inch was selected, it would appear on the screen for measurement as an area of 3.6 square inches contained by a circle rather more than 2 inches in diameter, and it was then comparatively easy to determine optically the stress-difference intensity at the centre of such a circle and also the directions of stress there. Again the sum of the stresses was determined for the area touched by the points of the needles, and the contact area could easily be brought down to 0.001 square inch, so that all the measurements required could be identified with an area on the surface of the plate, which for all practical purposes might be regarded as a point. Some interesting criticisms related to matters that had been left out of the Paper in order not to make it unduly lengthy, since the main object was to illustrate a method, without burdening the description with details. One of these, due to Professor Guest, related to shear stress, the existence of which was merely recognized in *Fig. 9*, and another related to the somewhat peculiar distribution of stress shown in *Fig. 7* near the left-hand abutment. In order to deal with both points fully, the Author's research assistant, Miss R. Levi, had since measured independently the stress-distribution near that abutment and under the same load. Although the isoclinics at the pin were hidden from view by the end shoe, yet the visible portion of the end of the bridge-frame showed enough bands to enable the directions of the stresses to be marked with considerable accuracy, and in the area near the pin all the possible isoclinic bands in the lower part of *Figs. 11* (p. 68) intersected at two points, indicating that $(P - Q) = 0$ at those places. Similar isoclinics also intersected the edges at four other points, from which it could be inferred not only that $(P - Q)$ was of zero value, but also that $P = Q = 0$, since at a free edge one stress must vanish. The upper diagram, constructed from those isoclinics, showed the directions of the principal stresses P and Q , and from that Figure it was evident that cross sections of the members must be under shear stress as well as normal stress. The arched rib, *Fig. 5*, also indicated the presence of shear stress. Another interesting point had been raised by Mr. Wilson, who had asked for an explanation of the vanishing of the stress at points A and B on the edges. If an inclined plane AB was drawn through those points, it was seen that the lines of principal stress intersected that plane at 45 degrees, showing that the stress on that plane was a pure shear which must therefore vanish at the ends of that section.

The Author. Its intensity was $\frac{1}{2} (P - Q)$ at any point, and could be read off at once from the stress picture. The values of $(P - Q)$ so determined were shown in *Figs. 12*. The distribution proved to be of parabolic

Figs. 11.

type, and, since that was one of the deep members, it was unlikely that the shear stress-distribution in any other part of the frame would be of the more complicated type shown in *Fig. 9*, which distribution was only found in very deep sections, as had been shown in

earlier Papers.¹ The upper part of *Figs. 12* showed that the stress- The Author.
distributions found along the edges confirmed substantially those
already given in *Fig. 7*, and they also brought out what was only

Figs. 12.

¹ E. G. Coker, "An Optical Determination of the Variation of Stress in a Thin Rectangular Plate Subjected to Shear," Proceedings Roy. Soc., Series A, vol. 86 (1912), p. 291; and C. E. Inglis, "Stress Distribution in a Rectangular Plate having two Opposing Edges Sheared in Opposite Directions," Proceedings Roy. Soc., Series A, vol. 103 (1923), p. 598.

The Author. shown partially in that Figure, that all the isoclinic nodes at the edges were points of zero stress. The more detailed work of measuring the stress-distribution near the pin was illustrated by stress-measurements at two sections on the vertical member and two on the parabolic rib, *Figs. 12*. Those measurements, with the directions given in *Figs. 11*, were sufficient to give completely the stress and its direction at any point of those sections. It was, however, not necessary to go further into those details here. Another matter related to the use of photo-elastic investigation for questions of loaded frameworks. Primarily it might be regarded as a useful adjunct for working out the stresses in a structure under design, but not yet built; and some criticism had been directed to that point by Professor Guest on the ground that the detailed construction at the joints was far too complicated for it to be possible. That did not appear to be a reasonable criticism, since no claim that the detailed analysis of joints was possible was advanced in the Paper. What was actually suggested, in conclusion No. 4, was that with the aid of suitable models the total direct loads, bending-moments, and shears could be determined at the cross sections required (which latter were for simplicity always of rectangular type), and that those data could then be used for the design of the bridge or other framework. This was advanced on the ground of its simplicity and the comparative ease and quickness with which the measurements could be made, as compared with the laborious calculations necessary in the case of a large frame, especially one of the indeterminate type, and the possibility of error in such work. The experimental method had also the advantage that large errors were unlikely in photo-elastic work, as the stress-distribution was visible and easily determined. Another plan mentioned in the Paper and also in the discussion was the ingenious method of Professor Beggs for obtaining influence lines by the aid of models similar to those used by the Author. That method was very useful, since the necessity of determining the form of an influence line by calculation was obviated, and the mathematical analysis was therefore shortened. The Author fully appreciated the valuable experiments that had been carried out by making strain-measurements on circles on the surfaces of bodies. Those gave results which were averages over the area taken and also depended on the degree of perfection of the elasticity of the material employed. The results obtained by Professors Batho and Wyss on steel were therefore particularly instructive, and the Author hoped that the work now being carried out by the former at Birmingham University on a truss would soon become available for

the information of engineers. Professor Batho was very insistent The Author. on the need for experiment, and the Author entirely agreed with him; but, of course, it was usually laborious and difficult to make experiments on actual structures, whereas a great deal could often be learned from a model without going to the trouble of dealing with an actual structure; while at the design stage of a new structure it was the only possible source of experimental information. In what he had said about the interferometer he was merely suggesting a possibility. He had made some experiments on interferometers, and it appeared to him that it might be possible to make a more robust instrument for work out of doors; but apparently, from what Professor Batho had said, that was not likely to be the case. He considered that Mr. Wilson's remark on the need of fatigue tests of structures was valuable. There was no reason why such tests should not be carried out; they would be far easier to make on models than on actual bridges or other frames, and might lead to a considerable addition to knowledge of the subject.