Ambient vibration measurements of the Humber Suspension Bridge and comparison with calculated characteristics

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The team from Bristol University have demonstrated that ambient vibration tests can be used to determine the natural frequencies and mode shapes of the lowest frequency modes of vibration of long span suspension bridges and so check the values used in design.

120. Although not mentioned in the Paper, a three-man team from the Building Research Establishment (BRE) also carried out ambient vibration tests on the Humber Bridge at the same time as the team from Bristol University. The BRE approach was slightly different in that signals from thirteen accelerometers were recorded simultaneously for about twelve hours. These thirteen accelerometers were positioned throughout one-half of the bridge to measure vertical motion, and a recording made. The accelerometers were then re-oriented and two further recordings made, one to measure lateral motion and the other to measure torsional motion. The accelerometers were then moved to the other half of the bridge and the process repeated. This enabled the vertical, lateral and torsional motion of the whole bridge to be obtained from just six recordings. However, the results obtained for the natural frequencies and mode shapes of the bridge are in good agreement with those presented in the Paper, particularly for the vertical modes where the results are almost identical.²⁷

121. The main reason for using the longest practical recording time was to try to minimize lack of frequency resolution in the analysis: a problem referred to in Appendix 2 of the Paper. The extremely low natural frequencies of long span suspension bridges, coupled with low damping values, can lead to enormous bias errors in the spectral analysis. Indeed, for any of the ambient vibration tests conducted on long span suspension bridges, if the damping values for all the mainspan deck modes are plotted against frequency, a characteristically shaped curve occurs. Figs 14 and 15 show this for the tests by Bristol University on the Humber Bridge and for the tests carried out on the Golden Gate Bridge.⁴ The lower the natural frequencies of a bridge, the more pronounced is the increase in apparent damping, so the results from shorter span bridges with higher natural frequencies show a similar but less pronounced curve. This phenomenon can be explained as the bias errors in the spectral analysis swamping the true values, in which case, these curves would not represent real damping values. The results of the 1985 tests by BRE also suffer from the same phenomenon,²⁷ although they

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also show that lower apparent damping values were obtained by analysing longer lengths of data with finer resolution.

122. Natural frequencies obtained from spectral analysis are usually accurate, even if the analysis has large bias errors. However, bias error increases with decreasing frequency if the damping values in the modes are constant. In § 89, the damping values derived from the spectral analysis are used to find out how many spectral lines are in the half-power bandwidth of a particular mode. However, the half-power bandwidth is itself dependent on the same frequency and damping values found in the analysis. Therefore, the results of the analysis are being used to justify the parameters used in the analysis. In the absence of any other source of damping data, this may seem reasonable, but it may give a false idea of the accuracy of the analysis.

123. Past experience on tall buildings and dams, for example, has shown that

Fig. 14. Frequency against apparent damping values for Humber Bridge (1985 tests by University of Bristol)

damping values taken from forced vibration tests, where the damping values are determined directly, have been much more reliable than those taken from ambient vibration tests. Until recently, data from ambient vibration tests were the only source of damping values measured on prototype long span suspension bridges. However, the forced vibration tests on Ohnaruto Bridge²⁴ have given us a value of 0.53% for the first vertical mode. While there are obvious differences between the two bridges, it is instructive to see what bias errors occur when using the frequencies given in the Paper and this 0.53% damping. Bendat and Piersol²⁸ give the normalized bias error of an autospectrum estimate (ϵ_β) as

$$\epsilon_\beta \simeq -\frac{1}{3} \left(\frac{Be}{Br} \right)^2$$

where Be is the bandwidth used in the analysis (0.003125 Hz) and Br is the half power bandwidth (HPB) of the mode in question. (The HPB can be expressed as $2f\zeta$, where f is the natural frequency in Hz and ζ is the ratio of actual damping to critical damping for the mode in question). If ζ is taken as 0.0053 then Br is $0.0106f$.

Fig. 15. Frequency against apparent damping values for Golden Gate Bridge (1982 tests by Abdel-Ghaffar et al.)

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124. This leads to only 0.39 spectral lines in the HPB of the first vertical mode which gives a bias error of over 200%. If the damping in the first lateral mode is also only 0.53%, the bias error is over 900%. Since bias errors always produce overestimates in damping, this is one explanation for the curves shown above.

125. The only way to reduce the bias error is to use finer resolution in the analysis, and this requires a much longer length of data for analysis. Two lines in the HPB would give a bias error of 8% which is probably acceptable. However, even to give only two spectral lines in the first lateral mode (with a frequency of 0.054 Hz and 0.53% damping) would require at least 3500 s or 1 h per sample. Consequently, if 50 averages are taken to give an acceptable variance error, 50 hours of data are required. However, it is unlikely that such a long length of data will be stationary. As is pointed out in Appendix 2 of the Paper, most signal processing theory is based on the assumption that the process being analysed is stationary, so the analysis of a long non-stationary recording may also lead to errors in the estimation of damping.

126. The only alternative to these ambient vibration tests is to conduct forced vibration tests where damping values can be determined directly. There is a need to conduct more of these forced vibration tests, preferably in conjunction with ambient vibration tests which do not suffer from the problems detailed above. In the meantime, designers should be careful about using damping values in their calculations which have been obtained from ambient vibration tests on long span suspension bridges.

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It would be of interest to know the effective model mass participating in each mode, thereby giving a measure of the significance of each mode. Also, it would be of interest to know the sum of the effective model masses, compared against the total structural mass, thereby giving a measure of the 'missing mass'. Have the Authors these figures available from the theoretical analysis?

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We thank *Mr Littler* for presenting a description of the BRE test and some data and observations we did not feel justified in including in our Paper. We welcomed the opportunity to compare our results with BRE and were reassured to find few differences between the independent estimates of modal frequencies and shapes.

129. The characteristic inverse relationship between damping estimates and natural frequencies has been observed within studies such as ours and from collected data from a variety of ambient vibration tests on suspension bridges.²⁹

130. Hanning windowing was used in both the Humber and Golden Gate Bridge studies, summarized in Figs 14 and 15, in order to minimize the effect of leakage, which is a particular problem for closely spaced modes. As explained in § 115, Hanning windowing, on account of its frequency domain smoothing effect, has a significant effect on damping estimates for spectral peaks spanning only a few spectral lines.

131. For an analogue spectrum analyser, bias has a similar smoothing effect on the data 'between' the spectral lines; but for a digital spectrum analyser, the windowing forces the signal to be periodic and so there can be no data between the spectral lines. The analogy of § 116 is not then strictly accurate and it is the windowing that results in the spreading of energy, in a similar way to bias.

132. Tanaka *et al.*³⁰ have simulated the effect of a Hamming window (similar to a Hanning window) in a numerical study of a single degree of freedom (SDOF) oscillator and have produced a measure of the error in damping estimates due to windowing if the half-power bandwidth (HPB) method is used. The important factor in the error is $f_0 T$, where f_0 is the modal frequency and T is the lag or sampling period; SDOF oscillator frequency response curves with the same damping are broader at higher resonant frequencies, as the width of the peak can be expressed by the HPB, which is $2\zeta f_0 T$.

133. Windowing errors result in biased estimates; unbiased errors may also be caused by the high variances in the spectral data.³¹ It can be difficult to quantify the individual and cumulative effects of variance and windowing errors on estimates of damping obtained either by measurement of the HPB or by curve-fitting as in § 35, and possibly the easiest method is by numerical simulation.

134. In one such simulation,³² the response to Gaussian white noise of a SDOF oscillator with 2% damping was analysed using the same spectrum analyser and windowing that was used for the analysis of the experimental data. The study showed that the normalized error $\varepsilon = \sigma/\mu$ for the damping estimates obtained by curve fitting approached the value expected for a single spectral estimate from a square-root auto power spectrum ($\varepsilon = 1/2\sqrt{BT}$). For the case of $f_0 T = 40$ (as for mode V1), overestimates of the mean value of damping decreased from 50% using the HPB method to 25% or less for a curve fit using 16 or more points, despite there being only 1.6 lines in the (true) HPB. This example reflects the extreme case for vertical modes; however, for the first lateral mode (L1) the error will be much greater. On the other hand, even though the measured HPB (and hence the number of lines it contains) will always be greater than the true HPB, a wider measured HPB almost certainly includes a proportionately smaller error; therefore, the damping estimates for the higher modes are likely to be much less in error.

135. The data used in the study were ideal (stationary and Gaussian), whereas the experimental data were not stationary by most accepted criteria and were often obscured by noise, but the results do indicate that errors in the estimates cannot by themselves fully explain the experimental trends. We feel unqualified to do more than suggest other possible explanations such as aerodynamic effects and hysteresis.³³

136. Naturally it would be useful to have damping estimates obtained from forced vibration studies, but effective hydraulic or mechanical shakers cannot be constructed without such resources as were available to the Ohnaruto bridge testers. The only alternative is 'snap-back' or step-relaxation testing, which was used to obtain an estimate of 2–3% damping for the first vertical mode of the Tagus Bridge.³⁴

137. No technique for estimating damping appears to be immune to error, although users of steady state and transient forced vibration have the benefit of a controlled source of excitation and do not need to resort to signal processing or time domain modelling. The experimental values of damping in Tables 1–3 were obtained from all possible estimates of the quantity and include almost every possible source of error. It might be fairer to use only the 'cleanest' data to give the most conservative (lowest) damping values and the results would be as described in reference 27 and Fig. 13, from which it does not seem possible to estimate a 'true' value of damping less than 2% for mode V1, even under the best signal processing conditions.

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138. We regret that we have not had time to find or regenerate data relating to the modal masses for the Humber Bridge, but we will be able to supply the data to anyone interested, including *Dr Maguire*, in the near future.

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