On the approximate prediction of elasto-plastic plate behaviour

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The Author has presented an apparently computationally efficient method for calculating the load-end shortening response of plates subjected to in-plane compression and loaded in the elasto-plastic range. However, the method uses coefficients which in themselves are based on pre-existing computer analyses. Thus, application of the method can at most be restricted to cases which have previously been considered elastically, or can be derived from such studies, e.g. long plates which buckle with approximately square waves. Cases which fall outside such investigations require elastic computer solutions, the derivation of coefficients from such solutions and then application of the procedure described in the Paper. The inference that this is necessarily faster and cheaper than a rigorous elasto-plastic analysis of the same problem using some of the methods referred to by the Author is debatable, particularly in relation to plate analysis.

49. With regard to the point that the coefficients are derived from elastic solutions of particular deformed shapes, how would the Author's method cope with an analysis in which a mode change occurred? Such an example would be a 3 : 1 plate having a dominant one-wave initial bow with a very small three-wave bow superimposed and subject to longitudinal compression. The behaviour of this would be dominated by the singlewave until buckling occurred and then the three-wave mode would assert itself.

50. Although the Author has derived procedures for handling initial distortions greater than 0.3 times the plate thickness, these are not presented in the Paper. It would be useful for anyone wishing to apply the Author's approach to have the modified procedures available as fabrication distortions will be of this order and greater for plates with width to thickness ratios of 60 and more.

51. I look forward to the possible extension of the Author's method to other plate problems. If formal justification of the method can be found, it might enlarge the range of structures for which elasto-plastic buckling analyses can be conducted as application of rigorous methods is currently limited by computer time requirements.

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Regarding the point made that the method relies on pre-existing elastic analysis, in the past 50 years a vast store of elastic plate investigations have accumulated from

Fig. 19

which an investigator can obtain information on a very wide range of plate behaviour. The expressions in the Paper cover a large spread of buckle half wavelengths, not simply approximately square waves as suggested by Dr Frieze, and cover many plate behaviour problems which are of practical importance. For investigators without direct access to elasto-plastic computer programs the advantages of the method presented are obvious. Even for a researcher with an elasto-plastic program there can be distinct savings in time. As Dr Frieze points out, computer time is a problem with rigorous analysis. Therefore an examination of the behaviour of a given type of plate, with a variety of different width to thickness ratios and yield stresses, by rigorous methods can involve very many computer runs, at many times the cost of a single run. This can be replaced by a single run with very high yield stress and width to thickness ratio to obtain the elastic analysis. Subsequent applications of the method in the Paper, at negligible computer time, can yield all the required results at a cost of a fraction of the overall time required for repeated analysis.

53. In the case of a 3:1 plate with initially dominant one-wave bow, as mentioned by Dr Frieze, the general approach described in the Paper (i.e. obtain elastic results and use the simple elasto-plastic modifications suggested) should yield accurate results. Elastic analyses of this type of problem have been carried out for many years. For this particular problem the initial one-wave bow has the effect of postponing the eventual three-wave natural buckling mode and producing an apparent increase in ultimate load. However, unless the one-wave imperfection were very large I would not recommend that any apparent strength increase be taken into account. In such a case sensitivity to variation in the imperfections in different modes can be very high and calculated advantages could be illusory.

54. On this basis I would suggest that, for safe design in the case where a plate is initially deformed into a mode higher than the nature buckling mode, each mode

should be examined independently with its own associated imperfection. At any stage of loading the mode which gives the more conservative prediction (i.e. lower load) should be taken as the governing mode.

55. This leads to very simple analysis of the problem using the expressions given in the Paper. An example is shown in Fig. 19 of a 3 : 1 plate with moderate imperfection of $0.188t$ in the one-wave mode and extremely small imperfection in the three-wave mode. Initially the one-wave mode governs and finally the three-wave mode. In the three-wave mode localization is encountered and taken into account using the method in the Paper. This particular value of imperfection was chosen to enable comparison with Dr Frieze's rigorous analysis in reference 5, and Fig. 19 shows that the agreement is very good.

56. Regarding the procedure derived for handling distortions greater than 0.3 times the plate thickness mentioned in the Paper, the modification required is simple, and is based on the recognition that beyond yield imperfection effects reduce in comparison with the elastically derived effects. To take this into account the modified imperfection can be considered to have a varying magnitude after yield, given by $w_0 = w_{op}(\epsilon/\epsilon_1)$. Using this varying imperfection produces negligible changes in the behaviour predicted for plates with small imperfections. For plates with large imperfections the growth in load after first yield is predicted accurately.

57. As Dr Frieze points out, for thin plates distortions can be high and the method suggested gives good agreement with his rigorous results. However, I would warn against acceptance of the results of any current calculation method in the case of plates with width to thickness ratio greater than 80 without experimental verification. For such thin plates it is known that first membrane yield gives a good indication of ultimate load, and failure by other mechanisms (e.g. so-called 'crinkly buckling') is often observed experimentally.

Corrigenda

On page 168 of the Paper the expression $\sigma_{AV} = \sigma_Y(b/b_e)$ after equation (6) should read $\sigma_{AV} = \sigma_Y(b_e/b)$.

On page 170 in equation (14) the term

$$\frac{E^*}{E} A \text{ should read } \frac{E^*}{E}$$

On page 173 in equation (24) the term $0.00284/(e - 0.5097)$ is not suitable for $e < 0.55$. A suitable replacement term for all values of e is $0.0833/(1 + 14e^4)$.

On page 179 in Fig. 15 the expression $(b - b_1)(\epsilon_x - \epsilon_1 + \epsilon_R)Et$ should read $(b - b_1)(\epsilon_x - \epsilon_1 + \epsilon_R)E/b$.