The stability of tapered and haunched beams

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In my opinion, haunch stability on pitched portal frames has always been the most arbitrary part of the design, the provision of stays being generally left to the discretion of the engineer. I feel, therefore, that the methods of analysis as proposed would be very useful if the following points could be resolved.

60. In modern industrial portal-type buildings full benefit is taken of wide-span decking to allow purlin spacings of 1.8 m and greater, thus reducing the number of purlins and expensive fixing time to a minimum. In § 12, supports, in the form of purlins are said to be required 'at sufficiently close intervals to prevent unrestrained lateral torsional buckling between supports'. Morris and Randall¹⁶ suggested that purlin spacings of approximately 1.3 m are required for economical universal beam sections in this region of a high positive B ratio. Is lateral torsional buckling between the supports therefore not the most critical design criterion for purlin spacings of 1.8 m and greater? If not, does the analysis as proposed still apply? What is the maximum suitable purlin spacing for this analysis, based on the economical sections theory in § 23?

61. In § 5 the Authors assume that 'lateral restraint is provided at the inner corner'. In my experience most designers and steelwork contractors do not provide a restraint to this position (the springing of the haunch) because of the cost of providing either an additional purlin, sheeting rail or longitudinal member. Similarly, stays from the eaves beam or purlins are not popular as they are long and look unsightly. Would it be correct to assume, therefore, that adequate torsional restraint to the position A is provided by a horizontal web stiffener at the bottom of the knee, bearing in mind that this position is generally stressed to a maximum?

62. If the answer is no then it is surely preferable to find an alternative solution to the haunch instability problem. A restraint is generally provided at the first sheeting rail below the eaves to prevent stanchion instability, in accordance with reference 15. The analysis could therefore be modified to cater for the unrestrained length from this position to the point of contraflexure in the rafter, thus achieving a more practical and cheaper solution.

63. I have always understood that plastic hinge positions must be restrained so as to ensure that adequate rotation can occur and prevent premature collapse. The analysis again assumes that purlins are sufficiently closely spaced for the beam to 'buckle by twisting about the axis of restraint' and, although

§ 35 suggests that the maximum rotation does not have to occur for the adequate load factor to be attained, would not the large spacing of purlins rule out the possibility of using this method of haunch analysis? Is it possible to assume a maximum safe purlin spacing?

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It is stated in § 6 that effective full lateral restraint can be assumed at the point of contraflexure. Should this not be the first purlin connection beyond the point of contraflexure?

65. It seems desirable to unify the earlier treatment of unrestrained members^{12,14,15} with the more flexible approach now proposed for rafters and columns restrained by purlins and sheeting rails. Earlier a semi-empirical expression for limiting slenderness under uniform plastic moment and axial load was developed, and a similar expression can be derived for elastic conditions. If the axial load component is (for uniformity) eliminated and the exact expression for the critical length under uniform elastic moment and the modified empirical expression for uniform plastic moment were used as basic slenderness references, would it then be correct to apply the same shape and moment factors c and k as are now proposed for restrained members?

66. The basic slenderness ratios (equations (14) and (15)) are based on a purlin or restraint axis position defined by $a/d = 0.75$. I have encountered numerous cases of purlins, sheeting rails and wall panels which lie both inside and well outside this position. It would be useful to have some guidance regarding the sensitivity of the member stability to variations in the eccentricity of restraint. In § 42 and reference 8 (discussion) it is recommended that the effect of axial forces may be allowed by adding $P(a + d/2)$ to the moments in equation (9). Is this sufficient to deal with the effect of varying eccentricity of restraint?

67. Could the Authors give numerical values of K for common moment diagrams and uniform sections?

68. The rafter stability problem is discussed largely in terms of the negative moment zones of portal rafters in the vicinity of the eaves or knee when subject to snow loading. The reversed moments due to factored wind (external suction and internal pressure) less dead load sometimes require investigation. Presumably the analysis is then equally applicable to the rafter lengths near midspan?

69. In the event of the limiting slenderness ratio being shown to be exceeded it will be necessary to provide intermediate full restraints in the form of stays or web stiffener plates designed to act in conjunction with the purlin cleats. It has been proposed⁸ that such restraint should be designed to resist a lateral force of 1% of the force in the compression flange. This is only 40% of the value that may be inferred from BS 449 for similar conditions. Can the Authors clarify this?

70. In reference 6 the stability of columns restrained by sheeting rails is considered using yield at the extreme fibre at midheight as the criterion of incipient instability. The treatment in the Paper for rafters and beams is presumably directly applicable to the simpler case of untapered columns. Does this supersede the earlier stress analysis? Does it in fact take into account results derived from that procedure or is it derived purely from considerations of post-buckling behaviour and perhaps more conservative for longer columns?

71. In § 5, it is emphasized that positive restraint should be provided at the

inner corner A in Fig. 1. This recommendation has often been made before and is consistently ignored by many designers largely because it is often singularly awkward to introduce a satisfactory restraint detail at the inner corner. There is also a widespread feeling that sufficient restraint may be provided by the inherent stiffness of the normal end plated connection, particularly when acting in conjunction with properly designed stiffener plates and a longitudinal member conveniently positioned at the outer corner of the frame. Has premature failure occurred in any tests because of instability of an unrestrained inner corner? Are there any circumstances where the omission of direct restraint to the inner corner is admissible or capable of satisfactory treatment by current theory? The pioneering portal frame tests reported by Baker *et al.*^{1,4} featured eaves beams on the outer face of the columns.

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Problems of purlin spacing, raised by Mr Newsome in §§ 60 and 63, are stressed throughout the Paper as being of prime importance. The purlins must be at intervals sufficiently close to prevent unrestrained torsional buckling between the supports. The limiting slenderness ratio for unrestrained sections, as given in reference 15, may be used to obtain the maximum spacing between purlins. Such spacing may indeed be well below typical present values which take full advantage of wide-span decking, allowing purlin spacings of 1.8 m and greater. To increase the number of purlins to satisfy the stability requirements of the rafter would be wasteful. We suggest that short lengths of purlin sections might be used to connect the sheeting to the rafter between the purlin positions such that the spacing is reduced to that permitted in reference 15. Such a solution would take full advantage of the wide-span decking available, and would reduce the additional fixing time to the minimum. These additional connections would be required only in those rafter lengths where plastic hinges are expected to form, and would ensure that the rafter ultimately failed by rotating about the axis of restraint and not by flexural-torsional buckling between lateral restraints.

73. Mr Newsome (§§ 61–62) and Mr Hairsine (§ 71) discuss the requirement for restraint to the springing of the haunch and the practical problems encountered in satisfying this requirement. A horizontal web stiffener in the column web at the bottom of the knee would, on its own, provide neither lateral nor torsional restraint to the outstand flange, its function being rather to prevent local instability of the web and flange of the column. The suggestion that one might deal with the problem of instability of the column and rafter by treating them as one unit extending from the first sheeting rail below the eaves to the point of contraflexure in the rafter would be sound in principle but complicated in practice, unless there happened to be a sheeting rail at the level of the horizontal web stiffener in the column at the bottom of the knee. Should this be the case, and provided the connection between the sheeting rail and column were a moment connection in the sheeting rail plane, then we would agree that such an arrangement would provide the restraint required for the inner corner A of the rafter-column joint, (Fig. 8(a)). If the sheeting rail is well below the level of the column web stiffener (Fig. 8(b)), the need to provide direct restraint to the inner corner A may be avoided provided

- (a) the top of the column remains elastic so that, in particular, the outer flange of the column between sheeting rails B and C remains elastic

Fig. 8. Rafter-column connection together with position of sheeting rails

- (b) the column-rafter joint is sufficiently rigid to ensure rafter-column continuity in the lateral direction, this being aided by the column stiffener AD
- (c) the stability of the rafter haunch is checked by considering its length extended to outside the column BD.

74. Tests on corner connections with deep haunches have shown the danger of considerable reductions of carrying capacity due to lateral instability when there is no restraint to the inner corner.¹⁷ In many laboratory tests, and in the Cambridge tests on portal frames, instability was not in evidence because of the small depths of the members and the relatively high torsion constants possessed by rolled sections at the bottom of the size range.

75. The point of contraflexure in the rafter is taken as the end of the haunched beam rather than the first purlin connection beyond that point, because it is the point beyond which the torsional buckling of the rafter is virtually ruled out. This results from the fact that the upper flange of the rafter, which becomes the compression flange, is close to the axis of restraint which practically prevents its lateral movement.

76. Mr Hairsine raises the question of whether the treatment for unrestrained members, given in references 12, 14 and 15, might not be unified with the treatment now given for restrained members. We agree that unification is needed and are undertaking this. Caution is needed in that the treatment of reference 15 is aimed at uniform members in which plastic hinges form only at one or both ends—never within the length—and further study will be necessary before it can be said that the same c and k factors can be applied.

77. Mr Hairsine also raises the question of the influence of the exact position of the axis of restraint. The effect of an axial force on the stability of the rafter may be allowed for by adding $P(a_1 + d/2)$ to the moments in equation (9) (not $P(a + d/2)$ as shown in § 42). However, this modification in itself is not sufficient to deal with the effect of varying the position of the axis of restraint when the actual a/d value is not 0.75 as assumed throughout the analysis. The restraint axis position defined by $a/d = 0.75$ was chosen on the basis of a survey of portal frames, and distance a refers to the distance between the fastenings in the purlin cleat and the centre line of the rafter. The analysis gives safe results if the actual value of a/d is less than 0.75, because in such cases the axis of restraint is closer to the tension flange than is assumed in the analysis, thus offering better

lateral restraint to the rafter. It is unlikely that the level of the fasteners in the purlin cleats would be more than $d/4$ from the tension flange of the rafter except for the very shallow UB sections, but even in such extreme cases it would not be true in the tapered part of the rafter. In cases where a/d significantly exceeds 0.75, it would be desirable to ensure that the cleat-purlin connection has a degree of moment resistance capacity (about an axis parallel to the rafter) by having two bolts—a situation which would in any case be likely to arise with the deeper connection.

78. Owing to the simplicity of obtaining the k values for uniform beams, we thought it would be a simple exercise to derive such values for common moment diagrams. Although we had hoped to provide k values for tapered and haunched beams, we abandoned the attempt because of the number of variables involved.

79. Mr Hairsine raises the problem of reversed moments in rafters due to wind loading. The lateral stability of the outstand flange in compression near the apex may certainly need investigation. It will usually be found that the frame behaves elastically under this combination of factored loading; the methods described in reference 6 may be used to check for lateral buckling. If plastic hinges are formed near the apex under this reversed loading, the analysis given in the Paper could be applied to determine whether or not stays to the lower flange were necessary.

80. In dealing with the design requirement for lateral stays (§ 69), the figures of 1% or $2\frac{1}{2}\%$ of axial force capacity are devices for attaining the necessary degree of stiffness rather than real requirements for capacity to withstand the derived forces. Plastic failure reduces St Venant torsional resistance to a much smaller degree than it reduces lateral flexural resistance, and so the restraint stiffness required to control torsional buckling is less than that required to restrain ordinary strut buckling. The $2\frac{1}{2}\%$ rule was derived empirically from the latter situation rather than the first.

81. Mr Hairsine refers (§ 70) to the use of the criterion of attainment of yield near mid-height in the treatment in reference 6 of lateral stability of restrained columns. The present analysis, based on considerations of post-buckling, does not supersede that treatment for cases where axial stresses are high, because post-buckling behaviour then becomes much less favourable and leads to a more catastrophic form of failure. However, the present treatment may be applied to columns in which the direct axial stress due to the factored axial load is only a small proportion of the material yield stress, or not more than $15\text{--}20\text{ N/mm}^2$.

82. Equation (14) should read

$$\frac{I_{cr}}{r_y} = \frac{(8.0 + 150f_y/E)(D/t_f)}{[4.4(f_y/E)(D/t_f)^2 - 1]^{1/2}}$$

References

16. MORRIS L. J. and RANDALL A. L. *Plastic design*. Constrado, London, 1979.
17. TOPRAC A. A. *et al.* Connections for welded continuous portal frames. *Weld. J.*, 1951, 30, 7 and 8; 1952, 31, 11.