The elastic analysis of cellular structures containing web openings

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The graphs presented in the Paper for determining effective shear and torsional areas will assist considerably in the analysis and design of cellular structures.

60. The use of concepts such as effective width, effective shear area and effective torsional area are essentially attempts to reproduce the three-dimensional behaviour of cellular structures in a two-dimensional analysis. Consequently, it is important to bear in mind their limitations, so that weaknesses of such analytical models are recognized and corrective measures taken when needed.

61. As the Authors have explained, the shear area facility provides a means of modelling the increased flexibility of cellular structures due to openings in the cross-section or in the web. In the two-dimensional grillage model, the use of the shear area facility leads to an increase in the computed deflexion over that for an identical analysis where shear area is not considered. Unfortunately, this increase in deflexion does not result in additional bending stresses. This is because the increase in deflexion due to shear is a linear function of span and therefore does not affect the curvature or the bending moments. For the cellular structure, however, the increase in deflexion is due to bending of the cell flanges.

62. The implication of §4 is that where openings are sufficiently large as to make the flanges of cellular structures flexible, there is a likelihood that a two-dimensional grillage model incorporating the shear area facility will underestimate bending stresses in the direction with the greater openings. However, the overall deflexions will be accurate.

63. For cellular bridge decks, the openings in the cross-section are generally larger and consequently the transverse bending stresses will tend to be underestimated.

64. The conclusions outlined in §5 may be verified by reference to Figs 12 and 23. For a solid web the deflexions from the grillage analysis agree almost exactly with three-dimensional finite element solution but the maximum transverse stresses are underestimated. The results for the cases with web openings of 12.5% and 31.25% do not bear this out. However, I suspect that the effective shear coefficient K_s may have been incorrectly computed. By reference to Table 1 it

may be seen that K_s has been calculated on the basis of shear deflexions of 0.24, 0.64 and 1.526 (first series) instead of 0.095, 0.495 and 1.381 as it should be if K_s is to be unity for no openings in the web. If K_s is recalculated, it is anticipated that results for these cases will also bear out the correctness of my conclusions.

65. It is possible to adjust the transverse stresses obtained from the grillage analysis by taking into account the accuracy of its deflexions. Stresses due to the relative settlement of webs are calculated and added to the grillage results. This approach was used in the examples in reference 7 and has the advantage that the transverse bending stresses are not underestimated.

Dr Evans and Dr Shanmugam

In the idealization of cellular structures into skeletal structures, such as grillages, the shear deflexion can be expressed in terms of effective shear area, as explained in reference 1. In this study, the effective shear area was assumed to be equal to the cross-sectional area of the web and the shear deflexion was found to have little influence on the bending stresses developed within the grillage beam elements.

67. The presence of openings in the webs results in a further increase in deflexion. In this case, removal of material near the neutral axis does not significantly affect the bending deflexions. The increase in deflexions is due primarily to the increase in web shear flexibility and, in the case of large openings, *vierendeel* action. In the proposed method these two effects are combined and represented by the single reduced effective shear area coefficient K_s .

68. The values of K_s in the Paper were calculated on the basis of this definition of K_s (§§6, 19 and 20). There are no errors in the computations. There may have been misunderstanding of the definition of K_s ; Mr Sen seems to suggest that there are no shear deflexions in girders without web openings.

69. Misunderstanding may have arisen from the fact that, for some girders, even when there are no openings, the value of K_s obtained from the proposed procedure does not equal unity.

70. For example, for the girder without openings in series 1 of Table 1, the total deflexion δ obtained from the finite element method is 0.602 mm. The bending deflexion δ_b was obtained from the elementary formula (§19) as 0.457 mm. In accordance with the definition, the deflexion due to shear ($\delta_s = \delta - \delta_b$) is 0.145 mm. Therefore, from the elementary formula for shear deflexion (§20) the effective shear area would be obtained as 1213 mm² with a corresponding K_s value of 0.86. However, the value of K_s for solid webs should be 1.0, as assumed in the Paper.

71. The difference between the assumed and calculated values of K_s in this case is due to the fact that elementary formulae are used to predict the behaviour of a complex structure. The value of K_s for the girder without web openings in series 3 of Table 1 is exactly equal to 1.0, thereby proving the accuracy of the formulae for longer girders.

72. When the web openings are very large, as in Figs 12 and 23, the additional stresses resulting from the *vierendeel* action can be obtained by considering the isolated web, as explained in reference 2, and superimposing the local stresses on the overall stresses obtained from the grillage analysis.

73. We agree with Mr Sen that it is important to realize the limitations of any approximate method of analysis. The grillage idealization provides a power-

ful and convenient method for the analysis of cellular structures. As its idealizations are based on a physical appreciation of the structural action, its limitations should be more apparent than in the case of techniques based on numerical approximations.

Reference

7. HIGHWAY ENGINEERING COMPUTER BRANCH, *User Guide: slab and pseudo-slab bridge decks*, Department of Transport, 1978, HECB/BI/7.