Elasto-plastic buckling in short thin-walled beams and columns

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The divergences between the Author's prediction and the experimental results of Little⁷ have not been fully explained. Indeed, the theoretical predictions are conservative, which is unusual for single layer plasticity approximations. By ignoring the elasto-plastic region, the onset of plasticity is delayed, and the peak loads predicted should be higher than the measured value. When the test specimen becomes fully plastic, so that the single layer model more closely follows the actual behaviour (as, for example, in the unloading regions of the load-deflexion curve), then the theoretical and experimental values should become asymptotic.

54. The yield surface used by the Author is Ilyushin's approximate yield function, and is not the best approximation available. Ilyushin determined the exact yield function many years ago, and although he used a deformation theory in the derivation, the result is still correct. (If the total strain terms in Ilyushin's derivation are replaced by plastic strain increments, and the deformation theory is replaced by the more respectable flow rule for a von Mises material, the same yield function results.)

55. This yield surface is a complex function of two parameters, and so does not lend itself to rapid calculation, even with modern computing techniques. However, it has two important properties: it is smooth, and it is convex. The approximate yield function proposed by Ilyushin is produced by passing two planes through four extreme points on the surface. As the true surface is convex, these planes lie almost wholly within the exact surface.

56. If

$$Q_T = \frac{1}{t^2 \sigma_0^2} (N_x^2 - N_x N_y + N_y^2 + 3N_{xy}^2)$$

$$Q_M = \frac{16}{t^4 \sigma_0^2} (M_x^2 - M_x M_y + M_y^2 + 3M_{xy}^2)$$

$$Q_{TM} = \frac{4}{t^3 \sigma_0^2} (M_x N_x + M_y N_y - \frac{1}{2} M_x N_y - \frac{1}{2} M_y N_x + 3M_{xy} N_{xy})$$

the planes are represented by

$$Q_T + Q_M + \frac{1}{\sqrt{3}} |Q_{TM}| = 1 \quad \dots \quad (14)$$

57. In cases where Q_{TM} is close to zero, there is a discontinuity in the slope of the surface, which gives rise to sudden changes in the resulting tangential elasto-plastic

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rigidities. Attempts to avoid this effect by introducing a third plane

$$Q_T + Q_M = 1$$

when $|Q_{TM}|$ is less than a certain number, as has been done by Crisfield,¹⁴ merely shift the discontinuity to a different position.

58. Ivanov¹⁹ has proposed a much better approximation to the yield surface, which has been discussed by Robinson²⁰

$$Q_T + \frac{1}{2}Q_M - \frac{\frac{1}{2}(Q_T Q_M - Q_{TM}^2)}{Q_T + 0.48 Q_M} + \sqrt{\frac{1}{2}Q_M^2 + Q_{TM}^2} = 1 \quad \dots (15)$$

This function is within 1% of the exact surface, whereas the plane approximation is only within 15%. Furthermore, it is everywhere smooth, so there is no problem in the evaluation of the normal to the surface.

59. Although equation (15) appears much more complex than equation (14), this is of little consequence when computers are used. Indeed, the fact that the same equation can be used everywhere can simplify the programming. As an experiment, both approximate yield functions were used in otherwise identical computer programs at Imperial College of Science and Technology, to solve the same problem of a simply supported rectangular plate under uniaxial applied displacements, with a small initial imperfection. The computer time needed differed only in the fifth significant figure.

60. Has the Author used the Ivanov yield function, or any other approximation, to improve the predictions of his method?

Dr Frieze

I agree that the Ivanov approximation to Ilyushin's exact yield surface is more accurate than the approximation proposed by Ilyushin himself. However, any inaccuracies expected from using this best available linear approximation to the exact surface rather than some high order one were not in evidence when comparisons were undertaken with a multi-layer solution.²¹ As a consequence of the good correlation shown by these particular results, I have not felt any need to alter my formulation. It would be interesting to compare Mr Burgoyne's results using the Ivanov approximation with the multi-layer analysis solutions.

62. In the light of these comparisons, it would seem likely that not even the multi-layer solution would demonstrate significantly closer agreement with the experimental results in question than I have achieved. If any agreement were obtained indicating that the flexural strength of a cross-section was 7% greater than its plastic moment of resistance, I would view such results with suspicion. Demonstrably the discrepancy is largely on the experimental side and only in a minor way is it affected by the choice of yield function.

References

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