Inelastic analysis of biaxially restrained columns

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The Authors have presented the Paper just before the final preparation of a new British Standard code for the use of composite construction in buildings. The part of this code concerning composite frames was synthesized from independent work on composite beams, composite columns and composite joints, with due regard being taken of modern methods in the design of steel frames. The Authors' work concerns the study of sub-frames consisting of composite columns restrained by steel beams and as such constitutes, I believe, the first tests on composite frames. Would the Authors indicate any observations from their work which are relevant to the new code?

36. In developing the design methods for composite frames for the code there appeared to be a lack of compatibility between the end rotations of beams and columns when the beams are relatively flexible and the columns heavy and slender. Lack of compatibility is most acute when the beam is of slender cross-section. In table 5.1 of the draft standard¹² upper limits are specified on permissible ratios of column stiffness to beam stiffness. Would the Authors comment on these values and give details of the moment-rotation characteristics of the joints in their own tests?

37. The Authors indicate that in reference 11 in the sub-frames tested the beams have sufficient strength to remain elastic when the columns fail. In the interests of economy, however, it is desirable to permit plastic design of the beam cross-section. The examination of column strengths under varying conditions of beam plasticity would therefore be a useful extension of this work, particularly if loads could be applied along the length of the beam to achieve a more realistic sub-frame condition.

38. A useful exercise could be conducted using the Authors' computer program with a view to rendering table 5.1 of the draft standard less conservative. End rotations could be found for a range of column slendernesses of 10-30 for beam stiffnesses between one tenth and one half of the column stiffness. If the program could be linked to a continuous composite beam program (e.g. that written by Johnson and Hope-Gill¹³), then refinements to other sections of the draft standard could be effected. Further work could be envisaged taking account of the tests on joints in composite frames currently being performed at the University of Warwick.

39. What is the stub-stanchion effect referred to in § 29? From Table 1 it can be deduced that it reduces the failure load by 11% when using accurate analytical models. Is such a reduction applicable in any situation when using normal design methods?

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The Authors have developed a method for the analysis of biaxially loaded restrained columns and it would be useful for parametric studies of reinforced concrete columns

DISCUSSION

and composite columns in which torsional effects are negligible—as assumed in the theory. However, in the theory some of the known effects which affect column strength, like residual stresses and strain hardening, have been neglected. Elastic unloading has also not been considered and in certain cases, depending on the load path followed, may be encountered. What percentage error would this involve?

41. The method of analysis is very similar to that developed by Virdi and Dowling¹⁴ for the computation of ultimate loads of biaxially restrained columns. However, there are differences in the mathematical approach for obtaining the equilibrium solution for the deflected shape. The Authors have introduced the forward integration scheme for the solution of second order differential equations instead of the direct differentiation procedure adopted by Virdi and Dowling. The results in Fig. 5 show close agreement between the two approaches. It is clear that a closed-form solution for the $M-N-\phi$ curves would drastically reduce the computational time. Such solutions are generally not available and it would be interesting to compare the computational efficiencies of the two approaches.

42. The present method has also been used to analyse bare steel columns. As torsional effects have been ignored such analysis should be carried out with discretion and no conclusions should be drawn because torsional effects cannot be dissociated from bending effects. Apart from very short columns ($1/r_y$ —minor axis slenderness ≤ 50) the interaction of torsional and bending effects for columns of medium slenderness ($1/r_y \approx 60$) and greater would be significant. Results of full-scale tests on H columns¹⁵ substantiate this. Adequate guidelines are not available about the extent of interaction of bending and torsion which should be investigated thoroughly.

Dr I. M. May and Professor R. P. Johnson

A recent reassessment of the experimental results has shown that the minor axis deflexions shown in Fig. 13 are incorrect and should be as given in Fig. 14.

44. Most of Mr Smith's questions relate to the design of composite frames. They are beyond the scope of this Paper, but relate to current research on joints and design methods for composite frames. Some results of preliminary studies are given in reference 16. The ultimate loads obtained from the eight tests carried out have been compared¹¹ with the proposed design method given in reference 17. This design method was shown to be satisfactory for both the serviceability and ultimate limit states.

45. In all the tests, the joints were designed to be fully rigid, and there was negligible rotation of the beam relative to the column.

46. In most laboratory tests on limited frames, it is necessary for practical reasons to apply the axial load through assumed pin joints that lie on extensions of the column axis beyond its points of intersection with the axes of the beams. When the column bends during the test, the line of action of this load no longer passes through the intersections of the beam and column axes. This divergence from what is normally assumed in analysis is the stub-stanchion effect.

47. Dr Sen is correct in stating that torsional effects were neglected in the analysis presented for reasons given in the Paper, although the forward integration technique can be used to solve problems including torsion. For steel columns with elastic rotational restraint about the minor axis, torsional effects are usually small. However, the problem of columns pin-ended about the minor axis and loaded to give extensive plastic zones about the major axis is indeed one in which torsion will play a significant part, and for which the analysis as presented would not be appropriate.

48. Residual stresses and strain hardening were not included because the analysis was originally devised for composite columns where the effects are negligible. Recent inclusion of generalized stress-strain curves for steel in the program have caused no difficulties in convergence. Our work and other studies in which elastic unloading has been neglected have shown good agreement with experimental studies. In general

Fig. 14. Column BC2: comparison of minor axis deflections at mid-height

it appears that theories that do not assume elastic unloading are unlikely to be greatly in error for practical columns.

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