Determination of maximum crack width in two-way reinforced concrete slabs

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The Authors have made an interesting attempt to re-interpret the test data produced by Nawy *et al.*¹⁻³ In order to be able to judge the merits of this new interpretation, it appears useful to try to answer two questions. First, what accuracy should one expect from a crack prediction formula? Second, does this new interpretation fit the facts better than Nawy's own interpretation of his data?

26. In order to answer the first question, it is useful to attempt to establish some overall measure by which to judge the ability of a proposed calculation method to reflect the influence of the various parameters which may affect behaviour. One approach which can give a simple measure of this is to compare the distribution of the errors produced by a particular formula (exp-calc) with the distribution of the original data. For example, consider Nawy's results for slabs having a steel stress of 60 kip/sq. in. The mean of all the measured cracks widths on all the slabs used by the Authors is 0.03789 in., and the standard deviation of these results is 0.01576, giving a coefficient of variation of 0.416. This is equivalent to saying that, if all variables are ignored and the crack width is simply assumed to have a constant value of 0.03789, the coefficient of variation of exp-calc will be 0.416. A formula which purports to take account of any variables within the test program must, to have any predictive value, give a significantly lower coefficient of variation than this. There is also a lower limit to the accuracy which a formula can achieve; this is defined by the inherent variability of the phenomenon considered and can be found by comparing the variability between identical specimens. A rough estimate of this for Nawy's slabs gives a coefficient of variation of 0.13. Thus the ideal formula would give a coefficient of variation of 0.13 and any formula giving a coefficient of variation of 0.4 or more has no predictive value. The Authors obtain a coefficient of variation of 0.41 for the case considered. Table 3 gives values for the variability of Nawy's data at the rest of the stress levels considered. It can be seen that, although for lower stress levels the coefficients in Table 2 are slightly below the values in Table 3, they are generally of the same order, suggesting that the Authors' formula is of little use in predicting the influence on cracking of the variables tested by Nawy.

27. The second question can be answered by reference to Table 4, which is comparable to Table 2 for values of w_{exp}/w_{calc} , derived using Nawy's own formula. The standard deviations and coefficients of variation are roughly half those given in Table 2. Nawy's formula is thus superior to the Authors' formula by a wide margin. Furthermore, Nawy's formula shows up favourably when tested against the criteria in § 26. It could be argued that a formula of lesser accuracy is justifiable in practical situations

DISCUSSION

Table 3. Variability of Nawy's data

Steel stress	Coefficient of variation of data
20	0.500
30	0.479
35	0.467
50	0.449
60	0.416

Table 4. Values of w_{exp}/w_{calc} at various stress levels in steel for 72 slabs using Nawy's formula³

Stress in reinforcement, kip/sq. in.	Average	Standard deviation	Coefficient of variation
20	0.953	0.22	0.23
30	0.940	0.20	0.21
35	0.967	0.21	0.22
50	1.025	0.24	0.24
60	1.141	0.30	0.26

if it is simpler to apply but the Authors' formula is, if anything, more complicated than Nawy's.

28. Consideration of these two questions must call the validity and usefulness of the proposed formula into question.

29. The applicability of Nawy's tests to practical situations should also be considered. He used model slabs roughly 1.5–1.8 m square and about 75 mm thick. They were mainly reinforced with welded mesh made from hard drawn wire. Two distinct types of cracking were observed: an orthogonal pattern of cracks which followed the lines of the reinforcement (i.e. the cracks all formed over the wires) and, for low percentages of reinforcement, cracks formed along only what eventually developed into the yield lines. Nawy explained this behaviour on the assumption that force was transferred from steel to concrete mainly at the welds in the mesh. This is probably reasonable for drawn wire because it has a much lower bond strength than normal rolled reinforcing bars. With certain weld spacings, cracks are induced at all welds; with larger spacings, insufficient force was transferred from the steel to the concrete to generate spread cracking and only the yield line formed. These types of behaviour are not typical of the behaviour of slabs reinforced with normal bar reinforcement⁶ and there is thus little reason to expect Nawy's results to be applicable to normal reinforced concrete slabs.

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Yield line and strip methods are being incorporated in the codes of practice of various countries. This will result in a reduction of the reinforcing steel and increase in the stresses under service loads. The problem of cracking and hence of corrosion assumes greater importance in this context. Adequate research has not been published on crack widths in two-way reinforced concrete slabs. The Authors are to be congratulated for their timely contribution on this aspect.

31. The Authors have presented an equation for the prediction of crack widths in two-way slabs considering the bond force on the bars and the anchorage forces due to cross wires. A similar approach was developed by Srinivasa Rao and Subrahmanyam⁷ for one-way slabs reinforced with welded wire fabric. It is good to see that a similar

approach has been extended to two-way slabs. However, we doubt if the Authors' equation is applicable only to two-way slabs reinforced with welded wire fabric, but believe it is also applicable in the case of regular bars. This is not reflected in the Paper.

32. In one-way slabs reinforced with welded fabric⁷ it was observed that the flexural rigidity of the transverse wire plays an important role in deciding if the bearing stress f_{bb} is uniform and if the entire length S_1 is effective. However, the Authors have assumed that the entire length S_1 of the transverse wire will have uniform bearing. Assumptions, without adequate proof, have also been made in the Paper regarding the distribution of the tensile stress on concrete area, bond characteristics of the bars and the bearing stress. It is felt that these assumptions might have contributed considerably to the large differences between the experimental and the calculated maximum crack widths.

33. The experimental maximum crack widths vary from +45% to -45% of the calculated maximum crack widths. Several points fall out of even this range (see Fig. 2). This is also the case with the equation due to Nawy and Blair³ (see their Figs 1.4(a)-(h)). To the design engineer such a large variation may not be acceptable even if the average of the value w_{exp}/w_{calc} is very near 1.00.

34. Recognizing the fact that the phenomenon of cracking itself is random, any approach to analyse test data and to calculate maximum crack widths should be associated with a definite probability. The generally accepted probability figure is 95%. We feel that such an approach, if adopted in all crack width studies, would lead to more reliable methods for the prediction of crack widths in two-way slabs.

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The object of the investigation described in the Paper was to arrive at a method which is sufficiently general for determining the crack widths in two-way reinforced concrete slabs and one which does not require the use of different constants for different support and loading conditions. As the test results of Nawy³ were the only results available to us they were analysed on the basis of the proposed method and a higher coefficient of variation was noticed.

36. We are aware that the proposed method gives a large coefficient of variation but wonder whether it would be reasonable to compare the results of Nawy's formula with those of the proposed analysis because Nawy's formula uses different values for the multiplier constant K which are obtained for the best fit of different groups of slabs. They would therefore yield a better agreement for the same test results than a method which is formulated to be applicable to all boundary and loading conditions.

37. An improvement in the prediction by the proposed method can be obtained by empirically modifying the effective concrete area in tension and the expression for f_b

Table 5. Values of w_{exp}/w_{calc} at various stress levels in steel for 72 slabs using equation (9)

Stress in reinforcement, kip/sq. in.	Average	Standard deviation	Coefficient of variation
20	0.9409	0.300	0.32
30	0.9182	0.266	0.29
35	0.9529	0.276	0.29
50	0.9779	0.284	0.29
60	1.0811	0.335	0.31

DISCUSSION

Table 6. Comparison of computed crack widths by proposed method and Nawy's method for flat slabs

Slab	Experiment crack width, in.	w_{calc} using proposed method, in.	w_{calc} using Nawy's method, in.*
2	0.016	0.021	0.020
3	0.016	0.024	0.023
4	0.016	0.018	0.017
5	0.016	0.010	0.010
6	0.016	0.019	0.018
7	0.016	0.018	0.019
8	0.016	0.019	0.018
9	0.016	0.022	0.022
	Average	0.0187	0.0184

* The value of KR in Nawy's equation was taken as 3.0×10^{-5} by Hawkins *et al.*⁹

given by equation (5). Thus, one could rewrite the crack width equation as

$$w_{\text{max}} = \frac{\alpha R (A_{\text{ct}})^m f_t}{\pi \phi_1 \left(\frac{M_{\text{cr}}}{M_u} \right)^n \frac{f_{\text{ub}}}{S_1} + \frac{\phi_2 f_t}{2S_2}} \epsilon_s \quad (9)$$

where α , m and n are constants to be determined for a statistical best fit of test results. Equation (9) would be dimensionally balanced only with α being assigned suitable dimensions. For certain assigned values† of $m = \frac{1}{2}$, $n = \frac{1}{2}$, $\alpha = 1.43$ and f_{ub} given by CP 110⁵ without 20% increase, there is improvement in the prediction of the proposed method⁸ as shown in Table 5. The coefficients of variation in Table 5 are smaller than those in Table 3 and closer to those in Table 4.

38. Doubts have been expressed as to whether the results of Nawy would be applicable to normal reinforced concrete slabs. This has been looked into by Nawy and Blair³ who applied their equations to slabs tested by other investigators and found good agreement between calculated and experimental crack widths.

39. Equation (9) has also been used⁸ to calculate the maximum crack widths in flat slabs reinforced with normal bars reported by Hawkins *et al.*⁹ Table 6 compares the experimental crack widths of these slabs with those obtained using equation (9) and the method proposed by Nawy. The proposed method and Nawy's method give almost the same result and overestimate the crack widths by 11.5% on average.

40. We think that § 37 and § 39 show the validity of the proposed approach, although it could be modified further on the lines suggested in § 37 to make it more useful.

41. We thank Dr Subramanyam and Mr Ganesh Babu for bringing our attention to reference 7 and are pleased to know that, working independently, both groups of investigators have attempted to include the bearing of the transverse bars in the resistance to cracking, although this is effected in different ways in the two studies. The proposed method is designed to be applicable in general to two-way reinforced concrete slabs irrespective of the usage of welded mesh or ordinary reinforcement and the support or loading condition. It is difficult to determine experimentally the values of k_t , k_b and f_{bb} and hence they can be only assumed. The values assumed in the Paper could have contributed to the greater spread in the agreement plot (Fig. 1) and to the higher values

† Not for the condition of a statistical best fit.

of the coefficients of variation (Table 2). Use of modifications similar to those suggested in § 37 improves the agreement considerably. Alternatively, one could try to determine the values of k_t , k_b and f_{bb} which yield statistical best agreement with test results.

42. In the studies on estimating crack widths, the two approaches used—estimating the maximum width and estimating a width which has an accepted probability of being exceeded—are well known.¹⁰ The use of the latter involves the measurement of the widths of all the cracks that form so that dependable distributions and averages can be determined. In the case of slabs this could be more difficult than in beams because every crack that forms, being spread over a length, has to be measured at a large number of places and at every stage of loading. Another difficulty which is faced in two-way slabs is an experimental determination of the strain in the steel bar at the location of crack width measurement. We feel that more experiments and test data are required regarding the crack widths of two-way reinforced concrete slabs so that dependable prediction methods can be developed for such cases.

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