Gates and shallow water waves

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The inclusion of junctions, control structures and the like in any scheme for the analysis of unsteady flows is a topic which has applications in a wide variety of circumstances. Some examples would include the operation of tidal barrages and the control of flows through complex networks such as river deltas or large urban drainage facilities. The Author has taken one particular method of analysing transient flows, the method of characteristics, and shown how the effects of sluice gates may be incorporated. His analysis shows not only how to include a particular gate operating policy but, perhaps more important, also how to derive an operating policy to meet certain restrictions or constraints under transient conditions.

52. In using the method of characteristics, the Author claims that it is the most rigorous method available for analysing transient flows. However, this method only maintains continuity in the limit as the mesh size approaches zero, whereas other methods will maintain continuity irrespective of the mesh size. In what sense does the Author mean that the method is rigorous?

53. It would seem from the Paper that the technique presented is applicable only to controls for which accurate and tractable analytic solutions are obtainable. In many cases, controls simply do not lend themselves to rigorous mathematical analysis. Although the Author points out that the technique could be applied to other gate problems, we feel that a more general approach to the whole question of control structures and junctions is desirable. It would seem that such generality may be achieved most easily by describing a control's discharge relationships as a table. The immediate advantages of this are

- (a) controls which cannot be analysed mathematically can be handled if a table or graph of the discharge relationships can be abstracted from field or calibration data
- (b) control structures can be lumped into broad groups for which the table representation is the same; this means that the same model can be used on widely differing systems with the minimum of re-programming, simply with new tables of discharge relationships.

Although this approach uses more computer core, this added expense is easily offset by the dramatic increase in flexibility.

54. The use of tables can be handled very effectively in an implicit scheme such as that developed by Gunaratnam and Perkins.⁶ Briefly, and omitting much important detail, consider the elements A and B shown in Fig. 10. The continuity equation for A may be written as

$$f(q_1, y_1, q_2, y_2) = RC \dots \dots \dots (48)$$

Fig. 10. Definition sketch for elements

and the momentum equation for A as

$$g(q_1, y_1, q_2, y_2) = RM \quad . \quad . \quad . \quad . \quad . \quad (49)$$

55. Describing an upstream boundary at 1 as, say, a discharge against time relationship gives

$$q_1 - h(t) = RV \quad . \quad . \quad . \quad . \quad . \quad (50)$$

56. Suppose the boundary at 2 is a control of some sort. In general terms

$$q_2 = q_3 = k(y_2, z_2, y_3, z_3, P)$$

57. For the sake of illustration, assume that discharge is independent of y_3 and z_3 . Then

$$q_2 - k(y_2, z_2, P) = RD \quad . \quad . \quad . \quad . \quad . \quad (51)$$

where q_i is the discharge at point i , y_i is the depth of flow at point i , z_i is the bottom elevation at point i , P is a set of parameters describing the characteristics of the control and its operation, f , g , h and k all represent functions and RC , RM , RV and RD are residuals which should be driven to zero.

Fig. 11. Coefficient of discharge C_D for sluice gate

58. There are now four equations and four unknowns. Solving by Newton's method

$$\begin{bmatrix} \frac{\partial f}{\partial y_1} & \frac{\partial f}{\partial q_1} & \frac{\partial f}{\partial y_2} & \frac{\partial f}{\partial q_2} \\ \frac{\partial g}{\partial y_1} & \frac{\partial g}{\partial q_1} & \frac{\partial g}{\partial y_2} & \frac{\partial g}{\partial q_2} \\ & 1 & & \\ & & \frac{\partial k}{\partial y_2} & 1 \end{bmatrix} \begin{bmatrix} \Delta y_1 \\ \Delta q_1 \\ \Delta y_2 \\ \Delta q_2 \end{bmatrix} = \begin{bmatrix} -RC \\ -RM \\ -RV \\ -RD \end{bmatrix}$$

or $\bar{A}\bar{B} = \bar{R}$.

59. All that is needed to include a control in this scheme is the residual RD and the partial derivative $\partial k/\partial y_2$, or the rate of change of discharge with the depth y_2 at the control. These may both be obtained from a table of discharge against upstream head. The table may be constructed directly from field data or from theoretical considerations.

60. In this illustration, the boundary at 2 may be a free-flowing sluice gate or any control in which the tailwater depth has no effect on discharge. The extension to a more complex control such as a submerged gate is straightforward, involving only the additional derivative $\partial k/\partial y_3$ and continuity and momentum equations for element B. Again $\partial k/\partial y_3$ could be obtained from a table or graph.

61. For a submerged sluice gate between points 2 and 3

$$k = WC_D \sqrt{[2g \max(y_2, y_3)]}$$

which allows for flows in both directions, where W is the gate opening (given in a table as a function of time) and C_D is the coefficient of discharge. C_D need not have an analytic expression. It can be obtained by a reading from a graph such as Fig. 11. From the graph the derivatives $\partial C_D/\partial y_3$ and $\partial C_D/\partial y_2$ may be obtained. Then, for $y_2 \geq y_3$

$$\begin{aligned} \frac{\partial k}{\partial y_2} &= (WC_D/2)\sqrt{(2g/y_2)} + \partial C_D/\partial y_2 W\sqrt{(2gy_2)} \\ \frac{\partial k}{\partial y_3} &= \partial C_D/\partial y_3 W\sqrt{(2gy_2)} \end{aligned}$$

62. Similar expressions may be found for $y_3 > y_2$. The transition from free to submerged flows may be described by tables and the flow conditions checked before setting up the matrix of partial derivatives A . For larger systems, matrix A is sparse and tightly banded so the system may be solved efficiently.

63. Could the Author explain the practical and computational advantages of using the method of characteristics? At first sight the use of tables with an implicit scheme seems to be a more suitable approach to the problems he has posed.

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The method of characteristics involves the integration of total differential equations along the propagation paths of waves or disturbances and in the course of computations the changing speeds of disturbances are accurately mapped. This may be done either with the fixed grid outlined in the Paper or by means of an irregular mesh, these computations taking place at the sluice gate, being similar for either of these mesh types. It is in this respect that I consider the technique to be rigorous. In circumstances where the transients may be generated artificially, as by the movement of a gate, it is particularly important that attention is given to calculation of the changing shape and steepness of resulting waves because a shock wave may well be formed. The method of characteristics will readily detect the occurrence of such a discontinuity and model its subsequent propagation. With other techniques, it is not possible to identify the point at which the shock forms and the analysis often requires the introduction of additional numerical or pseudo viscosity. In this respect the method of characteristics is seen to be more rigorous. Regarding the statement of Mr Franz and Mr Leytham on mesh size, this would be more correctly attributed to some numerical schemes based on the theory of characteristics, but it can hardly be applied to all techniques.

65. The treatment described in the Paper places least reliance on calibration curves or stored data. Between this approach and that outlined by Mr Franz and Mr Leytham may be a number of options depending on the availability of calibration data.

DISCUSSION

66. Since characteristic techniques can be applied either without or in conjunction with calibration curves, it is difficult to justify the claim of Mr Franz and Mr Leytham of increased flexibility for a technique which necessitates the acquisition of calibration curves before any calculations can begin. However, I would agree that some controls need to be defined by such curves.

67. The primary practical advantage of characteristic techniques in the present circumstances lies in their ability to detect the inception of shocks and to follow their progress. The use of tables as outlined is not restricted to the implicit scheme but could equally well be applied to any other numerical scheme, including one of the characteristic techniques. However, unlike the implicit method, the characteristic technique may also be employed as shown in the Paper, with little reliance on tabulated data. This has the computational advantage that any alteration in design of the control, say gate width, would adjust only a single number in the data, whereas with the tabular method a completely new set of tables would require to be generated for each design change before any simulation could be carried out.

Reference

6. GUNARATNAM D. J. and PERKINS F. E. *Numerical solution of unsteady flows in open channels*. Massachusetts Institute of Technology, Cambridge, Mass., 1970, Hydrodynamic laboratory report 127.