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DISCUSSION

Fatigue analysis of steel offshore structures

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The writer finds the Paper a very significant contribution to the literature on offshore structures. As it appears the immense effort invested in formulating the fatigue problem mathematically and methodically, is indeed worthy of high praise. Miner's criterion has been used and with some simplification and linearization of the functions that enter into Miner's criterion, a solution has been obtained for the damage integral according to Rayleigh distribution.

60. The closeness of the results of application of the deterministic and the spectral analyses is worth mentioning. The writer is sceptical about these two results. This scepticism may stem from the fact that the actual distribution of waves within a narrow wave spectrum is more indeterminate and complicated than the investigators assume. Therefore, these results cannot be guaranteed. No doubt the results would have been quite different had the intermediate functions used in the analysis been selected differently. In view of this comment, it is believed by the writer that the distribution of the significant wave heights (H_s) should be according to Weibull instead of Rayleigh.

61. It appears that the waves in the North Sea are better defined by a Weibull distribution function, whereas the distribution of the individual waves within a narrow spectrum defined by the significant wave and recorded within a period of 20 min is better approximated by a Rayleigh distribution. Therefore, to define a wave probabilistically, both the Weibull and Rayleigh distribution functions must be considered, and with this consideration, it is natural that the damage function must contain both these distribution functions.

62. The problem of the distribution of the individual waves within a wave spectrum has been studied by Longuet-Higgins. This study gives the ratio of the significant wave to the mean wave height corresponding to different percentages. Table 2 shows the percent of distribution of wave height within a wave spectrum defined by H_s .

63. For a 60 ft significant wave, less than 1% of the waves are 100 ft, whereas, about 45% of the waves are in the neighbourhood of 38 ft. This signifies the importance of the distribution of waves within a wave group. Although Rayleigh distribution renders satisfactory results for any wave group defined by H_s , to extend this to the entire set of H_s values would be mathematically very difficult if it can be done at all. Therefore, this difficulty immediately suggests the necessity of resorting to a numerical integration of the functions involved. Thus, it may be conceived that the probabilistic function is in fact a surface function with two variables as contained in the Weibull and Rayleigh distribution functions. Furthermore, since the wave period changes with the wave height, to find the percentage of time that a wave occurs within a storm becomes quite involved and time consuming. Hence, this also demands a numerical analysis. Therefore, it would be worthwhile to compare these results with the results obtained from the damage function containing both the Weibull and the Rayleigh distributions.

Table 2

H_s	H									
	0	0.01	0.05	0.10	0.20	0.25	0.30	0.333	0.40	0.50
20	33	27	27	20	17	16	15	14	13	
25	42	33	28	24	21	20	19	18	16	
30	50	40	34	29	26	25	22	21	19	
35	58	47	40	34	30	29	26	25	22	
40	67	53	45	39	34	33	30	28	25	
45	75	60	51	44	39	37	34	32	29	
50	82	67	56	49	43	41	37	35	32	
55	92	73	62	54	47	45	41	39	35	
60	100	80	68	59	52	44	45	42	38	

Mr Williams

Mr Erzen's scepticism regarding the closeness of the deterministic and spectral results is valid. The transfer function is not a constant function. Therefore, different forms of the energy spectrum corresponding to a particular significant wave height H_s , would yield different results. However, the emphasis intended in the Paper was that the application of dynamic amplification gave a lower fatigue life for the spectral method.

65. The distribution of the significant wave heights, H_s , was taken from observed data. The distribution of wave heights, H , or stress range, σ , within a particular sea state were in accordance with the Rayleigh distribution; see equations (35) and (57).

66. I would agree that for the analysis to be wholly probabilistic in a semi-empirical sense, the damage junction must contain both the Weibull distribution of significant wave heights, H_s , and the Rayleigh distribution of actual wave heights, H . This gives a double integral for fatigue damage that would require numerical integration except for the most simple cases, one of which is outlined below.

67. From the papers of the Oceanology International Conference in 1975 the Weibull distribution applicable to the northern North Sea is given by:

$$P(H_s) = \delta \left(\frac{H_s - x_0}{\beta} \right)^{\delta-1} e^{-[(H_s - x_0)/\beta]^\delta} \quad \dots \quad (82)$$

where

$$\delta = 1.31$$

$$\beta = 2.23$$

$$x_0 = 0.90 \text{ m}$$

are Weibull distribution parameters.

68. The number of waves dN , in a given sea state, H_s in the area $dH dH_s$ is given by:

$$dN = N_m p(H_s) p(H) dH dH_s \quad \dots \quad (83)$$

where N_m is the total number of waves in unit time.

69. If the stress range variation is represented by:

$$\sigma = aH^b \quad \dots \quad (84)$$

the total fatigue damage is obtained from equation (10), using equations (3), (35) and (84).

$$D = \frac{4N_m a^{-b/m_1}}{H_s^2 \sigma_1^{-1/m_1}} \int_0^\infty \int_0^\infty H^{1-(b/m_1)} e^{-2H^2/H_s^2} dH \delta \left(\frac{H_s - x_0}{\beta} \right)^{\delta-1} e^{-[(H_s - x_0)/\beta]^\delta} dH_s \quad (85)$$

$$D = \frac{N_m a^{-b/m_1} \Gamma[(-b/m_1 + 2)/2]}{\sigma_1^{-1/m_1} 2^{-b/2m_1}} \int_0^\infty H_s^{-b/m_1} \delta \left(\frac{H_s - x_0}{\beta} \right)^{\delta-1} e^{-[(H_s - x_0)/\beta]^\delta} dH_s \quad (86)$$

70. This integral can only be evaluated explicitly when $x_0=0$. Therefore

$$D = \frac{N_m a^{-b/m_1} \Gamma[(-b/m_1 + 2)/2]}{\sigma_1^{-1/m_1} 2^{-b/2m_1}} \int_0^\infty \delta \left(\frac{1}{\beta}\right)^{\delta-1} H_s^{a-1-b/m_1} e^{-(H_s/\beta)^\delta} dH \quad (87)$$

This integral has the Γ function solution (see equation (16))

$$D = \frac{N_m a^{-b/m_1} \Gamma[(-b/m_1 + 2)/2]}{\sigma_1^{-1/m_1} 2^{-b/2m_1} \beta^{\delta-1} \delta} \frac{\Gamma[(\delta-1-b/m_1+1)/\delta]}{(1/\beta)^\delta [(\delta-1-b/m_1+1)/\delta]} \quad (88)$$

or

$$D = \frac{N_m a^{-b/m_1} \beta^{(1-b/m_1)} \Gamma[(-b/m_1 + 2)/2] \Gamma[(1-b)/(m_1 \delta)]}{\sigma_1^{-1/m_1} 2^{-b/2m_1}} \quad (89)$$

71. The more general expression for fatigue damage including dynamic amplification function is given by:

$$D = \frac{4N_m}{H_s^2 \sigma_1^{-1/m_1}} \int_0^\infty \int_0^\infty H \{f(H)q(H)\}^{-1/m_1} e^{-2H^2/H_s^2} \delta \left(\frac{H_s - x_0}{\beta}\right)^{\delta-1} e^{-[(H_s - x_0)/\beta]^\delta} dH dH_s \quad (90)$$