Buckling and vibration of thin, flat-walled structures continuous over several spans

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Paragraphs 6-18 of the Paper are devoted to finding the continuous beam eigenvalues and eigenfunctions. This seems unnecessary because a continuous beam is a very simple plane frame and the free vibration of such frames is extensively treated in the literature. Thus equation (9) is the no-sway member equation, with α and γ commonly being denoted by (EI/IS) and (EI/ISC) , where S and SC are analogous to the well-known stability functions s and sc . Moreover, equation (11) is merely the stiffness matrix formulation of the simple plane frame which results from connecting two such members together to form a continuous beam. However, it is known that a structure can have natural frequencies in addition to those for which the determinant of its stiffness matrix is zero when, as in the Paper, the matrix coefficients are transcendental functions of μ_n . The existence of such frequencies is discussed in reference 24, which also gives a theoretically proven method which is guaranteed to find these and all other natural frequencies of a structure. The method has been extended to buckling problems.²⁵ Reference 26 gives a complete description of how the method can be used to find the natural frequencies of a plane frame.

31. Unfortunately, the Authors' method (§ 15) only finds those natural frequencies for which the determinant of the stiffness matrix (i.e. the matrix of equation (11) in their example) is zero. Therefore it seems that the Authors should adopt the method described in § 30 (which also overcomes the problem mentioned at the end of § 15) or an alternative, as otherwise eigenvalues will be missed for some problems.

32. The danger of missing eigenvalues does not arise for most problems, but is present for any beam with identical spans and clamped ends. Physical argument shows that such a beam will have natural frequencies coinciding with those of a single span with its ends clamped, and that the corresponding modes will involve zero rotations at the intermediate supports. Figs 3(b), 3(d) and 3(f) illustrate this situation for a beam with two identical spans. Moreover, the first six eigenvalues must clearly correspond to modes 1-6 in Fig. 3. Hence the method in the Paper will miss alternate eigenvalues by finding all those with anti-symmetric modes and missing all those with symmetric modes, because each symmetric mode corresponds to a non-trivial solution of an equation similar to equation (11) for which the vector of the θ s is null. Specifically, equation (11) applies to the problem of Fig. 3 when $l_2=l_1$ and $\theta_{1n}(0)=\theta_{2n}(l_2)=0$. Equation (11) can then be simplified to the scalar form

$$2\alpha(l_1)\theta_{2n}(0) = 0 \quad \dots \dots \dots (18)$$

and equating the appropriate determinant to zero gives

$$\alpha(l_1) = 0 \quad \dots \dots \dots (19)$$

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there are several half waves between supports and the finite strip method, with one strip per plate and a low value of n to reduce the number of degrees of freedom, to find the other eigenvalues.

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In §§ 30–33 Professor Williams points out that, in the particular case of a multispan beam with both ends clamped and equal spans, the eigenfrequencies corresponding to a zero displacement vector are missing. We fully agree. This was effectively detected soon after submission of the Paper. Since, in this case, the structure degenerates into substructures which are the different spans clamped, these eigenfrequencies can be easily determined²⁶ and introduced where they need to be in the growing sequence of μ_n obtained by finding the zeros of the determinant of the stiffness matrix. The method suggested in reference 24 could effectively be applied to check the number of eigenvalues below each μ_n and then the missing eigenvalues corresponding to a zero displacement vector could be inserted. This is unnecessary in all the cases where one end at least is not clamped, because the solution corresponding to a zero displacement vector is the trivial solution; consequently it is not worthwhile increasing the computer time by calculating roots which will not be used, as the only eigenfrequencies will be those obtained by the method presented in the Paper.

38. We are aware that α and β are equivalent to $(EI/I)S$ and $(EI/I)SC$, but the method in the Paper gives a quick and automatic way of determining the eigenfunctions Y_n necessary for the derivation of a multispan finite strip having any boundary conditions, number of spans, length of spans and so on.

39. With reference to §§ 34 and 35, it was stated in the Paper that n is proportional to α (§ 27), α being the number of half-waves for local buckling; it is then natural that n has to be large if α is large.

40. We do not deny the efficiency of the classical methods. First, we compare two numerical methods—finite element and finite strip—but we do not present any comparison between the efficiency of an exact and an approximate method. Second, the influence of the supports can be important in the case of short structures, and in the case of an overall buckling mode which will appear automatically in the present method as well as a local or torsional buckling mode, as no assumption is made about the type of buckling. Furthermore, the present method can easily handle orthotropic structures.

41. As regards § 36, the ultimate aim of the finite strip method applied to multispan structures being the stability analysis and static analysis of multispan flat walled structures, the number n has to be held large enough in view of

- (a) predicting an eventual local buckling mode in a stability analysis
- (b) approaching accurately the displacement and load distributions in a static analysis.

However, in dynamic analysis, the policy of using a low value of n to find the other eigenvalues without involving too many degrees of freedom can be adopted.

42. Whenever a new approach is presented it is the privilege of the user to combine the advantageous features of the new and the old methods.

References

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Errata

The last displacement function of equations (1) should read

$$w = \sum_{n=1}^r [(1 - 3x^2/b^2 + 2x^3/b^3)w_{in} + (x - 2x^2/b + x^3/b^2)\theta_{in} \\ + (3x^2/b^2 - 2x^3/b^3)w_{jn} + (x^3/b^2 - x^2/b)\theta_{jn}] \sum_{s=1}^S Y_{sn}$$

44. Equation (4) should read

$$\frac{d^4 Y_s}{dy_s^4} - \frac{m_s \omega^2}{EI_s} Y_s = 0$$

45. The expression for C_s in the penultimate line of § 7 should read

$$C_s = (m_s/EI_s)^{1/4} (m/EI)^{-1/4}$$

46. The term (2, 1) of the matrix of equation (7) should read

$$\beta(1 + s_1 s_2 - c_1 c_2)$$