Sediment transport theories: a review

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Additional notation

A	coefficient (threshold, Ackers and White ¹)
A_*	constant (Einstein ¹⁰)
B_*	constant (Einstein ¹⁰)
C_D	drag coefficient (Egiazaroff ⁸⁴)
D_α	effective sediment diameter in a mixture
D_i	particular grain size
D_m	mean diameter (geometrical) for roughness determination
D_{max}	maximum sediment size
D_o	threshold diameter within a mixture
e_b	bed load transport efficiency (Bagnold ³¹)
e_s	suspended load transport efficiency (Bagnold ³¹)
F_{cg}	sediment mobility, coarse grain (Ackers, ³ White ⁴)
F_{fg}	sediment mobility, fine grain (Ackers, ³ White ⁴)
F_{gr}	sediment mobility, general (Ackers, ³ White ⁴)
g_b	bed transport, dry weight per unit width per unit time
g_s	suspended transport, dry weight per unit width per unit time
g_t	total transport, dry weight per unit width per unit time
m	coefficient (Ackers, ³ White ⁴)
n	coefficient (transition, Ackers and White ¹)
R'_*	particle Reynolds' number with respect to the grain
T_b	work rate for bed load
T_s	work rate for suspended load
V_{yd}	velocity at a level $y=0.63D$
τ_{cg}	tractive shear associated with coarse grains
τ_{fg}	tractive shear associated with fine grains
ϕ	angle of dynamic friction

Mr P. Ackers, Hydraulics Consultant, Binnie and Partners

It may be difficult for the reader to set the review of the Paper in context, because without much background reading one would not appreciate the features which make the latest theory^{1,3} different from its predecessors. The following comments might assist.

73. In my view some earlier theories were over-complicated, bearing in mind that in every case they had to be reconciled with observation by incorporating

empirical coefficients. Some theories treated the transport of bed sediment as essentially a plane bed process, despite the facts that bed features (ripples and dunes) exist under many conditions and that sands, especially fine sands, travel partly in suspension. There are methods in the literature that allow for these effects by elaborations of the theory which introduce further empirical coefficients, but they were not always well established over a wide range of conditions. Also some researchers have relied entirely on a limited range of laboratory scale data when making empirical adjustments to their theories, and consequently there were doubts about the application of their methods to the range of conditions that the engineer meets in practice.

74. These considerations led me to break with some of the traditions of sediment transport theory, while retaining many features found in previous literature that seemed helpful. An important consideration was that the final result should be so framed that it was easy to apply in a direct fashion, to give the total load of bed sediment knowing the hydraulic and geometric properties of the flow.

75. There was also an analogy with the concept of rough turbulent, smooth turbulent and transitional flow in dealing with coarse sediment, fine sediment and intermediate sizes respectively. A further feature was the method of testing the quality of the new theory. First, it had to be demonstrated that different sediment sizes falling into these categories required different formulae. Second, if the theory had merit coefficient values derived solely from laboratory scale experiments over restricted ranges of depth/diameter ratio had still to apply with reasonable accuracy when tested against field data.

76. The main premises on which the theory was based were as follows.

- (a) Coarse materials travel as a bed process, with their mobility characterized by the ratio of grain shear stress to the immersed weight of a layer of grains.
- (b) The transport process is determined by its efficiency, defined as the ratio of rate at which work is done in moving a contact load to the stream power, based on the grain shear stress.
- (c) The grain stress relevant to coarse material is given by the rough-turbulent equation, with k_s related to median sediment diameter, as if the bed were plane (although it may have bed features).
- (d) Finer materials travel mainly in suspension, the concentration being dependent on stream turbulence; their mobility is characterized by the ratio of the overall shear stress (expressed as shear velocity) to their fall velocity.
- (e) The efficiency of the transport process in this case is described by the ratio of the work rate implicit in keeping the concentration of sediment in suspension to the stream power based on the total shear stress.
- (f) The total shear stress relevant to fine material is given by $\rho g m i$, and thus includes the stream power expended in overcoming form resistance as well as grain resistance.
- (g) Stoke's law determined the fall velocity of fine particles.
- (h) Governing formulae for the total transport of bed material for both coarse material and fine material were derived on the above basis.
- (j) The next important and novel premise was that the governing formula for transition material could be derived by a simple power combination of the formulae for the two boundaries, raising the coarse formula to the power $(1-n)$, and the fine formula to the power n , and taking the product. This transition formula then reduced to the coarse formula when $n=0$ and to the fine formula when $n=1$. For $0 < n < 1$ transition sizes were implied.
- (k) So that spurious correlations could not emerge from data analysis, the main variables were separated by algebraic manipulation of the dimensionless groups.
- (l) These variables reduced to non-dimensional groups representing sediment transport, stream velocity and sediment size.

- (m) Analysis must not be subjective. All data sets must be subjected to statistical analysis giving best fit values for n , A (an initial motion factor), C and m (coefficient and exponent respectively in the transport equation).
- (n) There would be no adjustment for wall effects; in effect the smooth-walled flumes used by experimenters were treated as if they were wide enough for wall effects to have little influence on the transport intensity.
- (o) Only if there was the predicted trend in n from 0 to 1 in data sets representing first coarse and then progressively finer sediment could the theory be considered successful.
- (p) The coefficients n , A , C and m were expected to be functions of D_{*r} , the dimensionless grain size. These functions must be derived empirically.

77. The basic theory is given in reference 3, and again briefly in appendix 1 of reference 1. Publication was only considered when the main analysis of data had produced convincing evidence in the method's favour. Fig. 3 of reference 1 shows that the expectations of (o) and (p) were justified, and the required empirical functions were deduced.

78. Only a preliminary examination of field data was included in reference 1, so it is particularly heartening to find that the Authors' later analysis of all available transport data from field as well as laboratory, and for lightweight materials as well as sands, has confirmed the applicability of those formulae over much wider ranges of conditions than those from which they were derived. The second and crucial test of the theory has therefore been successful.

79. The brevity of the Paper belies the great amount of careful analysis that the Authors have carried out. By taking all the available transport data and comparing them against all the main sediment equations, engineers concerned with the regime of rivers and canals have been provided with much-needed information about the reliability of the available equations. The Ackers and White method¹⁻⁴ comes out well, despite the simplicity of its application in engineering problems. However, the Authors will doubtless agree that the ability to make a fair prediction of sediment transport rates from the hydraulic properties of the flow is just one facet of the subject.

80. There remains much scope for detailed research on the physics of sediment movement. The method provides engineering answers but makes no claim to describe the mechanics of movement of individual grains. The analogy is the Colebrook-White friction equation rather than theories describing the internal mechanism of turbulent motion.

81. Bearing this in mind, in Table 1 the classification of sediment transport calculation methods into deterministic and stochastic seems artificial. While recognizing the motion of individual grains as a stochastic process, the overall transport rate of thousands—or even millions—of grains is well described by their average movements. This is a common feature of physics: the gas laws can be applied to describe the physics of gases even though the individual molecules are moving randomly. Of course, the scientist is rightly concerned with understanding the stochastic nature of molecular forces although the engineer will be happy applying his deterministic gas laws.

82. The science of sediment transport has progressed far since Gilbert's classic work,²⁹ but the engineer may have to wait some time yet for any major developments in methods stemming directly from fundamental research. In the meantime, engineering problems have to be solved by reference to empirical methods, although every encouragement should be given to more fundamental studies.

Professor J. R. D. Francis, Imperial College of Science and Technology, London
The Paper seems to show the superiority of the stream-power theories of Bagnold³² in predicting sediment transports. These theories, as reduced to empirical relations

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by Ackers and White,¹⁻⁴ have a solid basis of physical reasoning behind them and so are likely to be valid over a far wider range of parameters than earlier, less well-based theories. By comparing references in the Paper with those of Ackers and White,² wherein the Ackers-White equations are derived, it is clear that much data used for the Paper was also used for the derivation. Could the Authors show how much and what sort of new data was used for the Paper. Can this information also be given for one of the more traditional formulae?

Dr T. Maddock, US Geological Survey

The Authors have made a thorough review of the efficiency of a number of sediment transport equations. The result is most disappointing. The best of the equations is not very good. Except for one or two sets of data, it is doubtful that the discrepancies can be attributed to experimental error.

85. It is hard to understand why the Authors have not taken a good look at their initial assumptions of what constitute pertinent variables. They assume that sediment transport is a function of four dimensionless variables. About two, d/D and ρ_s/ρ , there is no question, but this is not so for the others.

86. The only place temperature effects can be evaluated is in the dimensionless grain size parameter $[g(s-1)/\nu^2]^{1/3}D$. This is inadequate. The parameter would require the temperature to affect all flows, whether the boundary were rough or smooth, and all sizes of sediment. Experimental data show no temperature effects for material coarser than about 1 mm.⁴⁰ Furthermore, studies with smaller sizes^{49,50} show variable temperature effects.

87. The second doubtful parameter is the mobility number. Generally the shear velocity has to be expressed as $(gdS)^{1/2}$. Both field and laboratory data show clearly that for exactly the same values of D , d and S more than one value of both velocity and sediment transport may be expected. However, single values of D , d and S are sliced or rearranged, the result will always be a constant. This is not remedied by a three-axis plot such as Yalin's.²⁰

88. The expected range in velocity is about 3 and the expected range in sediment transport is even greater.

89. Thus, if large quantities of recirculating flume data are used, it is impossible for any equation based on the parameters D_{gr} , Y , Z and s to do much better than 60–70% of the observations having a range of $\frac{1}{2}$ –2 times the mean. The problem is one of deficiency in pertinent parameters. Velocity or the equivalent of a Chezy coefficient cannot be ignored.

90. The reasons for the inadequacy of the Authors' parameters as applicable to laboratory data are given in the conclusion of a paper by Brooks⁵¹ and for field data by Leopold *et al.*⁵² The Authors are no more at fault than many others who have perpetuated the deficiency. One does not expect rapid progress in a difficult scientific field, but 20 years is slow even for sediment transport.

Mr M. R. Gourlay, Senior Lecturer in Civil Engineering, University of Queensland

The multitude of conflicting sediment transport theories and formulae has been exceeded only by the vast array of papers treating various aspects of this general subject. The end result has been utter confusion in the minds of most ordinary mortals whose business forces the subject upon them, as well as in the minds of those others such as academics who sometimes study it from choice. It is thus a great relief to read the Paper, which helps to reduce the confusion by careful comparison of some of the most commonly used and quoted sediment transport theories on a rational basis and tests their performance against most of the available data. It is also reassuring to find that the formula which I generally use and recommend—that of Engelund and Hansen¹¹—still maintains its position as one of the most reliable sediment transport theories.

92. References 53–55 and the original Meyer–Peter flume data could perhaps help expand the range of both field and laboratory data, particularly in regions where there are deficiencies.

93. Ferreira and Veigha de Cunha⁵³ measured bed load in 1965–66 at two locations on the Mondego River in Portugal: Coimbra (87 observations) and Vila Pouca (132 observations) with $D_{50}=2.3$ to 2.6 mm and $D_{gr}=60$.

94. On the Upper Rhine River, between Freistett and Worms, Hinrich⁵⁴ measured bed load in 1968–71 at several locations (53 observations), with $20 < D_{50} < 35$ mm and $500 < D_{gr} < 880$.

95. About 40 observations were made on the Rio Magdalena and Canal del Dique in Columbia⁵⁵ during 1970–72. Water temperature was not measured. For the Rio Magdalena $0.21 < D_{50} < 1.08$ mm and $5 < D_{gr} < 26$, and for the Canal del Dique $0.10 < D_{50} < 0.21$ mm and $2.5 < D_{gr} < 6$.

96. The original Meyer–Peter flume data measured at E.T.H., Zürich, gave $D_m=5.2$ and 28.65 mm for $s=2.68$, and $D_{gr}=130$ and 730 (20 and 33 tests were made respectively). Light and heavy weight data are given for $D_m=5.2$ mm (12 and 8 tests) and 66 tests with non-uniform size sediments where $D_m < 4.5$ mm.

97. I have not checked the completeness of the data available from these sources. It is certainly not all presented in the references quoted. However, the relevant tabulated data and other information for the Mondego River are obtainable from the LNEC in Lisbon, and there may be sufficient information concerning the Rhine data in reference 54. The Meyer–Peter data have been used by Chien²³ and Yalin.²⁶

98. It may be objected that the inclusion of the Meyer–Peter experimental data would unduly favour the Meyer–Peter and Müller formula.²² However, the same can be said concerning the Fort Collins data of Guy *et al.*,³⁸ some of which were used to determine the experimental constant in the Engelund and Hansen formula.¹¹ However, the latter verification used data from the 8 ft wide flume only, which represents only about half the total 185 observations attributed to Guy *et al.* in Table 2. Moreover, the Engelund and Hansen formula, although derived on the basis of lower flow regime conditions (i.e. essentially duned beds) was found to represent conditions involving dunes, transition, standing waves, anti-dunes and chute-and-pool flow equally well (Fig. 4.3.1 of reference 11).

99. Consideration of the Ackers and White theory, and the manner in which its constants were evaluated, shows that 925 individual sediment transport experiments including 173 with lightweight materials were used¹ in contrast with 95 points plotted on Engelund and Hansen's graph.¹¹ Moreover, Ackers and White did not consider any data for which the Froude number was greater than 0.8, thus neglecting most conditions with upper flow regime bed forms. On this basis it could be expected that the Ackers and White theory should perform much better than the Engelund and Hansen theory, particularly when it is realized that over 90% of the 1000 or more tests listed in Table 2 were apparently used in the derivation of its constants. Thus the good performance of the Ackers and White theory when tested against this large quantity of flume data is hardly surprising, and an even better performance might have been expected. However, the Engelund and Hansen theory had prior knowledge of less than 10% of the other tests and yet it gave practically the same performance. Could the Authors compare the distributions of errors as in Fig. 12 for laboratory and field data separately? Perhaps future investigators should pay more attention to the quality of the data which they use to support their theories rather than to amassing a large quantity of data of variable quality.

100. The Authors have noted that the breadth–depth ratio is distinctly different for field and flume data, but state that 'its influence on sediment transportation is thought to be minimal'. I can offer no specific evidence to prove or disprove this statement, but I am skeptical about its truth. Even if the breadth–depth ratio has minimal influence, what about the effect of the shape of the channel cross-section? For instance, various field observations show significant variations in bed load trans-

port across a channel, especially when the bed material is coarse,^{54,56,57} and this appears to have been confirmed in laboratory experiments.⁵⁸ There is also evidence that even suspended sediment concentration varies across a channel in response to variations in flow conditions.^{59,60} The former variations are most probably related to variations in the transporting forces exerted on the bed by the flow, and the latter may well be largely influenced by the adjacent reaches of the river and the variations in channel alignment in the horizontal plane, i.e. bends, constrictions, meandering and so on. Although those responsible for the original field measurements will in most cases have endeavoured to choose river cross-sections where these effects are minimal, this may not always be the case. Consequently users of such data ought wherever possible to obtain additional information concerning cross-section shape and location so that some estimate of the suitability of the data can be made. In this way some of the discrepancies may possibly be explained and the overall reliability of computations of sediment discharge improved. Did the Authors check the field data they used for the influence of any such factors which might make it unrepresentative, such as unusual cross-section shape or unsatisfactory location?

101. Leading on from the question of lateral variation of bed load transport for coarse materials to the actual performance of a sediment transport formula such as the Meyer-Peter and Müller equation,²² which is specifically intended for determination of bed load transport of gravels and which consistently underestimated the transport rate for such materials with $D_{gr} > 30$ by a factor of about 10, it is evident as indicated in the Paper that the value of the Shields' mobility number²¹ of 0.047 is unsatisfactory. Yalin²⁶ has already recognized this deficiency and suggests using a variable value for Y_{crit} derived from the Shields curve or a similar one for Y_{crit} such as Fig. 3. Tippner⁶¹ has found that Hinrich's⁵⁴ field observations on the Rhine indicate that Y_{crit} needs to be modified from the value of 0.047 specified in the formula. This inadequacy with respect to Y_{crit} for large particles was also recognized by French engineers some years ago. With coarse bed materials the phenomenon of partial bed load transport occurs where only a portion of the actual bed material present is transported.⁶² Empirical modifications to the critical mobility number were derived from field tests using tracers.⁶³ It would be interesting to see whether or not the performance of the Meyer-Peter and Müller equation is significantly improved by its application with these modifications, where appropriate, to the data used by the Authors.

102. The modifications to the Meyer-Peter and Müller formula referred to are only a fairly crude attempt to deal with the partial bed load transport process and its allied phenomenon of bed armouring or paving. In such situations there are distinct differences between the size distributions of transported material and the material remaining on the bed^{62,64,65} and the assumption of a uniform unchanging sediment composition appears to be hardly warranted. Consequently, the choice of a representative particle size becomes crucial and also well nigh impossible. Is it D_{35} , D_{50} , D_{90} or what? There is no clear consistent answer to this question except one that deals with the whole grain size distribution, e.g. as proposed by Gessler.^{64,65} Moreover, even for uniform materials there is definite experimental evidence from laboratory data⁶⁶⁻⁶⁸ that there is a change in sediment transport law when the mobility number Y is less than 0.05, and that in fact there is no real critical mobility number for zero sediment transport, as was postulated by Einstein years ago. From Table 3 it is evident, for instance, that the Aare River data lie in this zone. It seems to me that a more careful appraisal of these factors could result in better predictions of the transport rate for large materials, possibly without very much more actual experimental work, and therefore I concur with the last conclusion of the Paper.

103. Although the Meyer-Peter and Müller²² equation was found by the Authors consistently to underestimate the sediment discharge, the Einstein¹⁰ total load method was found to underestimate at small D_{gr} and overestimate at large D_{gr} , i.e. there appears to be a consistent trend for X_{calc}/X_{obs} to increase with D_{gr} . The Authors

have pointed out two possible weaknesses in Einstein's theory and I would emphasize the second one concerning the relationship between the average distance travelled in one movement of a bed particle and the particle size. I have always been struck by the apparent arbitrariness of Einstein's assumptions on this matter and wonder whether this is in fact the Achilles' heel of his theory. If this is so and, if the suggestion in reference 27 quoted by the Authors is correct, then it appears that there is room for careful experimental work on this problem. More accurate assumptions on this point may produce modifications to Einstein's monumental work which will result in a revised theory which is both based on sound physical principles and reliable over the full range of available data.

104. I am pleased to see that my preferred theory—that of Engelund and Hansen¹¹—performs almost as well as that of Ackers and White,¹⁻⁴ and in fact I feel that its overall performance is better. However, the practising engineer must bear in mind that even the best of the theories tested in the Paper predict the observed sediment discharge to within a range of 0.5–2 times its actual value for only two thirds of the data tested. The chances of a wrong estimate are still fairly large and it would seem good sense to use several methods when making an estimate of sediment discharge. On the basis of the Authors' results, it is suggested that the three top methods—Ackers and White,¹⁻⁴ Engelund and Hansen¹¹ and Rottner²⁵—be used and the results compared. The results could then be averaged if they are not significantly different, or the most discordant result rejected, particularly if it were the one from the apparently not so reliable Rottner formula, and the other two averaged. The Authors' results seem to indicate that it is hardly worth worrying about any of the others tested, particularly because the Einstein total load method is so much more involved in application compared with Engelund and Hansen's simple formula. However, it is possible that a method such as that of Colby,⁶⁹ which the Authors have not tested but which has performed reasonably well in a previous comparison,⁷ may also give reasonably reliable estimates of sediment discharge.

105. The Authors use the Rubey equation for estimating the fall velocity of natural sediments. In view of the deficiencies of this equation⁷⁰ and the availability of new data for fall velocity and/or drag coefficient of natural sediments,^{11, 71, 72} it is suggested that this should no longer be used. Admittedly the greatest errors in using it are only about 20% for coarse particles and probably less for smaller particles. Nevertheless more reliable experimental data are now available and should be used.

Dr White, Mr Milli and Mr Crabbe

Mr Ackers supplies clarification of some of the basic principles of the Ackers–White^{1, 3} sediment transport theory and gives details of its inherent assumptions. The basic desire was to produce a general purpose total load theory which could be easily applied by the practising engineer. For this reason the theory considers the movement of fine, coarse and intermediate sizes of material of any specific gravity and the empirical coefficients are based on data for a wide range of sediment and flow characteristics.

107. With reference to § 78 lightweight data from the US Waterways Experiment Station⁴⁷ were used in deriving the original functional relationships.⁴ However, field data were not used in the original development and subsequent comparisons with these data have been encouraging. They have also illustrated some of the difficulties associated with field conditions, including the problems associated with graded sediments and the reliability (or unreliability) of field data itself.

108. We agree that the development of a steady state sediment transport theory is only one facet of a large and complex subject (§ 79). The engineer is still faced with assessing or measuring the hydraulic resistance of a channel before he can predict transport rates and, in our opinion, the current available predictive methods leave much to be desired. The other major problem is that in nature there are rarely uniform, steady state conditions. Typical engineering problems relate to rates of accretion

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or degradation associated with proposed developments (usually involving complex geometrical situations) and this introduces acceleration and deceleration effects which may or may not vary with time.

109. The classification of the sediment transport theories in Table 1 into deterministic and stochastic was intended only to indicate which of the theories originate from considerations of the probability of movement of individual grains. Empirical coefficients are later introduced into the so-called stochastic methods and the form of the final product is often very similar to those theories which are derived entirely from considerations of the average values of relevant variables. This is the most disappointing feature of the stochastic methods and we would encourage fundamental research aimed at providing a better understanding of the mechanics of the subject, thus eliminating the need for this empiricism.

110. **Professor Francis** (§ 83) infers from the Paper that transport theories based on stream power concepts have a superior performance compared with methods based on other types of approach. The method of Engundel and Hansen¹¹ is not based on stream power concepts but it has been shown to have an excellent performance over a wide range of conditions. In our opinion the only weakness in this method is that it tends to overestimate transport rates at low shear values, largely because viscous forces are ignored. The method proposed by Ackers and White,¹⁻⁴ while bearing similarity with the stream power approach of Bagnold,³² has many facets which do not appear in the latter method. It is interesting to compare the two approaches.

111. If the work rate for bed load and suspended load transport are expressed as

$$T_b = g_b \frac{s-1}{s} \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$T_s = g_s \frac{s-1}{\epsilon} \frac{w}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Ackers and White assume transport efficiencies of

$$e_b = T_b / \tau_{cg} V \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$e_n = T_n / \tau_{fg} V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where τ_{cg} is the grain shear stress and τ_{fg} is the total shear stress. Thus $\tau_{\text{cg}}V$ is that part of the total stream power $\tau_{\text{fg}}V$ which is associated with grain shears. It is then assumed that

$$e_b = f^{(1)}(F_{cg}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$e_s = f^{(1)}(F_{t,s}) \quad (13)$$

where F_{cg} and F_{fg} are particle mobility numbers, expressed for fine and coarse material respectively. Transitional sizes are covered by

[illegible]

where

$$G_{gr} = \frac{g}{Vg_{\rho SD}} \left(\frac{v_*}{V} \right)^n \quad (15)$$

$$F_{\text{gr}} = \frac{v_*^n}{\sqrt{[gD(s-1)]}} \left\{ \frac{V}{\sqrt{32 \log(10d/D)}} \right\}^{1-n} \quad (16)$$

$$D_{\text{gr}} = D \left\{ \frac{g(s-1)}{v^2} \right\}^{1/3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where n is an exponent which achieves the transition between coarse and fine materials. Later, based on an analysis of experimental data, Ackers and White establish that

$$G_{gr} = C \left\{ \frac{F_{gr}}{A} - 1 \right\}^m \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

C, A, m and n being functions of D_{gr} .

Table 8. Summary of additional data used in appraisal studies

Investigator(s)	Date	Flume data with sand	Flume data with lightweight materials	Field data (location in brackets)
Macdougall ⁷⁴	1933	65*		
Chyn ⁷⁶	1935	32*		
O'Brien ⁷⁸	1936	83*		
Jorissen ⁷⁷	1938	26*		
Ho Pang-Yung ⁷⁸	1939	52*		
Water Board Authority, Geneva ¹⁸	1939			
Einstein ¹⁶	1944			43† (Aare) 62† (Mountain Creek) 19† (Goose Creek)
Meyer-Peter and Muller ²²	1948	53*		
Colby and Hembree ¹³	1955			
Hubbel and Matejka ¹⁴	1959			22† (Niobrara River)
Shinohara and Tsubaki ⁷⁹	1959			27† (Middle Loup River)
Singh ⁸⁰	1960	279*		36*† (Hii River)
Znamenskaya ⁸¹	1963	35*		
Chitales ⁸²	1966			
Hansen ¹⁹	1966			17* (Indian rivers)
East Pakistan Water and Power Development Authority ⁸³	1967	44*	8	1† (Skive-Karup)
Franco ⁴⁸	1968			
Toffaleti ¹⁵	1968			20† (Atchafalaya River) 19† (Tarbert Landing, Miss.) 21† (St Louis, Miss.) 54* (US rivers) 219*† (Mondego River) 27* (Indian canals) 24† (Elbow River)
Leopold ⁸⁴	1969			
Da Cunha ⁸⁵	1969			
Chaudhry <i>et al.</i> ⁸⁶	1970			
Hollingshead ¹⁷	1971			

Table 8—*contd*

Davis ⁸⁷ Mutter ⁸⁸ Samide ⁸⁹	1971 1971 1971	72* 16*		88* (Nordeg and Bragg Creek)
Gibbs and Neill ⁹⁰ Willis <i>et al.</i> ³⁴ White and Crabbe ⁶	1972 1972 1972	3* 78 7		4† (Paraguay River)
Straub ⁹¹ Sato and Kikkawa ⁹²	— —	18* 221*		
Totals		1084	8	703

* Indicates data obtained since submission of the Paper.

† Indicates sediment grading available.

Table 9. Data from Brooks⁵¹ and Brooks and Nomicos³⁵

Test	Depth, <i>m</i>	Mean velocity, <i>m/s</i>	Bed shear velocity, <i>m/s</i>	<i>D_{gr}</i>	<i>d/D₅₀</i>	ρ_b/ρ	Mobility <i>Y</i>	Bed friction factor λ	Froude number <i>Fr</i>	Bed condition
Brooks ⁵¹ 3	0.074	0.622	0.0366	4.3	460	2.65	0.517	0.0275	0.73	Smooth Dunes
Brooks ⁵¹ 9	0.075	0.411	0.0396	4.3	470	2.65	0.605	0.0742	0.47	Fiat Dunes
Brooks and Nomicos ³⁵ A	0.073	0.628	0.0323	4.1	490	2.65	0.429	0.0212	0.74	
Brooks and Nomicos ³⁵ C	0.073	0.277	0.0369	4.1	490	2.65	0.560	0.1420	0.33	

performance of the Ackers and White method is to be expected. When the Paper was written this was true in terms of the flume data but the field data were not included in the original analysis. Ackers and White were seeking to produce a general function and hence it seemed appropriate to base the functional relationships on as much data as possible. Flume data were chosen in preference to field data because of difficulties and hence uncertainties in the measurement of prototype transport rates, the problems associated with graded sediments and non-uniform flow configurations. In the Paper the main test of the Ackers-White method was in terms of field data. Further data have now been extracted from the literature⁷³ and the position is summarized in Tables 6-8.

114. We do not share Dr Maddock's faith in the reliability of measurements associated with sediment transport (§ 84). The problems are too numerous to mention in detail but include the apparent non-existence of steady-state conditions, the absence of repeatability, uncertainties in the measurement of depths, slopes and transport rates, and difficulties in the interpretation of the measurements, e.g. the calculation of depth where significant bed features exist.

115. Bearing these points in mind any large collection of data must contain significant random errors and no theory will ever take them all into account. The inference from Table 5 is not, for example, that the Engelund-Hansen method¹¹ gets within $\frac{1}{2}$ -2 times the actual transport rate 63 times out of 100 but that it will agree with observations on this basis.

116. Dr Maddock suggests that the variables used in the Ackers and White method¹⁻⁴ do not form a comprehensive set. It is suggested that temperature effects and bed formations (or boundary resistance) are inadequately represented.

117. With reference to the influence of temperature (§ 86), Taylor and Vanoni^{50, 67} have suggested that the effect could be related to the particle Reynolds number $X = v_* D/\nu$ and to a lesser extent the mobility $Y = v_*^3/gD(s-1)$, where D is the sediment diameter. As the dimensionless grain size D_{gr} is equal to $(X^2/Y)^{1/3}$ the same argument leads to the conclusion that D_{gr} and Y should control the process with the emphasis on D_{gr} .

118. Dr Maddock is wrong in suggesting that the D_{gr} parameter would require the temperature to affect all flows whether the boundary were rough or smooth and all sizes of sediment. In the Ackers and White theory¹ the D_{gr} parameter defines particle size. It distinguishes between fine, coarse and intermediate particle sizes and determines the coefficient values m , n , A and C in the general function. Ackers and White¹ described these functional relationships and suggested that for coarse materials m , n , A and C are constant. Coarse materials were defined by $D_{gr} > 60$, i.e. $D > 2.3$ mm for sand sizes at normal temperatures. Thus computed transport rates are independent of temperature for sand sizes greater than 2.3 mm and dependent on temperature variations for finer materials. This is entirely consistent with the findings of Franco,⁴⁸ and Taylor and Vanoni.^{49, 50, 67}

119. We assume that the essence of Dr Maddock's second criticism (§§ 87-88) is centred on the question of whether the group of seven variables ρ , ρ_s , ν , D , d , v_* and g defines the two-phase phenomenon uniquely, i.e. whether for one given group of values of D_{gr} , Y , d/D and ρ_s/ρ there exists only one transport capacity and one mean velocity. Yalin²⁸ argues that the consideration of additional variables is superfluous. It is argued that any additional variables are of a dependent character and result from the values of the basic seven independent variables, e.g. one cannot control channel resistance as a separate parameter. However, Brooks⁵¹ indicates that 'neither the velocity nor the sediment discharge concentration could be expressed as a single-valued function of the bed shear stress or any combination of depth and slope or bed hydraulic radius and slope. This finding is contrary to the almost universally held assumption that the suspended-sediment transporting capacity of a stream can be uniquely related to the geometry of the stream cross section, the slope of the channel and the size of the bed material'. He suggests that the difficulty arises from changes in the configuration of the bed forms and therefore the boundary resistance. In the light of Yalin's work²⁸ it

is useful to question Brooks' justification for his views as given in the reply to the discussion on his paper.⁵¹ He quoted four specific tests: tests 3 and 9 of his paper and A and C of the later work by Brooks and Nomicos.⁵⁵ These data are summarized in Table 9.

120. In order to show conclusively that the four groups D_{gr} , Y , d/D and ρ_s/ρ do not define the two-phase phenomenon uniquely it would be necessary to carry out a number of tests with the same value of these four quantities but with unmistakable differences in transport rates or frictional resistance. In this context we suggest that to draw firm conclusions from only four flume tests is risky. In any case Table 9 shows that in the four tests chosen the particle mobility differed considerably from test to test. Hence the Yalin argument would permit the observed changes in frictional characteristics without the need for additional parameters. It will be difficult to settle this argument conclusively either way.

121. In reply to **Mr Gourlay** the field data from the Mondego River, Portugal⁸⁵ and data from Peterson and Howells⁷⁹ have enabled us to extend the analysis to include more field data. Tables 7 and 8 summarize the data currently available and these have been used to produce separate distributions of errors for flume data with sand, flume data with lightweight materials and field data for both the Engelund–Hansen method¹¹ and that of Ackers and White.¹⁻⁴

122. The histograms are shown in Fig. 13 and the two theories show similar overall performance. This is surprising when one considers the totally different philosophy inherent in the methods. If anything, the Ackers–White approach¹⁻⁴ is more effective on flume data and the Engelund–Hansen approach¹¹ is more effective on field data. However, the differences are small.

123. With reference to § 101 we accept that the remarks in the Paper concerning the effect of the breadth/depth ratio gloss over some of the difficulties associated with field measurements. The remarks made are probably true where the cross-section of flow is approximately rectangular but are unlikely to be true where the cross-section and/or the velocity distributions are far from uniform. In the Paper we avoided using field data which departed from the normal concept of two-dimensional flow. The assessment was based on the original reports of the various studies. Unfortunately in the subse-

Fig. 13. Distribution of errors for flume, lightweight and field data

DISCUSSION

quent appraisal⁷³ the original reports were not always available. However, prototype data are scarce and if one sets very high standards the quantity of data remaining is disappointing.

124. It would be interesting to make a comprehensive study of the effect of cross-sectional shape on the sediment transport phenomenon. It would be even more interesting to try to interpret the results!

125. The influence of graded sediments is an important aspect of the sediment transport problem. Of the theories analysed in reference 5 only four seek to take the grading of a sediment into account. The others use an equivalent diameter which may be the D_{35} , D_{50} , D_m or other size taken from an analysis of a bed sample. This is clearly a simplification because grading curves with a different shape will almost certainly have a different effective diameter. Furthermore, the effective sediment size will probably vary with the transport rate, particularly in gravel rivers where there is a wide spectrum of sizes and where the sorting of material is most pronounced. It is useful to consider this point in two sections: the general problem of graded sediments in motion and problems with armouring and the associated threshold conditions.

126. The idea of an effective diameter related to the grading curve and the ambient flow conditions is shown in Fig. 14. Assuming that the graded sediment can be defined by the range of sizes between D_5 and D_{95} , say, sediment transport will occur within the zone defined by the limits $Y > Y_o$ and $D_{gr(5)} < D_{gr} < D_{gr(95)}$ if the armour plating effect is not significant.

127. Each size fraction has a value of Y_c which is a function of its size. However, the value of Y associated with each fraction will depend on the size of the fraction and the ambient flow conditions

[illegible]

[illegible]

and substituting

[illegible]

$$Y = \left(\frac{1}{\rho(s-1)v} \right)^{2/3} \frac{v_*^2}{D_{gr}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

i.e.

[illegible]

for a particular fluid and sediment density. Increasing the value of v_* from zero the values of Y for all the size fractions will increase until $Y_5 = Y_{5\infty}$. At this point the 5% fraction will start to move and the significant diameter D_s will be $D_{5\infty}$. At a higher value of v_* the conditions could be as shown in the line AB in Fig. 14. Several size fractions will be in motion and the significant diameter will be between D_5 and D_{50} and increasing with v_* , i.e.

$$D_\alpha = f(Y) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

128. As v_* increases the conditions $Y_{95} = Y_{095}$ will be reached and all the size fractions will then be in motion. This condition is illustrated by the line A'B' in Fig. 14 with a significant diameter D_{α}' larger than D_{α} . At higher shear velocities, and certainly as $Y \rightarrow \infty$, there is likely to be little change in the magnitude of D_{α} , i.e.

$$D_\alpha = \text{constant } (Y \rightarrow \infty) \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

129. This philosophy, which assumes that the individual size fractions have no influence on each other, thus leads to the broad conclusion that the significant or effective particle size in a graded material decreases with decreasing transport rates because the fractions moving are smaller than the bed material as a whole. However, the armouring effect where smaller particles hide behind, between or underneath the

Fig. 14. Change of effective diameter with stage

Fig. 15. Comparison of grain size distribution for field data

Fig. 16. Grading curves for moving and total sediments (Egiazaroff⁹⁴)

Fig. 17. Critical mobility numbers for graded sediments (Egiazaroff⁹⁴)

larger particles works in the opposite direction. It could be argued that if the bed is covered with material of the D_{95} size (assuming a sample in depth) the significant particle size is initially the D_{95} size not the D_5 size and the D_a falls from D_{95} to its final intermediate value as Y increases. Both views give the same answer when all the size fractions are in motion but there is plenty of scope for errors near threshold conditions. This presents serious problems for the engineer, particularly where the sediment is very coarse. Not only is there a wide range of sediment sizes in these rivers but, because of the large sediment sizes, the transport conditions are usually close to the threshold of movement. In fact, the larger the maximum sediment size the wider is the range of sizes. This is shown in Fig. 15 where the cumulative percentage by weight is plotted against the grain size ratio D_i/D_{50} for various rivers. As D_{50} rises so does the ratio D_{95}/D_{50} .

130. This argument leads to the conclusion that those theories which assume an effective grain size which is a constant value related only to the grading curve are likely to introduce errors close to the threshold conditions. One improvement would be to relate the effective grain size to the mobility as well as the grading curve of the bed material. Also the ratio D_{95}/D_5 or D_{90}/D_{10} would be a relevant parameter.

131. Although some theories attempt to take size grading into account they do so only in terms of established motion. Not one theory tested considers the initial movement of an armoured bed.

132. Egiazaroff⁹⁴ made an interesting evaluation of this problem in deriving his own sediment transport theory. He derived an expression for incipient motion of uniform sediments

$$Y_c = \frac{4}{3} \frac{1}{C_D} \left(\frac{v_*}{V_{yd}} \right)_c^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where C_D is the drag coefficient, V_{yd} is the velocity at level $y=0.63D$ and C denotes critical conditions. The ratio v_*/V_{yd} was evaluated using Einstein's equation¹⁰

$$\left(\frac{V_{yd}}{v_*} \right)_c = 5.75 \log \left(\frac{30.2}{k_s} 0.63 D x \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where

$$x = f(k_s/\delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

133. For an arbitrary individual grain size D_i in a mixture under rough turbulent conditions and with $k_s = D_m$, the geometrical average diameter for roughness based on the average figure for the material in motion and the total material (see Fig. 16) is

$$D_m = (D_{m1} \text{ material in motion} + D_{m2} \text{ total material})/2 \quad . \quad . \quad (32)$$

and equation (30) becomes

$$\left(\frac{V_{yd}}{v_*} \right) = 5.75 \log \left(\frac{19 D_i}{D_m} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Substituting this value into equation (29) and taking $C_D = 0.4$ gives

$$(Y_c)_i = \frac{0.1}{[\log(19 D_i/D_m)]^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

134. For the case of uniform material $D_i = D_m$ and equation (34) gives a value $Y_c = 0.06$. Fig. 17 shows the curve represented by equation (34) and the results from several experimental investigations. Equation (34) gives the threshold conditions for an individual size fraction within a graded sediment. For the entire mixture Egiazaroff⁹⁴ suggested the use of $D_i = D_{50}$ and thus

$$Y_c^{\text{mix}} = \frac{0.1}{\log(19 D_{50}/D_m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

As the geometric mean size D_m is usually larger than D_{50}

$$Y_o^{\text{mix}} > Y_o^{\text{uniform}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

135. According to Egiazaroff⁸⁴ the D_{50} value must be determined from the grading curve of the material in motion and equation (35) is valid only when there are motionless particles, i.e.

$$D_{\max} \geq D_0 = \frac{v_*'^2}{0.06 g(s-1)} \quad \dots \dots \dots (37)$$

When $D_{\max} < D_0$ no accumulation of large bed material can take place and $D_1 = D_0$ where D_0 is the equivalent threshold diameter for the mixture.

136. However, Neill⁹³ raised two fundamental objections to the Egiazaroff⁸⁴ philosophy. First, in his analytical derivation Egiazaroff applied the theoretical velocity profile at an elevation below the peaks of some of the larger particles. Neill argued that the physical picture in this area must be confused and the interaction between the flow and the smaller grain sizes must be dependent to a large extent on the disposition of the larger particles. Second, Egiazaroff's method of determining D_m demands a knowledge of the sediment travelling in suspension which cannot be deduced from a grading curve of the original bed mixture.

137. The importance of obtaining a satisfactory method of determining critical mobility criteria for mixtures cannot be overemphasized. Many theories base their computations of sediment transport on the concept of excess shear and this becomes crucial close to threshold conditions. A large proportion of the available data in the range $D_{gr} > 10$ constitutes bed load data and minor deviations in the value of Y_c can cause large errors in computed sediment transport rates. The theories of Shields,²¹ Meyer-Peter and Müller,²² Bagnold,^{31, 32} Laursen²⁸ and Yalin²⁶ are particularly susceptible to this effect.

138. The general function of Ackers and White¹ incorporates a threshold condition which was derived from the analysis of established movement data rather than observations of the detachment of individual grains. The curve they suggest is shown in Fig. 3 together with the classical Shields' curve. For rough turbulent conditions ($D_{gr} > 60$) there is a considerable difference between the two curves. Ackers and White suggest $Y_c = 0.028$ and Shields suggests $Y_c = 0.060$. Both figures were derived from uniform sediment data but the lower value improves the accuracy of the predictions of Ackers and White for the gravel river data from the Aare and Elbow rivers.

139. There is a need for further work to define, in a more precise way, the initial motion of a graded sediment and to investigate the established motion of a graded sediment at low transport rates.

140. We agree with Mr Gourlay's comments about Einstein's methods.¹⁰ There is room for a re-appraisal of some of the assumptions made by Einstein and this could lead to an improvement in predictions using the method. We have done this³⁰ using some of the ideas of Bishop *et al.*,²⁴ Yalin,²⁶ Christensen and Chiu.²⁷ The Einstein parameters A_* and B_* (assumed constant by Einstein) were related to D_{gr} and R_* respectively and a hiding factor based on the ratio D_{35}/D_{50} was introduced. It was also shown that Yalin's suggestion²⁶ regarding the limits of the Einstein entrainment integral improved the performance of the method. The revised approach³⁰ works well at medium transport rates for particle sizes defined by $D_{gr} > 2$, e.g. sand sizes in excess of 0.08 mm.

141. With reference to § 104 we are in favour of attacking sediment transport problems from as many angles as possible. This would include using two or more sediment transport theories to predict transport rates. However, it is also important to recognize the merits and limitations of individual theories and to apply them accordingly. For example, the Einstein approach¹⁸ will give excellent results for sand sizes of 0.5–1.0 mm but it would be unwise to use the method in a gravel river situation. Likewise the Bagnold approach³² will give excellent results within its limited range of applicability.

142. With regard to the use of Rubey's equation²⁰ for calculating the fall velocity of natural sediments, it is acknowledged that more recent, and possibly more accurate,

data are now available. However, the differences are relatively small and the Rubey equation has the advantage in a study of this nature that it permits direct calculation of fall velocities. It is thus very convenient for computer use.

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