Dynamic characteristics of three adjacent reinforced concrete buildings

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Referring to § 1, one such mathematical model which is useful within its limitations for the consideration of structure–foundation interaction involves treating the lateral vibrations of a tall slender building as if they were those of a vertical uniform cantilever whose base shear and moment have a linear relation to the base translation and rotation (Fig. 11). If the de-stabilizing effect of the displacement of the centre of gravity of the cantilever from its axis is neglected, and for many practical cases this can be done quite reasonably, the conventional equation for the mode shape of a long slender beam is given by

$$u = A \cos bx + B \sin bx + C \cosh bx + D \sinh bx$$

60. At the top of the structure the end conditions are

$$\left(\frac{d^2u}{dx^2}\right)_l = 0 \quad \left(\frac{d^3u}{dx^3}\right)_l = 0$$

while at its base

$$\frac{EI}{k_R} \left(\frac{d^2u}{dx^2}\right)_0 = \left(\frac{du}{dx}\right)_0 \quad \frac{EI}{k_T} \left(\frac{d^3u}{dx^3}\right)_0 = -u_0$$

where $b^4 = \frac{\pi^2 n^2 \sigma}{EI}$

- n frequency of vibration
- σ mass per unit length of the cantilever
- EI bending stiffness of the cantilever
- k_R rocking stiffness of the base
- k_T translation stiffness of the base
- l length of the cantilever

61. When these conditions are put into the mode shape equation the frequency equation is obtained

$$\frac{k_R}{EIb} (Q + R + 2) - \frac{EIb^3}{k_T} (Q + R - 2) - 2P \frac{k_R}{k_T} b^2 + 2 \frac{QR - 1}{P} = 0$$

where $P = \sin bl \cosh bl + \cos bl \sinh bl$
 $Q = \cos bl \cosh bl - \sin bl \sinh bl$
 $R = \cos bl \cosh bl + \sin bl \sinh bl$

62. Solutions of this equation can best be presented in the form of a set of interaction curves of $EI/k_R l$ against $EI/k_T l^3$ for various values of bl . Fig. 12 shows this relationship for the first mode. Similar graphs would apply to the higher modes but for those in which rocking and translation are in opposite phase the shape of the curves would be different. Fig. 13 shows the contribution of translation, rocking and bending to the total deflexion in the first mode. The modes in which the two foundation movements are out of phase would probably be difficult to set in motion by any excitation other than through the ground and difficult to identify in the response to an irregular ground motion.

DISCUSSION

Fig. 11. First mode for $EI/k_R l = 0.153$, $EI/k_T l^3 = 0.030$ (whence $b/l = 1.60$)

Fig. 12. Interaction curves for first mode

Fig. 13. Note that for specific values of b/l and $EI/k_R l$ the value of $EI/k_T l^3$ is determined by Fig. 12

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The analysis presented by Mr Howells is a useful method of considering foundation flexibility of uniform multi-storey structures.

64. The linear relation of base rotational and translational flexibility is a reasonable assumption when considering a narrow frequency range such as the response at and close to a particular normal mode frequency. However, it is necessary to consider a different linear relationship for each mode considered as the foundation flexibility is non-linear because it is dependent on the frequency and amplitude of vibration. The mass of the foundation should also be included, particularly when considering translation of the base of the structure.

65. Out of phase motion of the two degrees of freedom of the base of a structure (excluding antiphase motion) is a function of the damping associated with each degree of freedom and the relation between the excitation frequency and the normal mode frequencies of the system. In any practical system such as the structure tested, out of phase motion will always be present to some extent but, as indicated by Mr Howells, it will be difficult to identify in the response to an irregular ground motion.