Slab bridge design and drawing— an automated process

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Dr L. A. Clark, Cement and Concrete Association

The Author has presented an automated process which leads to the expedient and economic design of simply supported slab bridges, and in which particular attention is given to the arrangement of the reinforcing bars (or prestressing tendons) in the slab. I would like to both confirm and amplify the Author's comments on the latter consideration.

52. I have analysed 12 uniformly loaded simply supported skew slabs and three simply supported skew highway bridge slabs loaded with the Ministry of Transport's standard and abnormal loadings.⁴ The uniformly loaded slabs had skew angles of 15°, 30°, 45° and 60°, and aspect ratios of 0.4, 1.0 and 1.6. The three bridge slabs all consisted of two 3.7 m traffic lanes and two 1.5 m footpaths giving a total right breadth of 10.4 m. Bridge slab 1 had a skew angle of 45°, an aspect ratio of 1.0 giving a right span of 10.4 m, and a span to depth ratio of 20.8. Bridge slab 2 had a skew angle of 60°, an aspect ratio of 1.6 giving a right span of 6.5 m, and a span to depth ratio of 18.6. Bridge slab 3 had a skew angle of 30°, an aspect ratio of 1.0 giving a right span of 10.4 m, and a span to depth ratio of 20.8.

53. For the uniformly loaded slabs, the design moment triads at each reference point considered by Rusch and Hergenroder³ could be readily evaluated by using the graphs of influence coefficients prepared as a result of their experimental work. The moments of resistance required for the same four reinforcement arrangements as considered by the Author were then calculated using the same method as the Author; namely, that of Wood.¹³ Using these data a number of graphs have been prepared¹⁷ which show the relative moments of resistance required at each reference point for the four different steel arrangements for a range of skew angles and aspect ratios.

54. From a study of these graphs it was concluded that an orthogonal steel arrangement is invariably the most efficient. The better of the two orthogonal arrangements was the x - y one, except for slabs having a skew angle greater than about 30° and an aspect ratio less than about 1.0; for these cases, the u - v arrangement was preferable. For high skew angles, economies in the total amount of steel required can be obtained by placing the steel parallel and perpendicular to the abutments in the central regions of the slab and by changing to an arrangement which has a steel component parallel to the free edge at the free edges.

55. For each of the three bridge slabs considered, the design moments at each of the reference points covered by Rusch and Hergenroder³ were evaluated from the influence charts for the following load cases:

- (a) HA loading on both traffic lanes plus dead load;
- (b) the critical position of the HB vehicle in one traffic lane plus $\frac{1}{2}$ HA loading in the other traffic lane plus dead load;
- (c) the HB vehicle in the centre of the bridge plus dead load.

These load cases were then considered in combination with the footpath loading. The moments of resistance required for each of the four reinforcement arrangements for each load case were then calculated by Wood's method.¹³ It was then possible to select minimum moments of resistance at each reference point to resist all the

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different applied moment triads arising from the different load cases. The moments of resistance provided over each slab were obtained by inspecting the minimum moments of resistance and then dividing the slab into what was considered to be a reasonable number of regions for curtailment of the steel. As the Author has pointed out in § 41, the curtailment points are not clearly defined since the moments are usually known only at five points on the slab. A particular difficulty is the curtailment of the steel in the u or y direction as the supported edges are approached. When a finite element analysis, such as the Author's, is used then the curtailment points are more clearly defined.

56. The steel required over the slab was not calculated as it was felt that the moments of resistance gave a better comparison between the various reinforcement arrangements. The efficiencies of the various reinforcement arrangements have been compared on the basis of the moment volumes. The moment volume is defined as the moment of resistance provided multiplied by the area over which it acts.

57. In Tables 8–10 the moment volumes for the various reinforcement arrangements have been compared for each bridge slab. The combined arrangement referred to in Table 9 for bridge slab 2 consists of the central and obtuse corner regions of the slab reinforced with the x - y arrangement and the free edge with the u - y arrangement.

Table 8. Moment volumes for bridge slab 1, skew=45°, AR=1.0

Steel arrangement	Moment volume, kNm^2	Moment volume
		Min. moment volume
u - y	206 000	1.10
u - v	236 000	1.26
u - x	555 000	2.97
x - y	187 000	1.00

Table 9. Moment volumes for bridge slab 2, skew=60°, AR=1.6

Reinforcement arrangement	Moment volume, kNm^2	Moment volume
		Min. moment volume
u - y	55 500	1.00
u - v	73 500	1.32
u - x	674 000	12.13
x - y	61 900	1.11
Combined	56 700	1.02

Table 10. Moment volumes for bridge slab 3, skew=30°, AR=1.0

Reinforcement arrangement	Moment volume, kNm^2	Moment volume
		Min. moment volume
u - y	206 500	2.59
u - v	113 500	1.42
u - x	163 000	2.04
x - y	79 700	1.00

58. The main point to emerge from the tables is that considerable economies in the total amount of steel required can be obtained by using an orthogonal reinforcement arrangement instead of the more usual practice of placing the steel in directions that are parallel to the edges of the slab.

59. An inspection of the moments of resistance required at the various reference points for the four reinforcement arrangements for the three bridge slabs¹⁷ shows that, generally speaking, the steel arrangement that requires the least total moment of resistance for the above load cases is also that which requires the least total moment of resistance for a uniformly distributed load on a slab of the same geometry. This suggests that the graphs presented for a uniformly distributed load¹⁷ give a good guide to the reinforcement arrangement that should be adopted under a variety of loadings.

60. The British Oxygen Bridge considered by the Author has a skew angle of 48° and an aspect ratio of 0.825. From Table 6 it can be seen that the most efficient reinforcement arrangement is either the x - y or the u - y arrangement. Bridge slab 1 in my analysis had a skew angle of 45° and an aspect ratio of 1.0, and was loaded under HB as well as HA loading. From Table 8 it can be seen that the most efficient reinforcement arrangement is the x - y arrangement which requires 10% less steel than the u - y arrangement. However, I did not take account of laps, hooks, and shrinkage steel in the top of the slab as the Author has done. Probably the most important omission on my part is that of the bands of steel along the free edges which are required when using the x - y arrangement to prevent anchorage failures at the ends of the longitudinal bars which enter the free edge at an angle. Did the Author consider this steel when evaluating the quantity of steel for the x - y arrangement in Table 6? However, it can be said that the Author's results are in agreement with my own.

61. While discussing the efficiencies of various reinforcement arrangements I feel that Table 4 does not give a fair comparison of the efficiencies of the various arrangements; a much better comparison can be made from Table 5 or Table 6 which give the required moments of resistance and the cost respectively.

62. Finally, in § 28, the Author quite rightly states that a great deal of experimental work needs to be carried out on the effects of the various reinforcement arrangements on slab behaviour. Work is currently in hand at the Cement and Concrete Association to study this problem. Model skew slabs are being tested under the Ministry of Transport's abnormal loading⁴ to examine the crack control characteristics of different reinforcement arrangements. Some of the model slabs in the test series are being designed by yield line theory and some by the method described in this discussion.

Dr K. O. Kemp, University College, London

The only comment I should like to make concerns the procedures for choosing the reinforcement layouts. In § 32 the Author states that a working stress approach is being used but in reality the method adopted rests on the concept of plasticity and on the yield criterion for orthotropically reinforced slabs proposed independently by Kemp,¹⁸ Nielsen¹⁹ and Save.²⁰

64. Using this yield criterion, Wood¹³ has produced rules for the provision of orthogonal reinforcement which ensure that yield just occurs at each point of the slab and the sum amount of reinforcement (or moment of resistance) in the two orthogonal directions is minimized. Wood¹³ does not give the precise rules for the provision of minimum amounts of skew reinforcement but this extension has been carried out by Morley.²¹ It would be helpful if the Author could clarify whether he is using the rules for minimizing the sum amount of skew reinforcement as suggested by Morley.²¹ This sum reinforcement in the two (or four) directions would certainly seem to be a much better measure of the efficiency of the layout selected, than the ratio of the areas enclosed by the acting moment and moment of resistance curves as used by the Author, which could give a distorted view of the relative efficiencies of different layouts.

65. When a slab is to be reinforced to resist the moment fields produced by a number of different loading cases, the problem of optimizing the amounts of reinforcement is more complicated than the Author appears to suggest, and is essentially a problem in non-linear programming. Upper bounds on the reinforcement required can be obtained from envelopes of moment but if such upper bounds are not employed, it is important that the selected reinforcement layout be tested for each of the loading cases separately. It would be helpful if the Author would clarify the procedure he has used for selecting the optimum reinforcement with multiple loadings.

Mr M. J. Fothergill, Oscar Faber & Partners

The Author and his team are to be congratulated on producing such a comprehensive program, making use of advanced analytical techniques in conjunction with the most useful hardware for engineers, the plotter.

67. The support arrangement indicated in the figures is that of the edges being continuously restrained, presumably with rotational deflexion being unrestrained. Is any account taken of elasticity of supports which might occur in practice? If a slab is analysed under a particular loading with fully fixed restraints, and again with elastic restraints, there is a significant difference in the results, particularly in the distribution of reaction along a line support and so also in the stresses in the vicinity of a support as shown by Mehmehl and Weise.¹¹ The implication is that, since no practical supports are completely rigid in nature, an analysis carried out with rigid restraints is likely to be in error near the supports, a result borne out by comparisons between model tests and computer analyses.

68. The Paper discusses the application of dead load and highway and pedestrian live loadings which are undoubtedly the source of major principal stresses. However, minor principal stresses can be greatly affected by secondary effects, such as creep and shrinkage differential between deck and support, temperature effects in the deck and differential settlement of supports. Are these taken into account in the program? The principal stresses will be most affected by such loadings near the support where there is maximum restraint to the slab, particularly in a direction parallel to the line of support and where stresses due to the major loadings are likely to be low.

69. These two points taken together suggest that reduction in reinforcement close to supports should not be undertaken without thorough investigation and it is in these areas that most saving is indicated. Indeed, I have known a skew slab of this type to crack extensively, but fortunately not seriously, due to such second order effects.

70. The Paper describes the design of a reinforced concrete slab and a prestressed configuration consisting of precast, prestressed concrete beams in conjunction with an in situ reinforced concrete slab. Is the program capable of designing a prestressed concrete slab cast wholly in situ? Also, will the program cope with extra stiffening in edge beams, if required, or such geometric oddities as service ducts in the edge of the deck?

71. Comparisons are made in the Paper between complete hand calculations and complete computer designing. No mention is made of the more likely method of working at present; that of analysing a structure with the aid of a computer and detailing it by hand. Does the Author have any evidence that use of the program is more economic, either in cost of design or cost of materials, than working this way, which has the advantage of being more able to cope with non-standard situations?

Dr W. M. Jenkins and Mr K. K. Gangopadhyay, Teesside Polytechnic

We have been concerned with computer aided design of highway bridges and are investigating both elastic and ultimate load methods.

73. While the Author is to be congratulated on an interesting and useful approach to automatic design of slab bridges, it would appear that this approach is rather restricted in a number of important respects.

74. We would question the use of an isotropic element when an orthotropic slab

Table 11

Reinforcing system	Value of μ	Design case	Slab thickness, in.	Quantity of concrete, cu. ft/sq. ft	Quantity of steel, lb/sq. ft	Total cost, £/sq. ft	Remarks
$y-u$	0.5	1	18	1.5	22.8	1.37	Two values of μ are considered in order to compare the results
		2	27	2.25	18.8	1.51	
		Author	27	2.25	24.5	1.70	
	0.9	1	18	1.5	24.0	1.42	
		2	27	2.25	19.5	1.53	
		Author	27	2.25	24.5	1.70	
$u-v$	0.5	1	24	2.0	28.0	1.74	
		2	27	2.25	27.2	1.81	
		Author	27	2.25	27.5	1.81	
	0.9	1	24	2.0	27.0	1.71	
		2	27	2.25	25.5	1.75	
		Author	27	2.25	27.5	1.81	
$u-x$	—	1	25	2.083	36.0	2.06	Only nominal reinforcement has been used in transverse direction
		2	27	2.25	36.0	2.12	
		Author	27	2.25	68.5	3.21	

1. Mild steel reinforcement has been used since it would appear that mild steel was used in the Author's design.
2. The ratio of yield stress of steel to cube crushing strength of concrete at 28 days is assumed to be 8 for the design.
3. Value of μ gives the ratio of transverse moment of resistance to that in the longitudinal direction.
4. A load factor of two has been used in the design for both live and dead loads.
5. The loading is as assumed by the Author and also the prices of materials, 6.5 lb/sq. ft is included in the quantities as top steel.
6. In case 1, the slab thicknesses have been changed in places as shown in Table 11, while in case 2, the slab thickness is 2 ft 3 in. throughout which has been used finally by the Author.

element can be used at no extra computing cost. Such an element originally devised by Siddall²² and Jenkins²³ and further developed and applied by Lax²⁴ has been shown to give good results when applied to model tests on concrete bridges.²⁴

75. The adoption of a 4×4 mesh to represent the slab may not provide sufficient accuracy. In our work we have generally written programs in such a way that the number of elements transversely and longitudinally are left as items of data. In this way the designer can assess individual situations and prescribe suitable element sizes.

76. Certain other important practical points arise which are capable of treatment by the finite element method and these have been incorporated in Lax's program.^{24, 25} Slab-beam bridges, column supported bridges, support flexibility and varying slab thickness can all be handled by suitable computer routines.

77. A special slab-column element has been developed to enable column supported slabs to be analysed accurately without recourse to mesh refinement in the region of the column.

78. Recently we have extended our work on highway bridge design to study the use of ultimate load methods²⁶⁻²⁸ in skew slab bridges as we feel that this approach may produce more rapid and economical design on the computer. We are investigating the use of yield line methods in the design of skew bridges, possibly with non-parallel abutments, for both HA and HB loadings. If this approach is successful it is our intention to retain elastic analysis as a check on the final design at working load conditions.

79. In order to obtain a comparison with the Author's design of the British Oxygen Bridge (Fig. 4) we have designed this bridge using yield line theory and some comparative results are set out in Table 11.

Mr K. A. L. Johnson, Howard Humphreys Services

The Paper is most interesting and certainly points the way for the future. However there are a number of statements which require clarification.

81. The knife edge load of HA loading has been applied parallel to the abutments. The present BS 153 rule was never intended to cover the present case, as this means that the magnitude of the load increases with increasing skew. I consider the kinetic energy load should be applied normal to the line of the traffic lane, and of magnitude 27 000 lbf for each lane loaded.

82. The results for points 3 and 13 in Table 1 are interchanged.

83. In Table 2 a comparison of moments under a single point load is made between various methods of analysis and experimental results. Unless the following variables:

- (a) span/depth ratio;
- (b) shape of contact area;
- (c) contact area/depth ratio;
- (d) Poisson's ratio

are the same in all cases the comparison is of limited value.

84. The discussion on various placing systems of reinforcement is interesting. However, can bars that are not parallel to the free edges develop bond quickly enough to be effective in this area? Rather, I would favour a combination of a central area system together with free edge bands, i.e. a combination of the $u-v$ and $x-y$ systems. The optimum theoretical system is in the principal directions which vary remarkably little for various load combinations for solid slabs.

85. A program that does not allow for discrete bearings of a given vertical stiffness is of limited value in the analysis of highly skewed slabs. Large negative moments occur in the vicinity of the obtuse corner and for the size of element used the concentrations are not fully covered. These moments require top reinforcement parallel to the bearing lines and free edges, of size in some cases, equal to the central area reinforcement. Further without careful choice of vertical stiffness of the bearings it is possible to have negative reactions or uplift. Rusch and Hergenroder's point *E* does not adequately cover this area fully, either.

86. The presence of negative moments is pointed out in § 24(c) but no further treatment is given. In the example given top reinforcement is required for most of the bridge.

87. No mention is made of the design strength of concrete or the quality of reinforcement used in the design examples.

88. I would be interested in the Author's views on the following and how they could be covered by restraints in his program:

- (a) creep effects and control of cracking;
- (b) deflexion limitations;
- (c) ultimate load behaviour as a limiting criterion on reinforcement systems.

89. I have obtained good correlation with experimental work^{3,11} using grid analogy, and used it to design a deck slab very similar to the British Oxygen Bridge. The designs agree closely except for the above mentioned points.

Mr G. G. Aperghis

I wish to thank **Dr Clark** for both confirming and amplifying my comments on the arrangement of reinforcement in skew slab bridges. Considerable economies can be achieved by adapting 'unconventional' reinforcing patterns, such as $y-x$ or $y-u$, particularly as the angle of skew increases, and by varying the pattern in different regions of the slab. This has been pointed out by other writers also.

91. The conclusion reached by **Dr Clark** that the correct reinforcing pattern to be adopted for a slab bridge can be deduced from a comparison with a similar slab under uniformly distributed load is one I would support with the results of a number of test runs of the computer program.

92. It is argued that Table 4 gives a distorted view of the relative efficiencies of various reinforcing systems. This is so, and the table was used solely to dramatize the effect of the direction of reinforcement on slab behaviour. The program, in fact, produces a reinforcement factor for each system which is equal to the steel area (or moment of resistance) at each of the 15 design points multiplied by the slab area appropriate to each point.

93. While the computer program treats the question of isotropy, reinforcement behaviour, etc. in a theoretical manner, I am glad to hear that model tests are being carried out at the Cement and Concrete Association to examine crack control characteristics of different reinforcing systems. I shall be very interested to know to what extent the practical behaviour of slabs agrees with the theoretical approach described in the Paper.

94. **Dr Kemp** makes the point that the treatment of reinforcement in the program, though dealing with working stresses, is in fact based on yield criteria. Wood's rules¹⁸ are based on such criteria and I have used these rules, and an extension to skew reinforcement proposed by Armer in the discussion on Wood's Paper, to determine the amount of reinforcement in each direction under individual loading combinations. As each loading combination is considered at a design point, the quantities in two directions are determined to give the optimum total for that combination. If one or other quantity is less than the maximum stored for that direction, it is increased to this maximum and a reduction is computed for the other quantity. In this manner, the total amount of reinforcement provided for each loading combination may not necessarily be optimum for that combination but will give an overall 'optimum' for all combinations. In any case, the selected reinforcement layout will have been tested for each loading case.

95. Since the Paper was submitted for publication, a number of additions have been made to the computer program, including the consideration of elastic supports. I agree with **Mr Fothergill** on the effect these have, particularly on reactions. Manual methods of analysis, such as the common one of applying the experimental results of Rüscher and Hergenröder³ have tended to ignore this point.

96. Minor principal stresses are obtained by subtracting two quantities which are frequently of similar magnitude and, consequently, minor stresses are subject to greater uncertainty than major principal stresses. In slabs of 30° skew or more, minor stresses are negative in certain areas of the slab and top reinforcement is required. Fig. 11 shows that negative minor principal stresses can sometimes be as high as major principal stresses, particularly at the obtuse-angled corners. The practice, therefore, of placing top reinforcement near the supports is a good one.

97. **Mr Fothergill** and other writers have been concerned with the question of isotropy. This question was discussed in the Paper at length and it is left to the individual designer to decide whether his bridge may be analysed using the isotropic assumption. A prestressed concrete slab wholly cast in situ may well be a suitable

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slab. As stated in the Paper, orthotropy may be introduced into the program relatively easily. This is done by superimposing on the slab stiffness that of imaginary beams running parallel to the free edges. This approach could also be used for slab and beam construction. However, the determination of the correct orthotropic parameters to use in a skew slab is not, as has been pointed out in the Paper, an easy problem and must await more experimental treatment. Hopefully, the Cement and Concrete Association's investigations will throw more light on this point.

98. When the program was initially written, it was considered that it would be economical to produce drawings also. The problems of reinforcement patterns have since complicated the issue and it will only be when certain patterns have become accepted design practice that one can progress to a completely automated design though the software be now available.

99. **Dr Jenkins and Mr Gangopadhyay** have commented on the assumption of isotropy, and I have already pointed out that there is no difficulty in applying an orthotropic analysis, and cost is certainly not an obstacle. On other matters concerning the analysis, a 9×9 mesh is used of parallelogrammic finite elements which are intrinsically more accurate than non-conforming triangular elements. The mesh size is, in fact, a variable in the program.

100. The method used in the Paper to design the reinforcement is in fact based on yield line theory and it would be interesting to know how the results quoted in Table 8 are derived.

101. **Mr Johnson** has a valid comment concerning HA knife edged loads. With the ease with which computer programs can simulate actual traffic wheel loads, is there a case to be made for doing without knife edged loads altogether?

102. The results of Table 2 are intended to demonstrate the wide discrepancies in results obtained from different methods of analysis for concentrated forces. The variables referred to by Mr Johnson are rarely quoted. Fortunately, the effect of concentrated loads on slabs of this sort is rarely significant.

103. The question of discrete bearings, raised by Mr Johnson, has been treated already. I would add here that, prior to the treatment of bearings by the computer program, manual corrections were made to the computer moments and shears in line with the experimental results of Mehmehl and Weise.⁹

104. In conclusion, I should like to thank all contributors. It is to be hoped that future investigations, both theoretical and experimental, will clarify more some of the points that have been most argued.

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