Some results of quay-wall model tests on earth pressures

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The Author is to be congratulated for a valuable Paper which significantly increases our understanding of the active earth pressure problem. I wish to discuss the stress distributions observed on the wall in tests in which there was no surcharge on the sand and in which the wall was allowed to yield by rotation about its base. The strains within the sand mass will obviously significantly influence the stress measured on the wall during the test and so it is proposed to consider first the distribution of strain caused by wall movement.

88. One of the simplest assumptions that can be made about the distribution of strains adjacent to the wall is that the strain is constant in magnitude everywhere within the Rankine active zone and zero outside the Rankine zone. Ladanyi,^{28, 29} makes this assumption, primarily from a consideration of experimental data,³⁰ and so is able to determine the strain within the Rankine zone as a function of the angle of wall rotation ψ . He then adopts the suggestion⁶ that the earth pressure theories developed for conditions when the peak value of ϕ is mobilized are equally valid at an earlier state of loading when the mobilized value of ϕ is ϕ_m . Thus when the mobilized value of ϕ is ϕ_m the active pressure coefficient K_a is equal to $\tan^2(45 - \phi_m/2)$. The mobilized value of ϕ_m at any stage of wall yield is then determined from the relationship between ϕ_m and shear strain observed in the triaxial apparatus.

89. The outcome of Ladanyi's work is a prediction of the variation of the earth pressure coefficient K with wall yield from the initial condition of no lateral strain ($K=K_0$) to the fully active condition ($K=K_a$). The predicted curves are shown in Fig. 17 and are qualitatively remarkably similar to the data obtained by Terzaghi¹¹ and Matteotti, except that the amount of wall rotation to mobilize the fully active condition is severely overestimated. Furthermore, the assumption of uniform strain throughout the Rankine zone implies that the wall stress distribution is always linear, as ϕ_m is necessarily the same throughout the vertical height of the Rankine zone. However, Fig. 10 shows that the centre of pressure moves down from its initial position $H/3$ from the bottom of the wall as rotation proceeds and then moves up again towards the initial position as the active pressure is fully mobilized. Fig. 9 demonstrates this effect more clearly, for the pressure is initially linear, becomes concave after some wall rotation and then is approximately linear when the active pressure is achieved.

90. The main purpose of this discussion is to attempt an explanation for the movement of the centre of pressure which is observed. Ladanyi assumes that the wall is smooth and hence that the major principal stress is everywhere vertical. The test data (Fig. 10) indicate, for the particular test conditions imposed by Matteotti, that the angle of wall friction δ achieves values which are close to the values of ϕ_m . Thus a more realistic assumption might be that the wall was perfectly rough. Adoption of this assumption has the consequence that the patterns of displacement within the sand mass are more complex, and hence that the strains vary within the mass.

91. It is convenient to use a field of slip lines as a framework for an analysis of the strains within the mass and, for the simple case when the wall is smooth, the

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Fig. 17. Calculated relationship between the coefficient of earth pressure K and the angle of wall rotation ψ for a retaining wall supporting loose or dense sand, after Ladanyi²⁹

Fig. 18. Slip-line field when the wall is smooth

Fig. 19. Slip-line field when the wall is rough

relevant slip-line field is shown as Fig. 18. The slip-line field is considerably more complex when the wall is rough and depends on the stress-strain properties of the soil. For convenience of discussion, a simple approximate slip-line field will be adopted that takes account of the wall roughness. The pattern of the chosen slip-line field (Fig. 19) is closely similar to that proposed^{31,32} for passive problems and consists of a zone (ACD) where both families of slip lines are straight, and a zone of radial shear (ABC) in which one family of slip lines are straight and radial from A, while the second family are logarithmic spirals with pole A.

92. The mathematical determination of the strain distribution associated with the chosen slip-line field parallels that reported earlier,³² and it is proposed only to consider certain key points in this discussion.

93. First of all it is necessary to state that the slip lines shown in Fig. 19 define those lines or planes across which discontinuities in velocity can occur, and are inclined at an angle $(45^\circ - \nu/2)$ to the major principal compressive strain increment direction. (The angle of dilation ν is defined³³ as:

$$\sin^{-1} \left(-\frac{\dot{\epsilon}_1 + \dot{\epsilon}_3}{\dot{\epsilon}_1 - \dot{\epsilon}_3} \right)$$

They do not, in general, coincide with the stress characteristics, i.e. those planes along which the ratio of shear to normal stress reaches its maximum value of $\tan \phi_m$ and which are inclined at $(45^\circ - \phi_m/2)$ to the major principal stress direction. Further discussion of the differences between slip lines and stress characteristics is given elsewhere.^{32,34}

94. Choice of the form of the slip-line field, together with assumptions:

- (a) that the soil particles immediately adjacent to the wall do not move relative to the wall (i.e. the wall is rough); and
- (b) that there is no displacement of soil particles below the slip line BCD extending up from the toe of the wall,

enable a determination to be made of the strains within the deforming region. Fig. 20 shows calculated strain contours for a wall rotation of $\psi = 0.001$. (The

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value of ν has been taken as 20° , as found³⁵ for passive earth pressure tests, but, so long as the choice of ν is reasonable, the calculated strains are insensitive to the value of ν .) It can be shown that the shear strain $\gamma (= \epsilon_1 - \epsilon_3)$ at a point P within ACB (Fig. 19) is given by:

$$\gamma = \frac{\psi H \sec \nu}{r} \exp(-\theta \tan \nu) \quad (6)$$

where r is the radius of P from A, θ defines the angle BAP and H is the wall height. The strain calculations for the entire deforming region may be summarized simply as:

- (a) the shear strain at the edge BC of the zone of radial shear and everywhere within ACD is given by $\gamma = \psi \sec \nu$, and
- (b) on any radial line in the zone of radial shear (ABC), γ is proportional to ψ/r .

95. One of the interesting features of the calculated strain distribution is that the strain is substantially higher near the top of the wall than near the bottom. Elements of sand near the top of the wall will therefore mobilize their peak value of ϕ earlier than the sand near the bottom and thus it is to be expected that sand near the top of the wall will reach its fully active condition before sand at the bottom. At some intermediate value of wall rotation the stresses at the top of the wall will be close to the active values, while stresses near the bottom will have changed little from their initial K_0 values. The stress distribution will then be concave and of the form shown in Fig. 9 for $\psi = 0.0002$.

96. The wall stress data corresponding to $\psi = 0.0008$ (Fig. 9) indicate that the active stresses are measured from the sand surface down to the bottom of the wall at this rotation. The predicted strain field for the zone of radial shear is such that the predicted strain adjacent to the bottom of the wall at $\psi = 0.0008$ is identical to the predicted strain at a point adjacent to the wall but at a depth of $0.25H$ when $\psi = 0.0002$. It is to be anticipated, therefore, that as the active stresses were mobilized at the bottom of the wall by $\psi = 0.0008$, they will also be mobilized from the surface down to a depth $0.25H$ at $\psi = 0.0002$. Fig. 9 has been redrawn as Fig. 21 and the linear stress distributions corresponding to the active and K_0 stresses are marked. It can be seen that at $\psi = 0.0002$ the wall stresses are at their active values from the surface down to a depth of between $0.2H$ and $0.35H$, in good agreement with the predictions from the strain field.

97. The progressive nature of the mobilization of the active state explains both the fall of the centre of pressure below its initial position at a depth $2/3H$, as well as its subsequent rise to the initial position, as was observed by Terzaghi and Matteotti. The exact form of the stress field consequent upon the proposed slip-line field is the subject of further research.

98. The strain calculation partly explains Ladanyi's overestimate of the wall yield necessary to mobilize the fully active condition, for it is predicted that intense shear strains occur near the top of the wall, even at small angles of wall rotation. However it must be admitted that the slip-line field does not remove all the discrepancy, for the calculations indicate that there are relatively low strains in the body of the soil mass. The discrepancy between predicted and observed rotations to mobilize the fully active condition is likely to be partly a result of the use of inappropriate test data in the analysis. It is notable that the triaxial data quoted by Matteotti show a peak at axial strains of 1% in extension, while the strain to peak in compression is 7%. Thus correct stress paths must be followed in tests which are to be used to predict displacements to failure,³⁶ for details of test procedure often have a significant effect on strain behaviour, even though the peak ϕ values may be the same. Of course, triaxial data are not strictly relevant to the plane strain problem of active failure, while appropriate tests should start from an initial K_0 condition and should be at a suitable stress level.

Fig. 20. Predicted contours of shear strain in per cent at $\psi=0.001$ for $\nu=20^\circ$

Fig. 21. Pressure distributions observed by Matteotti

99. A further possible reason for the discrepancy is that if discontinuities, or line ruptures, occur within the sand mass, the strain within the discontinuities is likely to be an order of magnitude higher than the strain in a zone which is straining uniformly. Thus, if there is a discontinuity, the wall stresses would reach the fully active condition much earlier than would be predicted for the zone rupture. It is important in this respect that both Terzaghi and Matteotti used walls which were hinged at points below the bottom of the wall. In Terzaghi's experiments the wall rotated about a point $0.34H$ below the bottom of the wall, while Matteotti rotated the wall about a point

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0.10H below the bottom. Thus both investigators caused relative translation between the toe of the retaining wall and the base of the sand container. This discontinuity of displacement of the boundaries of the sand mass might well be propagated into the sand to form a line rupture along which the strains were large and thus it is possible that the fully active condition would be achieved earlier than would have been the case if true rotation about the toe had occurred.

Dr J. E. Gray

The key remark in this Paper occurs in § 59, that is regarding the porosity of the sand which was very little different from the critical. May we look forward to results for sand at different porosity values, in particular up to its densest state?

G. Matteotti

I greatly appreciate Dr P. L. Bransby's contribution to the discussion.

102. He discusses the stress-strain path during the tests without surcharge, and deeply analyses the earth pressure problems showing he is familiar with this subject.

103. His remarks are appropriate and I agree that some discrepancies are to be attributed to the model, in particular to the hinge set below the bottom which leads to a little lateral translation between the wall and the model base.

104. Nevertheless, the result retains its validity from the practical point of view in the design of quay-walls. To my way of thinking the main remark concerns the high value of δ developed by the quay-wall roughness, which may explain the stability of some quay-walls having been erroneously calculated.

105. Dr J. E. Gray would like to know the test results with sands of various porosities.

106. I do not think that the Coulomb theory may be invalidated in its practical aspects; however deeper studies on this subject are beginning at the Institute of Maritime Constructions of the University of Padua.

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