Computer analysis of pipe networks

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The Authors have shown that even after at least five years of considerable activity in this field, there is still need for discussion of the best method of approach to the computer work involved in pipe network design. This discussion revolves around two important points: the type of machine and, if digital computation is considered, the method of analysis.

29. The use of an analogue, such as WANDA, depends on the convenience of its availability both in time and place. In time—because reservations must be made months in advance, and once set up, a network should be studied continuously for several weeks—the analogue has no memory, so that switching problems is a lengthy process. In place, because, owing to the lack of versatility which the Authors have noted, analogues are few and far between. However, within its own sphere, the analogue is more versatile and adaptable to complex network computers; and the analogue's directness is its most commendable feature.

30. The digital computer has the great advantage of versatility and wide availability. Many programs are available for pipe network analysis, though it may be necessary to adapt them to the machine most convenient to one's use. But if one's network contains some feature which the programmer has not built into the selected programme, considerable time and effort may be needed to write and perfect additional sub-routines.

31. The Authors are using a batch processing machine, and it is probably necessary to reserve time in periods of a few minutes every hour or two, so that results can be assessed and new data prepared between runs. A more efficient method is the use of a time-sharing computer, linked by telephone to a terminal in one's own office. Since the computer is 'on-line', the interaction between engineer and machine approaches the directness of the analogue.

32. The Authors comment in § 2 on the possibility of automatic design, using digital computers. Yet the Paper goes on to describe fairly well known methods of analysis only. It is in the field of automatic design that fresh thought is needed, and I wonder if the Authors are familiar with the papers of Tong *et al.*¹⁰ and Raman.¹¹ They have shown that for any given demand loading, network layout, and pressure distribution, a uniquely economic set of pipe resistances and hence, diameters, may be determined without trial and error. This process would, of course, still require trials with several different network configurations, demand loadings, and possibly source availabilities. A contributor to this aspect of the argument was Abamov, at the IWSA Congress, Barcelona, 1966.¹²

33. The 'nodal' and 'loop' method of Hardy Cross both have their strong protagonists. I prefer the nodal method, for the reasons given by Dillingham.⁸ The Authors have shown that the specification of loops is not without its pitfalls; and in addition it leads to more complication in the insertion or removal of a pipe, and the need for dummy pipes if it is desired to include one which is not part of a loop. The problem of incorporating pumps is described both by Dillingham and by McCormick & Bellamy.¹³

34. The loop method makes more effective use of storage in the computer, so that larger networks can be tackled on the same machine. However, it is not necessary or even desirable to insert every pipe in the network as a separate entity: the very large quantity of data and results tends to obscure the wood with the trees. It is

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better to rationalise the system down to 100–200 pipes by neglecting relatively small diameters and inserting equivalents to series and parallel pipes. Where possible, a large network of interconnected zones should be studied a zone at a time. I understand that ICL have written a program to cater for some 4000 pipes in a gas network. In this case, it must have needed another programme to find what was significant in the results!

35. The Authors have dismissed the 'direct solution' on grounds of excessive core storage requirements. This method was first explored by Martin & Peters¹⁴ and they give an example of an interesting method of reducing the core storage requirements. The direct solution requires no preliminary estimates of pressure or flow to be made by the engineer, and its convergence is guaranteed.

36. My firm commissioned GEIS Ltd to write a program suited to their time-sharing system. It was to cater for 150 pipes, 15 pumps, 15 pressure reducing valves (which double as non-return valves if required), and upwards of 100 nodes with draw-offs or reservoirs. It was to start without preliminary estimates, and allow for pro rata variation of some or all demand draw-offs. A subsidiary version was also required which could accommodate 300 pipes when no pumps or control valves were present.

37. Unfortunately, the details of the method of analysis have been kept a commercial secret. The general principle was to combine the direct solution with the Hardy Cross nodal method, the latter being used to balance small sections of the network, followed by overall balance by the Newton direct method. Methods for improving convergence are similar to those described by the Authors in § 10.

38. The resulting programs have been applied to the highly complex water distribution system in Kingston, Jamaica, as well as several simpler cases. Some ill-conditioned networks have been encountered, but these were solved without any trouble apart from the need for a greater number of iterations. Convergence has not always occurred when pumps and/or pressure reducing valves were involved, but the time-sharing system allows the progress of the analysis to be observed, and rapid amendments to the data to be made. Non-convergence has always been indicative of an impossible situation.

39. In conclusion, I would like to ask the Authors if they have produced a program which is an improvement on currently available ICL network programs.

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In the introduction to their Paper, the Authors comment that the analogue computer has the advantage of greater convenience when setting up trial networks and modifying them. With the introduction of on-line systems and desk terminals, the digital computer should also have this advantage. It should be possible to make alterations to data quite easily, particularly simple ones, such as increasing draw-off or opening and closing valves, and obtain the results of the analysis immediately.

41. We agree with the Authors that the speed of convergence is relatively slow in the ill-conditioned network of Fig. 3, compared with the well-conditioned network; however, it is not difficult to obtain a solution to the same degree of accuracy for both networks. The terms 'well conditioned' and 'ill conditioned' are relative terms indicating that the rate of convergence to the final solution is slower in one than in the other. Clearly, it is possible to have networks which are worse conditioned and possibly divergent, if the initial guesses are chosen badly.

42. We have analysed the two examples of Fig. 3, using the nodal method, and feel that they illustrate a number of interesting points. Fig. 8 indicates the node numbering and the equivalent external flows in gal/min used in our computer analysis. The k values have been calculated using the formula¹⁵

$$k = \frac{10 \cdot 43L}{C^n D^{4 \cdot 87}}$$

Fig. 8. Comparison of k values for the examples given in Fig. 3Table 1(a). Variation of ΔQ_n with ϵH for the well-conditioned network

Node number	$\epsilon H = 0.1$ ft		$\epsilon H = 0.01$ ft		$\epsilon H = 0.001$ ft	
	$ \Delta Q_n $ gal/min	$ \Delta H_n $ ft	$ \Delta Q_n $ gal/min	$ \Delta H_n $ ft	$ \Delta Q_n $ gal/min	$ \Delta H_n $ ft
1	22.30	0.09	0.01	0.000	0.17	0.0007
2	25.73	0.09	2.14	0.007	0.20	0.0007
3	39.65	0.09	3.94	0.007	0.31	0.0007
4	3.02	0.01	0.05	0.001	0.00	0.0000
5	0.05	0.00	1.67	0.009	0.01	0.0000
6	3.03	0.05	0.29	0.004	0.02	0.0003
7	0.05	0.00	0.66	0.006	0.07	0.0006
8	4.70	0.08	0.01	0.000	0.02	0.0003
9	2.81	0.09	0.00	0.000	0.20	0.0006
10	6.58	0.08	0.10	0.001	0.01	0.0002

Table 1(b). Variation of ΔQ_n with ϵH for the ill-conditioned network

Node number	$\epsilon H = 0.1$ ft		$\epsilon H = 0.01$ ft		$\epsilon H = 0.001$ ft	
	$ \Delta Q_n $ gal/min	$ \Delta H_n $ ft	$ \Delta Q_n $ gal/min	$ \Delta H_n $ ft	$ \Delta Q_n $ gal/min	$ \Delta H_n $ ft
1	14.03	0.07	1.35	0.006	0.13	0.0007
2	26.54	0.08	2.56	0.007	0.28	0.0008
3	17.35	0.10	1.79	0.009	0.18	0.0010
4	11.17	0.04	1.23	0.005	0.14	0.0005
5	20.03	0.05	2.08	0.005	0.20	0.0005
6	17.08	0.07	1.82	0.008	0.18	0.0008
7	8.03	0.01	2.61	0.002	0.09	0.0001
8	109.03	0.10	11.15	0.009	1.09	0.0009
9	0.08	0.00	0.00	0.000	0.00	0.0000
10	16.11	0.09	0.17	0.001	0.17	0.0010

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Table 2. Effect of initial guess for the pressure at the nodes on the speed of convergence (for example of Fig. 3)

Maximum allowable residual head at each node ϵH : ft	Number of relaxation cycles required for the solution of:			
	well-conditioned example		ill-conditioned example	
	Initial guess A	Initial guess B	Initial guess A	Initial guess C
0.1	17	6	53	2
0.01	23	10	72	11
0.001	27	15	90	31
0.0001	31	19	109	49

Initial guess A is essentially arbitrary.

Initial guess B is a 'good' guess for the well-conditioned case.

Initial guess C is a 'good' guess for the ill-conditioned case.

Fig. 9. Network example

where D is the pipe dia. in inches and L the length in feet, n has been assumed to be equal to 2 and C to be 100. For the ill-conditioned case the k values are smaller (due to the decrease in dia. of the pipe) than for the well-conditioned case for four of the five pipes which are different, and so the head losses along these pipelines are expected to be smaller. Therefore, to achieve an acceptable solution, the maximum allowable residual head ϵH at any node (stated in the input data) must be smaller in the ill-conditioned case. This is illustrated in Table 1 where the values of ΔQ_n for different values of ϵH are given for both cases; the initial pressure guesses are the same for each case. Clearly, the value of ϵH chosen for the ill-conditioned example must be approximately one third the corresponding value for the well-conditioned case. The accuracy criterion could be placed upon the $|\Delta Q_n|$ rather than the $|\Delta H_n|$ but this may lead to anomalies in other examples.

43. In order to reduce the time needed to obtain a solution and possibly to improve the rate of convergence it is very important to have a good initial guess for the pressure at the nodes, or the head loss along the pipes. This, of course, is not easy, but we have found that a simple but quite effective way is to estimate the head loss along each pipe (from the usual equation $h=kQ^n$) using the calculated value of k together with an estimated value of Q based on the given flow data and values of k . Table 2 gives the total number of relaxation cycles required to achieve a solution, for various values of ϵH , with arbitrary initial pressure guesses (case A) and the corresponding figures when a 'good' initial guess (cases B and C) are used. The figures are presented for both the well-conditioned and the ill-conditioned example of Fig. 3. The arbitrary initial guess is the same as was used for the results which were used to formulate Table 1. As perhaps expected, the improvement (due to changing the initial guesses) to the speed of convergence is restricted to the early part of the iteration process, i.e. the number of cycles needed to improve the result from $\epsilon H=0.01$ ft to $\epsilon H=0.0001$ ft is substantially unchanged.

44. We have given considerable attention to the nodal method and have developed three Algol programs which differ in the way the relaxation takes place. Briefly, the methods are:

- (i) Straightforward node-by-node relaxation with each node being balanced before moving to the next node;
- (ii) Node-by-node relaxation but only a stated number of relaxations at each node before moving to the next node;
- (iii) Simultaneous relaxation of all the nodes.

45. These three programs enable comparisons to be made of the speed of convergence with each program. The input data for all three programs is the same, so that the data need only be prepared once, even if all three programs are eventually used to obtain a solution. This work is continuing, but, by way of illustration the simple network shown in Fig. 9 has been analysed using method (i) and also with method (iii) with the value of C taken as 100 for all pipes. It was found that to obtain a solution with the same initial guesses and to the same degree of accuracy, method (iii) took 9% less time (the total head corrections were 4% less) than method (i). However, method (iii) is not faster for all networks (and indeed it may be slower) so it is not possible to make any general conclusion at this stage.

46. We agree in general with the conclusions of the Authors, that the loop method of analysis is the most suitable for use on digital computers and provides a rapid convergence more often than the nodal method; however, this does not make the loop method an ideal one. In some cases it is preferable to use the nodal method rather than the loop method because of the given data or configuration of the system, indeed, the nodal method can provide a quicker solution if the system incorporates pumps and storage tanks or reservoirs. Bellamy¹⁶ and McCormick¹⁷ consider the nodal method to be more versatile than the loop method and have therefore used it as the basis of their pipe network programs.

47. The Authors' technique of 'open' loops when there is more than one node at which the head is specified (i.e., $M>1$), adds extra input data preparation and storage as well as increasing the time of computation. In our opinion if $M>1$ then it may well be advantageous to use the nodal method.

48. When using the loop method of analysis we have found that the speed of convergence for a solution is improved by using simultaneous relaxation of all the loops included in the network instead of successive loop-by-loop relaxation. For example, the time required to analyse a network of 28 pipes and 9 loops was 36% less using the simultaneous relaxation program compared with the loop-by-loop relaxation program to the same degree of accuracy and with the same initial guesses. Another advantage of the simultaneous relaxation technique is the straightforward preparation of input data without the complication of dividing the network into principal and subsidiary loops suggested by the Authors.

Fig. 10. Variation of $H_{n/h}$ with $Q_{n/q}$ for a single pipe $n=2, 1.85$ and 2.423

49. The quadratic form for the head (flow) correction in the nodal (loop) method is an important improvement and appears to improve the convergence quite markedly. It would be useful if the Authors could give some indication of the time saved because since the quadratic form takes longer to evaluate than the linear one, not all the improvement in convergence is a real saving in time.

50. We have some reservations about putting n equal to 2 all the time. For many practical flow conditions the value of n may be sufficiently different than 2 to warrant its consideration at least for the last few cycles of relaxation. In this connexion it is interesting to consider the effect of flow in corrugated pipes where Allen¹⁸ and others have observed that n can be as high as 2.4. Fig. 10 compares the linear correction for the nodal method for the cases of $n=2.423$ and $n=1.85$ with the quadratic correction when $n=2$. We feel it may be beneficial to allow the value of n to be varied. If the quadratic form of correction is used then a procedure could be used to solve equation (7) or (12) and n could be a formal parameter in the procedure heading.

Mr B. Andrews, Applied Computer Systems, Ltd

I have worked on computer programs for the analysis of very large low pressure gas distribution networks for the last four years, and on both the basic methods of analysis discussed in the Paper. My first experience was with a program using a nodal solution for pressures. Most nodal solutions use a node-by-node relaxation, but this particular program attempted a simultaneous nodal balance by forming the complete set of non-linear simultaneous nodal pressure equations. In general terms, the pressures P at the nodes can be related to the outflows D by the matrix equation:

$$[K][P] = [D]$$

P and D are vectors of length N , where N is the number of nodes, and the matrix of coefficients K , contains elements of the form $1/k|Q|^{n-1}$ with the flow Q in a pipe being related to the pressure drop Δp in the pipe by the expression:

$$\Delta p = kQ^n$$

The program worked as follows:

- (a) assume a flow pattern;
- (b) compute $[K]$;
- (c) solve for $[P]$;
- (d) compute the new flow pattern from $[P]$;
- (e) repeat from step (b) if the maximum pressure change is too high;
- (f) print solution.

52. The equations were solved iteratively by a method similar to Gauss-Seidel and four or five re-balances with a new ' K ' matrix were required. The program was abandoned because the method produced ill-conditioned equations and only networks with less than 100 nodes could be guaranteed to converge satisfactorily.

53. The second program I worked on used a loop analysis. Here again, the usual method is a loop by loop relaxation, but this particular program used a simultaneous loop balance. The program is available through ICSL and is used by a number of Gas Boards in Great Britain. I have not seen any mention of a simultaneous loop balance approach, so some figures comparing the method with a loop by loop relaxation might be of interest. I would point out, however, that a loop balance of any type is likely to be at least ten times faster than a nodal solution. Briefly, the loop analysis methods are described below.

Simple loop by loop balance

54. The method involves the sequential correction of the flows in the independent loops in the network. For each loop, taken in turn, the flow correction to balance the loop is:

$$q = \sum k|Q|^{n-1}Q/n \sum k|Q|^{n-1}$$

where the summation extends to all the pipes in the loop. The process is repeated until the flow corrections in all the loops are small.

Simultaneous loop balance

55. The method involves the solution of the matrix equation:

$$[F][q] = [p]$$

where q is a vector of the flow corrections to the loops and p is a vector of the out-of-balance pressure drops in the loops. The F matrix contains elements of the form $k|Q|^{n-1}$. The program works as follows:

- (i) read the data;
- (ii) compute the loop information;
- (iii) assign a flow distribution to produce nodal flow continuity;
- (iv) compute F ;
- (v) solve for q ;
- (vi) apply flow corrections;
- (vii) to step (iv) if flow corrections are too large;
- (viii) print solution.

56. Up to eight re-balances with a revised F matrix are required on very large problems with more than 4000 pipes and three or four with networks having less than 1000 pipes. The equations are solved by accelerated Gauss-Seidel with a maximum of 72 iterations, and a minimum of 30 iterations.

57. The computer time associated with the two methods is very interesting. The loop by loop balance requires, for each pass through the system, the calculation for each loop of:

$$q = \sum k|Q|^{n-1}Q/n \sum k|Q|^{n-1}$$

58. The correction q is then made to each pipe in the loop. The method is, therefore, two pass. One to calculate the correction, and the second to apply it. The number of operations per loop depends on the average number of pipes per loop.

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In contrast, the simultaneous balance requires the assembly of the F matrix, which takes about the same time as four simple passes through the loops, and the row by row estimation of the corrections q . q is estimated by calculations of the form:

$$pI - \sum_{J=1}^{J=\text{Loops}} F_{I,J} qJ$$

where I refers to the I th row of p and F , and J the J th column of F and J th row of q . The number of operations per loop depends on the average number of elements per row in the F matrix. The time to make one pass through the equations is about one quarter that for one pass on a loop-by-loop relaxation. This is because there are, on average, twice as many pipes per loop as there are elements per row and the simple balance is two pass. For example, a network of 2000 pipes would require up to six re-calculations of F with 72 iterations for the first three solutions and 30 iterations for the remainder. The equivalent number of simple balances would be:

$$6 \times 4 + (3 \times 72 + 3 \times 30)/4 = 101$$

59. In fact, this amount of calculation would produce a very accurate solution which would only be equalled by at least 200 simple balances. I have found that the most economic solutions are obtained by choosing loops as small as possible. Small loops are more independent than large loops, and so produce fewer elements in the F matrix. Sparse matrices tend to be well conditioned as well as requiring less computation per iteration. I have developed an algorithm which produces a very good set of loops based on this criterion and requires a small amount of computation.

60. There are techniques for speeding up a loop analysis which can be applied to both methods. One method is to avoid correcting 'balanced' loops. This is indicated by small values in the p vector, and can be confirmed by monitoring the progress of the q vector. A second method is to use an accelerating factor. This is done by using accelerated Gauss-Seidel in the simultaneous approach, but can be very dangerous with a simple balance. An accelerating factor less than the optimum will always improve Gauss-Seidel, but a factor greater than the optimum causes instability. Methods of estimating acceleration factors for Gauss-Seidel are well known, but I would think it would be very dangerous to 'accelerate' a simple balance on a large problem.

Mr J. Cowan, Senior Lecturer in Civil Engineering, Heriot-Watt University

The main emphasis appeared to be directed towards efficiency and effectiveness of method, and it was therefore unfortunate that the performances were described in terms of number of iterations; for the time taken per iteration will vary from one method to another, and, although the error will be imperceptible when the only change is the introduction of a simple over-relaxation parameter, in other cases it may be of more importance. The use of the quadratic in the nodal method is presumably accompanied by a noticeable increase in the period of iteration.

62. I was disappointed to find that a range of performances had not been quoted, since the rather unpredictable nature of certain methods of solution must give cause for concern. Many research workers have given examples of inconsistent convergence behaviour when a program for network analysis has been applied to a number of problems. The prospective client for such a program is only interested in the *individual* performance of any given method when applied to his own particular problem, and is not so concerned with the overall, average or optimum behaviour of that method over a large sample of problems. As we are not yet able to assess methods of solution in terms of overall reliability, I would welcome a description of the accuracy of any performance correlation made between any two methods. It is equally important to give an indication of any variation in the pattern of relative performance with increasing network size; instances could be quoted of methods which are superior on small and medium sized networks, but inferior on large problems.

63. The Paper appears to suggest that the use of a simplified exponential law with $n=2$ entails negligible inaccuracy and has the advantage of improving the convergence rate. It would be of interest to see detailed figures to support this statement. Improvement of convergence, within limits, must come second to accuracy of solution, and there are certainly examples on record in which the use of a more accurate flow formula has been held to yield a significantly different solution. It is open to question if a highly accurate value for f is required for minor pipes where flow is in the transition zone; for this reason the Colebrook-White formula may well be set aside in favour of an exponential law. The value of this exponent then requires determination.

64. Finally, the use of over-relaxation in its many forms has been justified by a record of successes, rather than by a formal proof. Having shared the Authors' experience that a larger value of parameter is required for optimum convergence in problems of increased complexity, I suspect that further improvement may be obtained by varying the parameter value in different loops or nodes. This method has already proved successful in dealing with biharmonic equations arising from stress analysis.

Mr G. G. Boyne, Department of Civil Engineering, Heriot-Watt University

The extrapolation process is based on the assumption of linear dependence of errors and changes in head. An alternative extrapolation procedure which has been successfully used when dealing with gas networks makes use of the fact that the successive head corrections form a convergent geometric series with an almost constant convergence ratio r once the process has settled. It was found that r has almost the same value for each node on the same iteration and, from iteration to iteration, r varied little. This property of the method makes it possible to extrapolate to a solution with one large step once r has assumed a reasonably constant value. It was also found that the successive error terms ΔQ_n exhibited the same convergence characteristics as the successive head corrections ΔH_n , the convergence ratio being almost the same. Hence, in estimating the value of r for the system either of the following expressions can be used:

$$r_x = \frac{\sum |\Delta H_n|_x}{\sum |\Delta H_n|_{x-1}} \quad \text{or} \quad r_x = \frac{\sum |\Delta Q_n|_x}{\sum |\Delta Q_n|_{x-1}}$$

where x =current iteration number. This r value is then an average value for the system. The procedure is then to iterate until the value of r_x has assumed a reasonably constant value. The geometric series formed by the successive head corrections is then summed to infinity to give a head correction which when applied should theoretically give zero error. This correction is $\Delta H_{n,x+1} = \Delta H_{n,x}/1-r$ and is applied to every node. Since the assumption of a constant r value is not strictly true the new head values do not constitute the solution. However, the application of several normal correction cycles is usually sufficient to assure the restoration of the geometric convergence from which it is possible to apply another extrapolation step. A typical network having 48 nodes and 56 pipes requiring 1200 iterations with no convergence aids required only 278 iterations when treated by this method.

66. The Paper suggests that the successive over-relaxation technique is of limited use owing to the difficulties in choosing a suitable value for a particular problem. However, if a method could be developed which commences with a successive over-relaxation factor of 1.0 and gradually amends this value on the basis of the convergence rate of previous iterations, this problem could be overcome. Such a method should be more effective than ordinary successive over-relaxation with the optimum factor since it can account for the variation in the optimum factor throughout the iteration cycle brought about by the non-linear nature of the network equations. I developed a method along these lines which used approximations to the maximum eigenvalues of the network equations to estimate the convergence rate of the process

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for the current successive over-relaxation factor and to estimate an improved successive over-relaxation factor giving an improved convergence rate. The method proved very successful for several small networks but was unreliable for some larger networks. The method, however, deserves to be studied further to see if a reliable method for all networks can be obtained. An indication of its performance on a small network of 19 nodes and 32 pipes is given by the fact that 175 iterations were required with no convergence aids, 60 iterations using the extrapolative method outlined, and 39 iterations using the method of successively modifying the successive over-relaxation factor.

67. A further improvement in convergence rate can be obtained by combining the two methods of successive over-relaxation and extrapolation. If this is done the value selected for the successive over-relaxation factor must be below that value for which instability occurs. Application of this combined method to the 48 node 56 pipe network produced the following results:

Factor . . .	1.0	1.025	1.050	1.075	1.10	1.11	1.12
Iterations . .	278	259	258	243	235	243	no convergence

68. These comments referred only to the nodal method of analysis although the methods discussed are equally applicable to the non-linear loop equations arising from the loop method of analysis.

Mr W. M. Morrison, International Computers Ltd

We also, as in § 14, introduce an 'open' loop for each supply node after the first and of course Euler's Theorem gives the number of *independent* loops. There are a large number of different sets of independent loops and various ways of determining them. The convergence of the 1900 algorithm is strong enough that finding a suitable set for convergence reasons has not been necessary. Indeed the Mk 1 program took a set which merely depended on the order the pipe connexions were fed into the program. In the Mk 2 and Mk 3 we had to abandon the loop finding procedure used in the Mk 1, since it depended on the pipe/node incidence matrix and even cleverly packed, this required too much core store. Nevertheless we gave considerable thought to finding an independent set of loops such that the loop-loop matrix had maximum sparsity, i.e. the loops should be selected in such a way that the number of loops 'connected' to any given loop is kept small. As a rule we find it possible to obtain very good loops in this sense very quickly, i.e. hundreds per minute on a slow computer, automatically. In our view it is quite unacceptable to ask an engineer user to select and specify loops, even though a number of programs demand this. In a large network the manual procedure is prone to various errors and in any case is a simple computation from the node connexion list. Intuitively, loops selected for sparsity as above, should also avoid any convergence problems.

70. With reference to § 17, we prefer to compute an equivalent for parallel pipes, rather than to introduce another loop.

71. Routines have been written to perform Gaussian Elimination mentioned in § 18 with sparse storage techniques to take advantage of the very many zero elements. In certain circumstances core storage can be less than for Gauss-Seidel methods and direct methods are quicker and less troublesome. At present the ICL Mk 3 program actually uses an accelerated SOR (modified Gauss-Seidel) algorithm to solve its equations, since the sparse Gaussian Elimination procedure was developed only last year. The Mk 3 program would be classified as a direct solution method despite its use of SOR for solving equations.

72. One difference between our Mk 2 and the Mk 3 is that the 'open' loops are conceptually different. Again, we were trying to reduce core store. The Mk 2 finds a loop from a 'reference' supply point, i.e. fixed pressure point, to each of the other supply points. The Mk 3 algorithm does *not* put any reference supply node into *all* 'open' loops. Instead it seeks to find a nearer supply point to reduce the number of

elements in the loop-loop matrix for open loops. There are also 'independence' considerations here of course.

73. The Authors appear to be saying, in § 20, that this type of network has less loops than nodes and hence the loop matrix is of smaller order than the node matrix. This is offset to some extent by mesh analysis always being more complicated than nodal analysis in my experience. In this particular case the convergence problem seems to suggest that a mesh analysis approach is likely to remain better.

74. In my view this problem is quite suitable for interactive CAD provided the network is not too large. I am sure Water Boards would like to have an interrogation typewriter on a program with many more facilities and handling say 200 nodes, entirely in core. The only snag is of course, limited resources. Similarly there is further scope for advancement in trying to optimize supply point conditions, within the pressure and flow limitations of these nodes. Finally, there is network design and advances are being made in this field. Our pipe sizing program is restricted to essentially simple tree networks, but Engineering Computations have produced a program.¹⁹ We also have a simple pipe-line design algorithm devised for an irrigation system design problem.

75. We expect to incorporate other network elements, e.g., pumps, valves, etc., in an enlarged pipe network analysis program in due course, after we have determined the best means of speeding up the calculation. At present the Mk 3 program is somewhat slow but its solution is highly precise. We have yet to see a program capable of similar accuracy. Essentially the algorithm focuses attention on an independent set of loop-closing pipes or links. If these links were removed the network would be a tree structure. Every pipe flow in the network can be expressed in terms of the flow in these link pipes and the algorithm linearizes the pressure/flow relation and concentrates on finding the link flows. Once found the pressures are easily computed. An excellent balanced initial flow distribution is found by theoretically removing the links. The link flows are of course also loop flows.

Mr D. G. P. Iszatt, Ministry of Public Building and Works

Whilst this Paper admirably reviews the methods of analysis used in most existing programs, the Authors' preoccupation with increasing the speed of convergence as a prerequisite to the development of an automatic design may not be justified.

77. I consider that there is a much greater need to establish the logical criteria upon which the design process will be based. My limited experience in writing 'design' programs suggests that considerable time and effort will be required to determine the bases on which engineers in practice decide between various design materials, pipe diameters, etc. The translation of engineering judgement into logical statements will prove taxing, at the least, and where judgement borders on prejudice, impossible.

78. The main justification for the emphasis to be placed upon speed of convergence appears to be the assertion that it will be necessary to select the best network from a very large number of feasible networks. However, even in the unusual circumstance of having a new system to design the engineer rarely finds there are more than a handful of feasible networks, this is due to topography location of sources and demands, the need for wayleaves, site access for plant and materials, etc. When an existing network is to be extended or modified then comparatively few alternatives are available.

79. Whether the program automatically determines the feasible solutions or merely selects the most attractive from a range of solutions presented to it, the engineer will be faced with preparation of enormous quantities of data, always an error-prone process. A complete description of the site, both terrain and subsoil data will be required together with full details of available materials, construction costs, and so on.

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80. At the present state of computer aided design I consider that it is more practical for the engineer to submit a limited number of feasible networks to the computer for analysis and to compare the results on the basis of cost, speed and ease of construction or other appropriate criterion using manual methods. Many existing and commercially available programs permit the analysis of multiple demand cases in a single run and some permit the geometry of the network to be amended. The use of a remote access terminal or computer time sharing system permits numerous load cases and geometries to be analysed in rapid succession.

81. The foregoing is not intended to detract from the Authors' efforts since there is a very real need in all branches of civil engineering for 'design' programs to supersede the existing analysis programs.

Mr R. J. Maher, Research and Development Officer, The Australian Gas Light Company

Since the publication of the University of Illinois Bulletin 286, in 1936,²⁰ my firm has had continuous experience with the quantitative analysis and design of fluid-carrying networks, particularly for the distribution of gas.

83. Initially, the analyses were done by manual and desk calculator algorithms. This experience was most impressive with regard to the development of a 'cost effectiveness' outlook. I can recall an early and ambitious analysis in which 14 man-weeks were absorbed in executing the iterative calculations.

84. It was established, soon after starting, that the mesh analysis method was far more economic than the nodal method. Further, it was noted that convergence was very much faster on some analyses than was usual. This was investigated; the incentive being possibly reduced labour for calculation if the bases for this more rapid convergence could be defined.

85. It was determined that loops which had as common elements pipes of relatively high resistance were slower in convergence than those where the lowest resistance pipes were the common, or coupling pipes. Loops were defined by following the lowest resistance pipes throughout the network and using them as loop coupling pipes as far as possible. High resistance pipes were used only once as non-coupling pipes. In one case the simple redefinition of loops on this basis reduced the calculation time to approximately 20% of the time used in the original analysis where loops were defined exactly as seen on the system maps.

86. The advent of the IBM 626 punched card calculator permitted the mechanization of the calculations and a tremendous advance in the economics of network analysis and design. An additional circumscribing loop was defined in analyses done in the machine to ensure all pipes appeared in two loops. The algebraic sum of all flows in all loops remained zero as an arithmetic check. Having reduced the calculations to hours and minutes instead of months and weeks, it was clear that the major costs in analysing and designing networks was in defining and setting up the problem, and in analysing and acting on the results. The considerable cost advantages of the mesh analysis method were retained, and the problem of automatic definition of a 'best' set of loops was tackled.

87. The method outlined by Travers⁴ has been particularly effective and does not suffer any problems. From a cost effectiveness point of view it is many thousands of times better than any manual method.

88. I am surprised that the Authors express doubts and state preferences for a human execution of this straightforward piece of logic. Travers states that a network of 660 pipes required only 57 seconds to determine a near optimal set of loops and assign the initial flows. In more recent times I have noticed 125 pipes assigned to a near optimal set of loops in not more than one second, and 960 pipes similarly assigned in 45 seconds. Man cannot compete with this on a time or cost basis. The only way in which these loops are nonoptimal is that they are not based on the apparent resistances of the pipes under their true flow conditions. Travers has indicated that the

optimal set of loops have the lowest sum for the loop resistances. This conclusion has not been found inaccurate in any actual studies.

89. The linear form of the correcting flow has been accepted as satisfactory and accelerating factors have been tested and dropped. In practice it was found that a full correction on every loop on every iteration was not efficient. The loops which are near balance are left, while effort is put into those that are poorly balanced. After several partial iterations a complete iteration is made and attention again directed to those most out of balance. By keeping the algorithm simple and compact, it is found that more storage is available for larger problems and greater speed in reaching convergence is attained. The matter of having multiple nodes at specified pressure has been handled by superimposing a tree of auxiliary pipes which connects all head specified nodes. This introduces additional loops. The auxiliary pipes have constant resistances. All node flows are summed to one node flow at the root of this superimposed tree.

90. In large practical networks, such as that operated by The Australian Gas Light Company, there are approximately 50 000 pipes, and the majority have loads spaced throughout their length. Analyses are required regularly with 1000 pipes involved. The problem of naming and describing such systems manually is slow and tedious so, as in the case of the loop forming need, this matter has been handed over to the computer.

91. If a customer, a pipe, or a node is specified, and the extent of the system to be analysed is stated in terms of the number of pipes involved or within the limits of specified nodes, the computer will build up from a magnetic file a description of the pipes, their loads, and their connexions, until the specified extent of the analysis is reached. These data are then used by the computer to form the 'best' set of loops and the analysis proceeds.

92. On completion of the analysis the computer compares the results against specified limits of pressure and determines whether modifications to the network are required. If modifications are required a number of recommendations are printed as output for consideration. It is important to realize that it is possible to vary the resistance of pipes as well as to vary flows. Current work is directed toward completing the automatic optimization of the cost of network modifications.

93. Another important matter is the adoption of geographical coordinates for node designators to permit the automatic plotting of the network map and the related flow and pressure data in graphical form for more rapid assimilation. At no time have I seen a benefit in presenting network analysis to a computer in matrix form. The inevitable consequence is the sparsity of the matrices and the poor cost effectiveness achieved, even with special programming to reduce the sparsity. By storing only non-zero data in the form of compact lists or strings these difficulties are avoided. The sparsity of the matrix involved illustrates the relatively small coupling which occurs between non-adjacent loops, and this is one of the factors which are important in the stability and convergence speed in a reasonably defined set of loops. With a very powerful set of algorithms operating under a specially written control program, which in turn operates under the operating system in an IBM 360/40 computer with 64 000 bytes of storage, it is routine to automatically handle analyses with 1000 pipes involved.

94. A general description of much of the above work was presented by the Institute of Gas Engineers.²¹

Messrs Barlow & Markland

We are pleased at the range of comments made by contributors to the discussion and for the many helpful observations.

96. **Mr Webber** has usefully reminded us of the analogue computer's directness, but we would agree with **Mr Al-Nassri** and **Dr Halliwell** that improvement in methods of communication with digital machines is rapidly bringing them to a point where

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they will be equally direct. For instance, Fig. 11 shows a small section of a digital solution of a network, as drawn using the graph plotter attached to the machine; this form of presentation is clearly more effective than tabulated results.

97. Many contributors have discussed the relative merits of nodal and loop methods. The deciding factor which, in our view, makes nodal analysis less attractive, is that of ill-conditioning; in our somewhat limited experience the loop method has not yet given this trouble, while the nodal method has clearly been found to do so by a number of writers and contributors to the discussion. Messrs Al-Nassri and Halliwell rightly point out that the term 'ill-conditioned' is a relative one, and we are pleased to see that they obtained a satisfactory solution to the problem which had troubled us. Our experience with that particular problem, using the Colebrook-White formula, was that an acceptable flow closure error of 0.00134 was reached after 55 iterations when the pipe diameters gave a well-conditioned network. However, for the pipe diameters which produced an ill-conditioned problem, the closure error was still over 0.633 after 150 iterations, and was reducing very slowly.

Fig. 11. Section of network showing node numbers and elevations, pipe diameters and flow rates. (Traced from computer output)

98. Mr Webber's experience with networks on which pumps and pressure-reducing valves have led to ill-conditioning is rather similar to our own. It seems to be particularly difficult to deal infallibly with networks in which certain regions are supplied only by a number of pressure-reducing valves acting in parallel.

99. It appears to be extremely difficult to make meaningful comparison between different methods on the matter of speed. The number of cycles required to reach an acceptable solution, by any of the methods discussed, depends on the set accuracy limits. As mentioned in the discussion, the method which best suits one problem does not necessarily best suit the next. Moreover, the storage capacity required by various methods must also be considered, as well as speed. We therefore very much welcome Mr Andrews's description of the simultaneous loop balance method and his assessment of the number of operations it requires in comparison with the loop-by-loop method. However, we do not understand why there are about twice as many pipes/loop as elements per row of the F matrix, as for a loop in the interior of any network there will presumably be $(p + 1)$ elements in the row corresponding to a loop with p pipes. The frequency of revisions to the F matrix is an important factor in determining the total number of operations.

100. We have found now that the speed of the loop-by-loop correction method

is much improved by use of *more* than the required Euler number of loops as given by the equation (10). The addition of a few loops, containing mains with large diameters and covering as much as possible of the area of the network, reduces the required number of passes very considerably. A further advantage of the loop-by-loop method over the direct method of solution, is the ease of dealing with pumps and pressure-reducing valves, which require conditional statements in the program to deal with backing up and other problems.

101. **Mr Cowan** rightly draws attention to the interest of the client, who is concerned essentially with whether his particular problem can be solved by the method available to him. We know of no completely satisfactory means of assessing performance of various methods, but we are ready to co-operate in any attempt to establish suitable criteria. We are prepared to run test problems of various sizes using our program in comparison with others. At present, we do not know how the ICL program rates against our own. We should explain that in § 8 we recommend the use of $n=2$ only for determining corrections, retaining an accurate expression for computation of frictional head loss. We normally use either Colebrook-White or Hazen Williams formulae when Re is in excess of 2000.

102. We are indebted to **Mr Boyne** for his ingenious extrapolation technique, which might be used with advantage in problems other than network analysis, and for his useful observations on over-relaxation combined with extrapolation. We are also grateful to Mr Webber for his references to papers on design rather than analysis. We have such programs available, both for *ab initio* design and for modification of a deficient network, and hope to describe these shortly. However, any design will ultimately be checked by analysis under various loading conditions, so we see the need for satisfactory analytical programs as well.

103. We are interested to know of **Mr Morrison's** progress on network design.

104. We sympathize with **Mr Iszatt's** plea for more logical criteria for design. We do realize that many factors as well as cost often need be taken into consideration but our present design programs seek only to reduce cost. Where an existing network is to be modified, we agree that it is usually best to submit various proposed modifications in turn; in this event, the user can compare different solutions on any basis he considers appropriate. This procedure does emphasize the need for a rapid program.

105. We are grateful to **Mr Maher** for his most useful contribution to the discussion. We had not previously appreciated the full advantage of the method of automatic loop assignment from the paper by Travers.⁴ In the very large networks with which Mr Maher is dealing, manual choice of loops is a very considerable chore, and we have now adopted Travers' method. Where there are pumps and pressure-reducing valves, however, we require a certain amount of route specification by the user. We agree completely that data is best stored in compact strings.

106. Finally, we thank Mr Al-Nassri and Dr Halliwell for their detailed comments. We agree that the correct value of n could be used to good effect in some circumstances, for instance, if the Hazen Williams equation is used, and n remains constant throughout. We are doubtful as to whether it is worth recalculating it from our equation (6) if the Colebrook-White expression is adopted.

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