Analysis of pretensioned cable systems

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It is with great interest we have read the above Paper. Pretensioned cable structures are eminently suited for large spans for which there is now an increasing demand.

26. The difficulties in analysing them arise from the fact that, assuming the cables to have no bending stiffness, they are often mechanisms and not structures and thus exhibit a high degree of non-linearity. For the truss analysed in the Paper the degree of mechanical freedom is 19 if we assume the truss to be weightless.

27. We have recently investigated a large number of cable girders and trusses of the types shown in Fig. 9 in an attempt to reach a design criteria for optimum stiffness and economy. In doing so we have varied the following parameters:

- (a) span
- (b) curvature of cables
- (c) diameter of cables
- (d) intensity of prestress
- (e) intensity of loading on half and full span.

The work has been carried out on an Elliott 503 electronic computer. The method of solution is iterative and based upon the minimization of the total potential energy of the assemblies. The correctness of the theory and the method of solution has been verified by experimental work on flat and doubly curved nets, cable girders and cable trusses.

28. The investigation of cable girders and trusses has shown that:

- (a) for increasing equal loads on all the upper node points the non-linearity of the load displacement curves decreases with increasing curvature of the cables and, for a dip of the load-carrying cables of more than 6% of the span, the curves are very nearly linear;
- (b) for increasing equal loads applied to the upper node points on one half of the span only the non-linearity of the load/deflexion curves increases with increasing curvature of the cables;
- (c) the non-linearity of the load displacement curves decreases with increasing intensity of prestress.

29. The degree of accuracy achieved using the method of influence coefficients cannot be estimated in advance, and will therefore depend upon the following factors:

- (a) the geometry
- (b) the loading pattern and its intensity
- (c) the intensity of prestress.

The work we have carried out indicates that for applied loads equal to the pretension force in the cables the method of influence coefficients could for any geometry yield errors in the calculated displacements of the order of 30–50% from the actual ones and considerably more in the displacements of the unloaded nodes in the case of loading on one half of the span only. That this may be so is indicated when studying the displacement curves in the Paper where the maximum ratio of applied load on half the span to the pretension in the load carrying cable is approximately 0.31.

Fig. 9. (a) cable girder ; (b) cable truss

30. It is to be regretted that the general theory and especially the method of solution of the resulting equations is not given, as others working in this field (such as Möllman) have found that when applying the same approach to similar problems such as prestressed nets the method diverges when the effect of the non-linearity becomes significant.

31. Several design studies which we have carried out have shown that when using cable girders and trusses as the load-carrying elements in roofs with spans from 120 to 500 ft the saving in cost is considerable when compared with more conventional forms of space structures, such as domes and grids.

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I welcome the Paper by Krishna and Sparkes, whose work I have been aware of for some time. Indeed it was after being shown their test rig that I decided to adopt a similar structural form to demonstrate a numerical technique for solving non-linear structural problems which I was then currently developing.¹

33. With this type of structure it is essential to realize that it is inherently very flexible. Therefore, when the structure is loaded rather large displacements can result, which makes the deformed geometry significantly different from the unloaded state. The directions of the internal and external stress resultants thus change, and account must be taken of this when forming the equilibrium and compatibility equations. This results in sets of governing equations which are algebraic and non-linear, and whose solution is a far from trivial exercise in numerical analysis.

34. The assumptions made by the Authors in their influence coefficient method are those which are commonly adopted in suspension bridge analysis, and also with this type of roof structure. They were also used by myself, so a direct comparison can be made between the calculations.

35. It is worth noting that the non-linear behaviour is assumed to arise solely from changes in geometry. The cables from which an actual structure would be constructed, would, in practice exhibit a non-linear stress/strain law. This, if it were not eliminated by pretensioning the cables up to some value in excess of their working load, would have to be taken account of in the analysis. Pretensioning has the effect of linearizing the stress/strain law for a stranded cable.

36. With any non-linear system it is in general true that if the disturbing influence is made sufficiently small, the response, if continuous, can be based on the linearized characteristics of the initial undisturbed state. This fact is used in the well known flexibility coefficient method for analysing suspension bridges, and is the basis of the Authors' method. It would be interesting to know, therefore, whether the Authors found that with smaller loads their influence coefficient method tended to give better results than with larger loads. Since the magnitude of error involved in their method is of some interest, it would be helpful if the Authors could give some guide on how this is related to prestress, member sizes and geometrical configuration.

37. Unfortunately the Authors describe only very briefly their more general numerical method for solving cable network problems. This is a pity, since I regard this aspect of the problem as the real challenge in non-linear analysis. In a comparative study of several different methods which have been proposed for solving non-linear problems, it has been found that all had markedly different convergence characteristics. Since solution time, and hence cost, are closely related to speed of convergence, this feature of a method is well worth looking at in detail. In some methods which have been proposed answers will in fact diverge if the disturbing forces are not kept small. It would therefore be of value if the Authors could describe briefly the numerical technique employed. A graphical representation for showing the convergence of one or two typical variables with successive alterations would also be of considerable interest. Is the iteration routine used a first- or second-order process?

38. The Authors are to be congratulated on their experimental investigation which must have called for a high degree of precision in the fabricating and operating of the test rig. The problems of measuring tension and displacement met with on a small-scale rig can be quite formidable. They can be made less difficult if a model of larger scale is used. This advantage is offset, however, by the extra material required by a larger rig. In my own test rig the scale was about three times that of the Authors'. Tensions were measured by calibrated electrical resistance strain gauges, and displacements by dial gauges.

39. Using my own method of analysis I have worked out a few of the cases. These are shown in Fig. 10 superimposed upon those given by the Authors. Fig. 10(a) shows the change in hanger tensions due to a 5 lb load applied to the quarter point, and Fig. 10(b) the deflexion of the upper cable due to 5 lb at the quarter point and 5 lb at hanger 10. The changes in hanger tension due to point loads applied to the upper cable at hangers 2-9 simulating a uniformly distributed load of 0.5 lb/in. are shown in Fig. 10(c), and the upper cable deflexion for this loading is shown in Fig. 10(d). From these diagrams it will be seen that, in general, agreement is good. These results required about two or three cycles of iteration to converge answers to within 1% of their final values. Each analysis took about two seconds of computation time on Atlas.

40. The variations in hanger tensions obtained by the general method are shown in Figs 3, 6 and 8 as a continuous function across the span. Since the hangers are discrete elements this will not be the case. This fact emerges most clearly under more irregular loading conditions (see Fig. 11 of ref. 1).

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The Authors thank the contributors to the discussion of their Paper. The contributions are most interesting and of much value.

42. The influence coefficient method² for the analysis of the pretensioned system makes assumptions similar to those made in the analysis of suspension bridges. A study of Figs 3, 4, 6, 7 and 8 will show that the error between the computed results of this method and the experimental values reduces as the ratio of applied load to pretension reduces. Calculations have shown that the effect of member sizes on the resulting forces and deflexions is small provided the load:pretension ratio is kept the

Fig. 10

same. The Authors' findings are in general agreement with some of the information given by Buchholdt and McMillan.

43. The more general method of analysis used is based upon a paper by Eras and Elze.³ The final equation of equilibrium (their equation 25) of forces resolved along the X axis at a node k is,

$$\sum_i \left[S_{i0} \cdot \frac{\Delta u_i}{s_{i0}} + (EF_i - S_{i0}) \cdot \frac{\Delta x_i}{s_{i0}} \cdot e_{i1} \right] = -X_k - K_{xk} \quad \dots \quad (1)$$

where,

$$K_{xk} = \sum_i (EF_i - S_{i0}) \cdot \left\{ \frac{\Delta u_i}{s_{i0}} \cdot e_{i1} + \frac{\Delta x_i + \Delta u_i}{2s_{i0}} \cdot \left[e_{i2} - 3e_{i1}^2 \left(1 + \frac{e_{i2}}{e_{i1}} - \frac{5}{3} e_{i1} \right) \right] \right\}$$

$$e_{i1} = \frac{1}{s_{i0}^2} (\Delta x_i \cdot \Delta u_i + \Delta y_i \cdot \Delta v_i + \Delta z_i \cdot \Delta w_i)$$

$$e_{i2} = \frac{1}{s_{i0}^3} (\Delta u_i^2 + \Delta v_i^2 + \Delta w_i^2)$$

where k is any node in the system and i the nodes adjacent to it;

S_{i0} = initial tension in an element $k-i$;

ΔS_{i0} = change in S_{i0} due to applied loads;

s_{i0} = initial length of an element $k-i$;

E = Young's modulus of elasticity;

F_i = cross-sectional area of an element $k-i$;

X_k = component of applied load at node k , along the x axis;

$x_k, y_k, z_k, x_i, y_i, z_i$ = co-ordinates of nodes k and i respectively;

$u_k, v_k, w_k, u_i, v_i, w_i$ = deflexions of nodes k and i respectively, due to applied loads;

$\Delta u_i = u_i - u_k$;

$\Delta x_i = x_i - x_k$;

and $\Delta v_i, \Delta w_i, \Delta y_i$ and Δz_i can be defined similarly.

44. It will be seen that the left-hand side of equation (1) contains the unknown deflexion terms and their coefficients, X_k is the loading term and K_{xk} contains the higher powers of the deflexion terms (the non-linear terms). Two such equations of equilibrium will be necessary for forces resolved along the two axes lying in the plane of the structure. The equations have to be set up for each junction between the hangers and the upper and lower main wires, and the resulting set of the algebraic non-linear equations have then to be solved by means of a suitable numerical procedure.

45. The numerical procedure adopted was as follows. If the square stiffness matrix (left-hand side of equation (1)) for the structure be denoted by A , the unknown column matrix by $\bar{U}$, the loading matrix by P and the column matrix, containing the non-linear terms K_{xk} , by K , then

$$A \cdot \bar{U} = P - K \quad \dots \quad (2)$$

or

$$\bar{U} = A^{-1} \cdot (P - K) \quad \dots \quad (3)$$

46. The solution of equation (3) can be obtained by an iterative procedure, assuming suitable values of K in successive cycles. The advantage in this procedure is that the matrix A is a constant for a particular structure and therefore the inversion of this matrix has to be obtained only once. Subsequent computations in different

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cycles of iteration and for any number of loading cases involve only the multiplication of A^{-1} with the column matrix $(P-K)$, which is an operation much simpler than the inversion of A .

47. Two procedures were adopted for obtaining the value of K in successive cycles of iteration and are given in Table 1. The starting value of K was in each procedure zero.

Table 1

Cycle of iteration	Value for K	
	Procedure 1	Procedure 2
1	0	0
2	$K^{(1)}$	$K^{(1)}$
3	$K^{(2)}$	$\frac{1}{2}(K^{(2)} + K^{(1)})$
4	$K^{(3)}$	$\frac{1}{2}(K^{(3)} + K^{(2)})$
...	...	...
...	...	...
...	...	...
n	$K^{(n-1)}$	$\frac{1}{2}(K^{(n-1)} + K^{(n-2)})$

48. The values of vertical deflexion v under an applied concentrated load, $p = 2.5$ lb, obtained in successive cycles using the two procedures, are given in Fig. 11. The results show that faster convergence to linearity is obtained by using procedure 2.

49. Figs 12 and 13 give results for vertical deflexion v and change in upper wire tension ΔS_0 , for some cases of applied loads. For the range of loads shown, the results converge to within 2% accuracy in six cycles of iteration. It may thus appear that this method requires more cycles than the method used by Poskitt. However, this fact has to be seen against the simplicity of using equation (3).

50. The Authors feel that this is a suitable approach for the degree of non-linearity tackled in the present cases of loading. Where higher magnitudes of loads and hence non-linearity have to be tackled, this approach is likely to yield slowly

Fig. 11. Comparative results for the two iterative procedures: vertical load of 2.5 lb applied at hanger 10

Fig. 12. Vertical loads of 1.5 lb each applied to the upper wire at hangers 2, 3, 4, 5, 6, 7, 8 and 9

Fig. 13. Vertical load P , applied to the upper wire at hanger 10

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converging results and in some cases the results may be divergent. However, for the maximum loads considered, the maximum deflexions are of the order of $1/250$ of the span (see Figs 4 and 7), and it is not envisaged that deflexions larger than this will be permissible in practice.

References

1. POSKITT T. J. Numerical solution of non-linear structures. *Proc. Am. Soc. civ. Engrs*, 1967, **93** (ST4) (Aug.) 69–94.
2. SPARKES S. R. and KRISHNA P. An influence coefficient method for pretensioned cable systems. To be published.
3. ERAS G. and ELZE H. *Zur Berechnung und statisch Vortielhaften Formgebung von Seinnetzwerken*. North Holland Publishing Co., Amsterdam, 1963.