Framed structures: the instability problem

by

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Professor Arthur Bolton (Heriot-Watt University) complimented Dr Gregory on his review of recent work on the instability problem and the clear way in which his examples were set out.

71. Instability was a most important factor in structural safety and not enough consideration was given to the problem either in design codes or by engineers on site. There had been numerous failures in recent years in which instability had played its part and led to sudden collapse. It was this lack of warning which made instability failures so dangerous.

72. When such failures had occurred in scaffolding, temporary structures, tower cranes or jib cranes they were often regarded as being merely an unfortunate result of poor site practice. If a building frame collapsed before the cladding was completed, many engineers brushed away any suggestion of structural inadequacy by referring to it as an erection failure. On occasions there was no doubt that the cause of failure was poor bracing whilst the frame was still partially erected, but sometimes the frame was almost complete when it failed. This showed that the structure on which all the calculations of strength and stiffness had been carried out could not carry even its self-weight without the support of the cladding. The danger in such a situation was that the degree of safety of many completed buildings was unknown, since no satisfactory calculations of the stiffness of the cladding could be made.

73. A basic cause of the trouble was that the strut formula used in B.S. 449, for example, did not refer to struts which could sway but to pin-ended struts, in spite of the various diagrams and stated effective lengths which might mislead the unwary. It was surely time for design codes to give better guidance on the question of instability. The Writer had said this on several occasions, one of which by coincidence was in a reply to a discussion by Dr Gregory²³ on one of his papers²⁴ written more than seven years before.

74. The method of analysis described by Dr Gregory (§§ 58, 59) was derived by comparing the behaviour of a complete frame with that of a pin-ended strut. Perhaps it was fair to comment that it would be impracticable to measure the imperfections of every frame built and therefore no design code, as such, could be based on individual eccentricities and their magnification by the factors $1/(1 - W/W_n \text{ crit.})$. There was however, a more serious doubt as to whether the whole process of calculating several 'critical' loads and using them in magnifying factors was permissible.

75. To take the pin-ended strut as a simple example, it was often claimed that there were an infinite number of critical loads given by $n^2 Q$ where Q was the Euler load and n could take any integral value 1, 2, 3 . . . In the Writer's opinion there was only one critical load when n had the value unity. As Dr Gregory stated in § 6, all others were unstable and only arose if the strut was supported at one or more points

along its length. There were, however, unstable forms neglected in the expression $n^2 Q$. If, for example, a pin-ended strut was laterally supported at a point $0.4 l$ from one end, the critical load was approximately $1.34 Q$. This meant that the mode shape consisting of two half sine waves of length $0.4 l$ and $0.6 l$ respectively was magnified indefinitely when the axial load was $1.34 Q$ even though none of the series of magnifiers corresponding to $1/(1 - P/n^2 Q)$ showed an infinity at $1.34 Q$.

76. This seemed to indicate that Fig. 1(b) should have included a line DE parallel to AC and with OD equal to $1.34 Q$. It was apparent, however, that the support point was chosen arbitrarily at $0.4 l$ from one end of the strut, which could have been supported at any one of an infinite number of points. Hence Fig. 1(b) could be considered to consist of a straight vertical line OA with a closely spaced field of horizontal lines above AC. This corresponded with the real world, in which no pin-ended strut would carry an axial load greater than the Euler load, but showed an indefinite deflexion if such a load were applied.

77. In a similar way, for more complex structures, there was always at least one mode of deformation which led to infinite magnification at any load above the critical load. It was difficult to decide how to choose proper modes and magnifiers for loads below the critical under these circumstances.

78. There was some doubt therefore about Dr Gregory's insistence on the necessity to calculate 'higher critical' loads and modes. There was no doubt, however, that it was possible to get excellent, if not exact, agreement between experiments and calculations based on overall plastic behaviour and buckling in the appropriate critical mode. B.S. 449 could not approach the order of accuracy now possible by these modern methods.

Dr A. Jennings (Queens University, Belfast) wrote that Dr Gregory had developed an interesting method of formulating the elastic stability analysis of framed structures as a latent root problem.

80. Latent root methods had been used considerably in the solution of vibration problems. Normally a matrix was formed whose latent roots defined the natural frequencies of vibration and whose latent vectors defined the corresponding modes of vibration. In fact the application to vibration problems had probably been a significant stimulus in the development of numerical methods of determining latent roots and vectors.

81. Because latent root methods were employed far less in the field of elastic stability, the Writer had developed a method of defining a matrix whose latent roots and vectors determined the buckling loads and associated deformation modes of a structural frame subject to a prescribed proportional loading.¹³ In contrast, Dr Gregory's method did not use latent roots to predict the buckling loads directly; instead they were used as a vehicle for determining the degree of stability at a specific loading condition. The actual values of the latent roots and vectors of the stiffness matrix were not unique for any structure under a given loading condition as they depended on the displacement variables used to define the stiffness matrix. At other than the critical load conditions, the latent roots and vectors would seem to have little significance either to the structural analyst or to the designer. The usefulness of the Author's method must therefore be judged mainly by its relative numerical efficiency and reliability when compared with other methods of determining the buckling loads and associated deformation modes, namely the methods considered in §§ 30-34 and the Writer's latent root method.

82. The amount of computation required to determine all the latent roots and vectors of a matrix was far greater than that required to perform operations such as inversion or evaluation of the determinant by Gaussian elimination. Dr Gregory's claim (§ 44) that his process was very rapid must therefore rely heavily on the parallel shift and intensification procedure for determining the lowest latent roots and vectors.

During the process of iterating for the lowest latent root and vector, the factor by which the component of this vector was intensified at each iteration was $(g - \lambda_1)/(g - \lambda_2)$. It was therefore apparent that if λ_2 was close to λ_1 , or if g was large, then iteration would be slow. Furthermore, if iteration was curtailed too soon, the estimate of the lowest latent root would tend to be too high.

83. The parallel shift parameter g was determined from the value of the highest latent root, and hence although this route had no effect on the stability it had a considerable effect on the speed of convergence of the intensification process. The example in Fig. 7(d) was a particular case where the intensification factor would be fairly low and λ_1 would be required with high accuracy because of its nearness to zero throughout the range of loading. Hence, unless a particularly good estimate for the latent vector was presented at the start of each iteration, the method was not likely to be very rapid numerically. Because the speed of convergence depended on the largest latent root of the stiffness matrix, it seemed essential that displacements which did not involve introducing into the stiffness matrix very large terms arising from the axial stiffness of the members should be used. As the simplest methods of constructing stiffness matrices for frames of arbitrary geometrical form on a digital computer involved including the axial stiffnesses of the members, this was a restriction of the method which should be noted. The largest latent vector of the stiffness matrix did not affect the convergence rate of the iteration process of the Writer's method. The examples in Figs 7(c) and (d) were cases for which the Writer's method would be particularly suitable because the joint rotation and sway modes alone would be quite sufficient to define the first buckling load and deformation mode to a reasonable accuracy.

84. In § 43 the Author stated that an advantage of the proposed method was that initial guesses to the latent vectors could be used to reduce the amount of iteration required. Whereas it was certainly possible to do this, care must be exercised to ensure that the introduced vector was definitely an approximation to the appropriate latent vector. In the Writer's view it was preferable in most cases to carry out iteration using an arbitrary starting vector. This meant that any prior knowledge of the required latent vector could be used to check the numerical results. If the lowest latent roots were well separated, then iteration would, in any case, be fast, whereas if the lowest latent roots were very close to each other, it seemed possible that the initial guess might approximate to the wrong latent vector. If so, a very large number of iterations would be required to regenerate the correct latent vector, or, in the worst circumstances, the wrong latent vector would be predicted.

85. One circumstance in which there was a moderate chance of predicting the wrong vector occurred with a symmetric frame under symmetric loading if a symmetric initial guess was used when the required latent vector was anti-symmetric (or *vice versa*). Another circumstance where this might occur was when the vector obtained from one loading case was used as an initial guess for the next loading case, as suggested in § 43, and a crossover between the lowest latent roots occurred between the two loading cases. This situation did not arise in any of the examples given by Dr Gregory but was not an impossible situation (see Fig. 10). In this example the lowest latent roots of the stiffness matrix in the earlier stages of loading were two anti-symmetric modes involving large sway components. These latent roots remained the lowest until near the buckling load, but were suddenly overtaken by a symmetric root which was the actual cause of buckling.

86. Dr Gregory suggested (§ 64) a treatment of the post-yielding behaviour by determining critically the plastic collapse load, its associated mode of deformation and the collapse bending-moment diagram. If plastic collapse happened to involve only partial collapse of the structure, then using the rigid-plastic theory the bending-moment diagram for the complete frame was indeterminate. For the case of a symmetric frame under symmetric loading, collapse might occur by an antisymmetric mode of instability forming after only a small proportion of the plastic hinges had been

formed. If this happened, the deformation mode causing collapse would bear no resemblance to the plastic collapse mechanism. Thus the proposed method might well be limited in application to certain types of frames or loading conditions. The elastic-plastic collapse load could be determined without any undue difficulty by predicting the formation of successive hinges, taking continual account of the effects of reduced stiffness until the structure became unstable.²⁵

87. In conclusion Dr Gregory might be interested in the method the Writer had developed of obtaining the dominant and subdominant latent roots and vectors of a symmetric matrix.²⁶ This was effectively an extension of the intensification process so that various trial vectors which were converging on to all the largest latent vectors were operated on simultaneously. This method was likely to be an improvement on the existing methods of determining latent roots and vectors for the type of requirement given in the Paper.

The Author was grateful for Professor Bolton's comments on the Paper and for pointing out the dangers of the failure of structures by instability, the need for further study, and the desirability of eventually incorporating in codes of practice the results of research. However, in spite of the many criticisms which could be made of existing codes of practice, codes such as B.S. 449 still remained, in the Author's opinion, a fairly sensible guide to design (apart from the obvious exceptions such as struts forming parts of frames which could sway readily). In the present state of knowledge, the Author believed that the best that could be done was to attempt to pass on to designing engineers a better appreciation of the actual deformations of unstable structures. The main point to be learned was that structures buckled as a whole and that consideration of isolated compression members was not adequate. This did not mean, however, that the concept of effective length was out of date: when a structure buckled it did so as a whole, but individual members did experience changing curvature or even reversal of curvature and hence did have effective lengths. The Author would offer the following advice to designers: (a) sketch carefully likely modes of instability of the structure being designed; (b) use the sketch, and also the advice given in codes of practice, to estimate effective lengths of compression members; (c) design the members using the code provisions; (d) consider appropriate means of bracing the structure and adjust the design if necessary; (e) use a method of analysis such as that presented in the original Paper to find the first, second, third, etc., modes of instability. The first was, of course, the gravest mode and the one to be designed against. However, under test, structures often deformed in combined modes, and details of several modes were needed for the interpretation of tests. The final step was to apply some disturbing loads or moments to the structure sufficient to excite the first mode, i.e. sufficient to give deflexions of similar magnitude to the imperfections or crookedness likely to exist in the structure when built, which could be readily estimated, and to analyse the structure and compute the load to cause first yield, using a load factor to give the working load, and comparing with the design load.

89. The designer thus concentrated his attention on the gravest mode. The fact that a real structure deformed in a combination of modes made it sometimes a little stronger, as tests had shown.

90. Professor Bolton had questioned whether calculated higher modes had any relevance at all, and had pointed out that a pin-ended column could be supported at any point in its length, thus giving critical loads not included in $n^2\pi^2 EI/l^2$, where $n=1, 2, 3$, etc. However, it was clear that these extra modes required lateral force at the point of support, whereas the natural modes did not (though they did require that the support had adequate stiffness).

91. In practice, the method given in the Paper permitted analysis of any elastic structure, supported or braced in any way, and it gave the force necessary to brace any point, which was useful to designers.

92. As Professor Bolton pointed out, it was impracticable (and indeed impossible) to measure the imperfections of every frame, and hence no design code could be based on individual eccentricities and their magnifications. As a result of tests on many structures and models, the Author believed that a practical compromise was to consider a perfect structure and to apply some external loads sufficient to excite deformations having the same pattern as the gravest mode of the structure *as a whole*, thereby replacing crookedness and eccentricities by an equivalent loading, which was easier to handle in the analysis of the deformation. The choice of the magnitude of the imperfections to use was an important one, but not insuperable. It was wise to perform the analysis for a range of values which covered fabrication and erection tolerances. Measurement on a few structures in Tasmania had shown that $0.003 l/r$ was likely to be an over-estimate of crookedness.

93. The Author had advocated a useful alternative procedure, namely to adopt a reasonable allowance for imperfections at the design stage, to specify this on the drawings, and to ensure that the specification was met during erection. This procedure encouraged the contractor to take care where it was needed, and time and trouble was not wasted elsewhere.

94. Professor Bolton also mentioned the mounting evidence regarding the adequacy of post-yielding studies of instability problems. It was to be hoped that further work would enable these important ideas to be passed on to designers in the near future.

95. The Author was grateful to Professor Bolton for pointing out two errors in the Paper. In Fig. 6(c) the joint D should be shown pinned, and the symbol k had been omitted in the upper rows of the matrix in Fig. 6(d).

96. Dr Jennings had kindly made some valid and useful comments on computation, and had set out some limits and restrictions on the methods given in the Paper, and indicated where trouble would be experienced in the numerical work. Against these drawbacks could be set the advantages of the methods given: that the designer always maintained an appreciation of the physical meaning of his calculations as they related to actual or approximate buckling modes, which could be sketched from time to time. The Author would advise the designer always to try first a calculation which had physical meaning. If numerical trouble was experienced, this generally meant that two modes lay close together, and this was useful information: small changes in member stiffness, or changes in the imperfections pattern, might cause the calculated second mode to be the gravest in practice, and this contingency must be allowed for in the design. However, with complex structures, a computer would be used in any case, and there were various efficient means of accurately extracting the needed latent roots and vectors, as Dr Jennings had pointed out.

97. Regarding post-yielding studies, it was very clear that unexpected modes of collapse had to be guarded against, and that a structure could begin to deform plastically in one mode, then suddenly change to another. Practising designers would welcome more help on how to anticipate such behaviour so that it was not missed in their calculations. It was useful to be able to define an upper limit to a load-deformation curve, but the drawing of such an upper limit involved assumptions about the deformations, and the designer was always worried that he might have missed a governing mode. However, elastic studies such as those given in the Paper would indicate all the likely elastic deformations, which would usually excite the relevant post-yield behaviour, or at least make the designer aware of it.

98. Fig. 10 presented some interesting results, and the Author was grateful to Dr Jennings for preparing them. For such a structure a computer was necessary, and it was very useful to the designer to have the whole plot, as given. The designer must then face up to the question of deciding what buckling mode he would take as governing the elastic portion of the loading path of the structure or, since two very different modes had their critical loads so close together, it would be safer to design against both. Here was a case where the designer needed accurate information on at least two buckling loads and modes.

REFERENCES

23. GREGORY M. Discussion on BOLTON A., ref. 24. *Struct. Engr*, 1960, 38 (7) 239-241.
24. BOLTON A. A convergence technique for determining the *elastic* critical load of rigidly jointed structures. *Struct. Engr*, 1959, 37 (8) 233-242.
25. JENNINGS A. and MAJID K. I. An elastic-plastic analysis by computer for framed structures loaded up to collapse. *Struct. Engr*, 1965 (Dec.) 407-412.
26. JENNINGS A. A direct aeration method of obtaining latent roots and vectors of a symmetric matrix. *Proc. Camb. Phil. Soc.* (in press).