Elastic stability of practical multi-storey frames

by

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Professor M. R. Horne (University of Manchester) wrote that Dr Stevens had performed a most useful service in demonstrating the nature of the critical loads of regular, multi-storey frames designed according to commonly accepted criteria. He was doubtless correct in his statement that the method of design would make no difference to the general trend of results, and it was probably of little significance that the strength design criteria that he adopted (use of safe stresses according to B.S. 449) was irrational as applied to rigid frames. The nature of the strength design criterion would, however, become of primary importance if one were to proceed to the next step of assessing the significance of the elastic critical load.

42. The Author mentioned the Merchant interaction formula, which gave as an estimate of the failure load, λ_F , the value

$$\lambda_F = \frac{\lambda_P}{1 + \frac{\lambda_P}{\lambda_C}}$$

where λ_P is the rigid-plastic failure load factor and λ_C the elastic critical load factor. In the Author's notation, $\lambda_C = \alpha$. For practical purposes, λ_F as given by the formula could be regarded as a lower bound. It tended to become conservative when the plastic collapse mode differed appreciably from the elastic critical mode. For tall frames with only a few bays, the Merchant formula gave a reasonably close estimate of the failure load, but tended to be conservative for frames with large numbers of bays in which rigid-plastic failure was by local beam collapse while the elastic critical mode was, as always, a sway mode.

43. It was possible to arrive at some general conclusions about the probable failure loads of the frames investigated by the Author. With a small number of bays, deflexion criteria control the design and λ_P was therefore high, i.e. significantly in excess of 1.75, which would be the value of λ_P for a simply supported beam. Since λ_C was very high, λ_F was virtually the same as λ_P . With a certain number of bays (depending on the span, storey height, frame spacing, number of storeys and loading) the design criterion became the maximum stress under combined loading, and rigid-plastic failure was according to the combined mode. The value of λ_P would probably be in the region of 2, and (from the Author's curves) λ_C would lie between 5 and 10. Due to the similarity between the plastic mode and the elastic critical mode, the Merchant formula gave a good estimate of failure, so that λ_F could be expected to be some 10–20% below λ_P . With a large number of bays, λ_P remained much the same, but the Merchant formula became conservative, so the failure load should rise slightly.

44. The conclusion might therefore be that, using conventional methods, the most unfavourable frames were those in which the maximum stresses and the permissible deflexions were both reached under the specified loading conditions. On general grounds, this was not surprising. Although the criteria of strength and limiting deflexions were, for convenience, treated separately, they were not independent, being linked by the effect of deformations on failure loads. It therefore stood to reason that

frames in which both were in danger of being violated if a slight reduction in sections was made were the most critical. An important factor not discussed in arriving at these conclusions was the relationship between the stress criterion of strength as used by the Author and the rigid-plastic failure load factor λ_p . This was likely to have a variable effect up to about 50%. Although λ_p might be in the region of 2 for all frames in which sway deflexion was not the ruling criterion, the value would certainly vary from 1.75 up to about 2.6.

45. It would be most interesting if the above conjectural remarks could be tested by the calculation of the elastic-plastic failure loads of a few of the frames considered by Dr Stevens. This would provide a valuable commentary on the significance of the results he had presented.

The Author, in reply, wrote that the contribution by Professor Horne extended the scope of the Paper considerably beyond that contained in the brief reference to elastic-plastic failure modes in § 34. This was a logical extension and the conclusions reached from a consideration of the Merchant interaction formula provide a valuable guide to probable failure loads. The range of probable load factors estimated by Professor Horne was quite large and gave weight to his suggestion that frames, designed to satisfy the criteria adopted in the Paper, should be analysed to determine the actual elastic-plastic collapse load.

47. This had not yet been done by the Author although a programme was at present being developed which it was hoped would enable the analysis of large multi-storey frames to be made, taking into account the effects of deformations and strain hardening. There might be programmes of sufficient capacity already available for the type of analysis required; if this were so, the Author would welcome further information.