Design of channels for supercritical flow

by

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Mr A. M. R. Montagu (Sir M. MacDonald and Partners) considered that Mr Harrison's Paper was of great interest, particularly to those who were or had been in touch with the design of the lined section of the remodelled Alphin Brook. To those few, additional information on the dimensions of the model would have been of the greatest interest. That the model was on a scale 1:24 seemed to be the sum of information on this head (§ 18). The reasons for deciding on this scale would be of interest to future designers.

30. Presumably the slope was such that the Froude number at maximum designed discharge was the same in model and prototype. In a geometrical model, the depth would be to the same scale and this would determine the velocity in the model. The slope of the model must therefore provide this velocity. This in turn raised the question of the coefficient of rugosity.

31. Clearly, if the slope of the model was the same as in the prototype, the roughness must be very considerably reduced. How was this determined, and equally important (and far more difficult), how was the required 'roughness' secured on the surface of the model envelope? If, on the other hand, a slope had been selected other than that of the prototype, how was the selected slope determined and the required 'roughness' ascertained and applied to the model.

32. What would be the quantitative effect of an error in estimation of the coefficient of rugosity? If the value of N (Manning) for concrete in the prototype was assumed to be 0.013 or the corresponding value of $k=0.005$ in the Colebrook-White formula, variations due to ageing or to variations in finish during construction might cause appreciable variation in the delicate balance of the wave pattern. Such considerations led Mr Montagu to the conclusion that every work of this type should be tested on a model with the same meticulous care as evidenced by Mr Harrison's Paper.

33. One small point of definition and nomenclature: to Mr Montagu, the standing wave and hydraulic jump were synonymous terms defining the phenomenon of change from supercritical to subcritical flow accompanied by loss of surplus energy. Mr Harrison could not be referring to this phenomenon when he used these and terms such as gravity, stationary, oscillatory, shock or cross waves. In the synopsis 'standing wave' was used: in the notation, 'cross wave or oblique hydraulic jump' was used in the definition of β . 'Standing wave' appeared in §§ 11, 18, 19, 27(a) and in Table 1: 'hydraulic jump' in §§ 12, 13 and 24(a) and (b). Would the term 'ripple' be unsuitable to cover the whole class of this phenomenon?

34. The design of the Alphin Brook did not end with the lined channel. The problem of the Alphin Culvert and the subsequent return of the stream to subcritical flow were very important. Mr Montagu hoped that an equally informative paper on the problems in this respect, the solutions to which had been attained under Mr Harrison's control at the Hydraulics Research Station, would follow in due course.

Mr K. V. H. Smith (Department of Civil Engineering, Southampton University) congratulated the Author on some interesting experimental work in connexion with a proposed improvement scheme for the Alphin Brook.

36. Before referring to the substance of the Paper, Mr Smith wished to comment on the prototype proposals. It was apparently proposed, on economic grounds, to take advantage of natural ground slopes and convey water in these channels at velocities (17–22 ft/s, for example) which surely had no parallel in an open channel passing through any urban area of England. In the event of a contingency such as the partial collapse of a bank at the same time as a major flood flow, quite fantastic damage might well occur.

37. The proposed cross section for the greater part of the channel was trapezoidal. Because of unavoidable circumstances, such as curves in the channel, surface waves were bound to be propagated. Sloping side walls normally aggravated wave effects in supercritical flow and he would have thought that a rectangular cross section, though more expensive, would have been amply justified. Had the Author offered advice on this point? The wave pattern shown in Fig. 9 did not inspire confidence, and a large 'freeboard' would surely have been used in the prototype. A comparison with the flow in an equivalent rectangular section would have been very valuable.

38. Referring to the experimental and theoretical values of β given in Table 1, the similarity of values was certainly very striking. The relative insensitivity of β to the exact method of calculation was, however, clearly demonstrated. Observations on the prototype channel would also be of interest, if these were available.

39. The Author had gone to some lengths to produce design data for a supercritical channel contraction with straight converging side walls. However, it should be appreciated that such a design would produce a transition conforming to Fig. 4 (i.e. downstream wave fronts terminating at the end of the contraction, points C and C') for only one discharge, with its appropriate value of F_1 . At any other discharge this coincidence would not occur, and the usual reflected waves would be transmitted downstream to a greater or less extent.

40. In such a situation Mr Smith suggested that it might be preferable to use a transition of the type shown in Fig. 12, which, although having a somewhat greater angle of turnthrough θ , was relatively insensitive to large variations of discharge such as would obviously occur in the Alphin Brook.

FIG. 12

Mr T. C. Kabell (Ministry of Water Development, Rhodesia) wrote that he was most grateful for the publication of the results of Mr Harrison's research work and for the design curves, which would be very useful for hydraulic design engineers. He was particularly interested in the section dealing with transitions for supercritical flow, and hoped that Mr Harrison would extend his research to cover contracting transitions from a rectangular to a trapezoidal section, which were frequently incorporated in chute spillways.

42. Spillway transitions had to carry a wide range of flood flows, with a correspondingly wide range of entrance Froude numbers (F_1), and it was desirable to achieve smooth flow conditions throughout this range. As Mr Harrison stated (§ 6), it was most practicable in excavation to have chutes of trapezoidal cross section. Moreover, only a trapezoidal chute with correctly designed side-slopes would be able to effect smooth flow at all discharges.

43. The range of values for F_1 would commonly be between 1.5 for maximum flow and 8.0 for low flow. As could be seen from Fig. 5, the ratio $b_3:b_1$ for $F_1 = 1.5$ must be no smaller than 0.90 if flow was to remain supercritical. For higher values of F_1 the ratio $b_3:b_1$ should be smaller, with a correspondingly larger deflexion angle θ . This immediately suggested a trapezoidal chute, if the value of b_3 was taken as being applicable to the water surface width at the downstream end of the transition.

44. In 1963, Mr Kabell was engaged on preliminary design and model testing for the chute spillway of Manjirenji Dam. Radial gates controlled the flow and there was a warped (straight-line) transition from the rectangular control section to the trapezoidal chute. In design of the transition and chute, use was made of the data given by Ippen and Dawson.³ Values of θ and $b_3:b_1$ were determined for two flow conditions: maximum flood, and probable annual flood. For a given value of b_1 , the values of θ and b_3 for maximum flood defined the length of transition L . With b_1 and L known, mutually compatible values of θ and b_3 were found for the annual flood flow. The invert grade of the transition and the side slopes and bottom width of the chute had to be designed to meet these requirements for smooth flow conditions at all stages. This design was tested in a hydraulic model and found to give smooth flow over a wide range of floods.

45. Although this design was largely empirical and the calculations were done on a trial and error basis, it was felt that the problem was amenable to the type of analysis and computation described in Mr Harrison's Paper.

FIG. 13: AVILL RELIEF CHANNEL: AERIAL VIEW

FIG. 14

FIG. 15

Mr J. D. Winders (C. H. Dobbie and Partners) considered that Mr Harrison's Paper was of great interest in respect of the amplification of superelevation on bends and his method had the great advantage of simple application. However, in the Paper, the problem of oblique standing waves was presented with some prominence and the theory that a pattern would be formed which was related to the Froude number was proposed. The results given in Table 1 showed that for a Froude number of 1.698 an average wave height of 1.72 ft was expected to be produced in the prototype channel proposed for Alphin Brook.

47. The River Avill Relief Channel had almost identical physical properties, i.e. a bed width of 18 ft, side slopes of 1:1 and a bed slope of 0.0077 on the upper length and 0.0057 on the downstream length. The bed and sides of the channel were protected by concrete. The channel was constructed for the Somerset River Authority whose Consulting Engineers were C. H. Dobbie and Partners for whom Mr Winders had worked on the design and subsequent site supervision. The channel was opened to flow in 1963, since when several floods had been observed and he felt it was useful to compare the working of this channel with the model analysis of the channel described in the Paper.

48. Fig. 13 was an aerial photograph showing the general layout of the channel, the intake being in the foreground and the outfall to the Bristol Channel just off the top left hand edge of the photograph.

49. Fig. 14 showed the influence of the first bend below the intake. At this stage the flow was about 430 cusecs and the oblique standing waves caused by the bend were clearly shown. It should be noted however that only a relatively short distance below the bend there was no evidence of standing waves. Fig. 15 was taken during the same flood just downstream of the railway bridge (the outfall to the Bristol Channel could be seen in the distance) and also showed no evidence of a standing wave pattern though the Froude number in this case was 1.95. No photographs were available of higher flows but in no case had a pattern of standing waves been noted on the lengths of channel beyond the zone of influence of the bend.

50. In all cases, however, the surface of the water had been disturbed and the influence was in all probability due to air shear. In the prototype high velocities would generate sufficiently high shears in still air for the propagation of surface waves, but their significance was small and would not in any way influence the design of the channel.

51. It would therefore seem probable that there was a strong scale effect in the generation of standing wave patterns and the problem of scaled wind shears did not appear to have been investigated. The Author's comments on these two points were sought.

52. Finally, it would be most useful if fully instrumented data obtained from the floods in the prototype channel could be compared with the model analysis and presented as a supplementary Paper.

The Author, in reply, observed that three of those contributing to the discussion had commented on various aspects of the Alphin Brook flood relief scheme and on the model investigation of the supercritical channel which had been briefly mentioned in the Paper. Full details of this investigation had been presented elsewhere⁹ and it was not his intention, therefore, to prepare a further paper on the subject. He felt that the novel features of the design had been presented in the present Paper; the other aspects of the investigation, which included the study of flow in culverts and the design of a stilling basin on a bend, were interesting but not sufficiently unusual to warrant publication. However, he hoped that some observations would be made on the prototype channel after completion so that a comparison could be made with the model predictions.

54. Mr Smith's remarks on the desirability of constructing a channel with supercritical flow through an urban area were beyond the scope of the Paper. It seemed,

however, that such a scheme would be quite acceptable provided that the design discharge was not exceeded, that particular care was taken in the structural design to reduce the risk of bank collapse and that adequate fencing was erected to prevent access to the channel.

55. Mr Winders pointed out the remarkable similarity between the dimensions of the supercritical sections of the River Avill and the proposed Alphin Brook relief channels; these were:

	Alphin Brook	River Avill
Bed width . . .	17.2 ft	18 ft
Side slopes . . .	1:1	1:1
Bed slope . . .	0.00544	0.0077 and 0.0057
Length . . .	2100 ft	3600 ft

56. A comparison between the two channels was thus quite justified but it had to be remembered that Mr Winders's photographs (Figs 14 and 15) showed the Avill at a flow of 430 cusecs whereas Figs 9 and 11 showed the Alphin Brook at a flow of 4000 cusecs. Unfortunately the model was no longer in existence so it was not possible to determine whether a stationary wave pattern could be observed in the Alphin Brook at 430 cusecs. However if it was assumed that, at the least, the wave heights were reduced in proportion to the reduction in depth, then it should be expected (from Fig. 11) that the maximum wave height immediately beyond the bend would be 6 in., reducing to nil in a length of 100 ft. Since the wave length measured along the wall would be about 40 ft, it would be unlikely that the second wave crest beyond the bend would be detectable.

57. In Fig. 14 the first trough and crest on the left bank could be seen but whether a second crest existed was difficult to ascertain from the photograph. As Mr Winders suggested, Fig. 15 was taken too far downstream of the last bend for any standing wave pattern to be evident at 430 cusecs.

58. In any case the standing wave pattern in the Avill channel was partially masked by the transitory waves produced by air shear: a rough calculation indicated that these waves could be 3 in. high at the downstream end of the channel. It was true that such waves were a scale effect and were not formed in the model channel of Alphin Brook, since the maximum water velocity ($4\frac{1}{2}$ ft/s) was too low. However it was not thought that the presence of such waves would affect the stationary wave pattern, since their wavelength and celerity were different. Furthermore the height of the transitory waves would not vary appreciably with discharge so that at high flows the height would be small in relation to the height of the stationary waves at the bends.

59. In answer to Mr Montagu, the model scale was chosen as the smallest which would have the same Chezy coefficient or friction factor as the prototype at maximum discharge. The Colebrook-White flow formula could be written¹⁰:

$$\frac{C}{\sqrt{32g}} = \frac{v}{\sqrt{32gRi}} = -\log \left[\frac{k}{14.8R} + \frac{1.255\nu}{R\sqrt{(32gRi)}} \right] \quad \dots (31)$$

where C = the Chezy coefficient, i = the hydraulic gradient, k = the roughness height, R = the hydraulic mean depth and ν = the kinematic viscosity. In the prototype the second term within the square brackets was negligible compared with the first, i.e. the flow was rough turbulent because the criterion $vk/\nu > 807$ given by Ackers¹¹ was satisfied. The smallest model could be achieved by constructing it so that the flow was smooth turbulent, i.e. so that the first term was negligible relative to the second in the square brackets. Thus, equating the Chezy coefficients in model and prototype:

$$\frac{k_P}{14.8R_P} = \frac{1.255\nu_M}{R_M\sqrt{(32gR_Mi_M)}}$$

where subscripts P and M referred to prototype and model respectively.

60. Rearranging this equation, the scale R_P/R_M was given by

$$\frac{R_P}{R_M} = \left[\frac{k_P \sqrt{(32g i_M)}}{14.8 \times 1.255 \nu_M} \right]^{2/3} R_P^{1/3}$$

Assuming $k_P = 0.005$ ft for the prototype concrete lined channel, $\nu_M = 1.23 \times 10^{-5}$ sq. ft/s (water at 15°C), and taking a calculated R_P of 4.8 ft and a slope the same in the model as in the prototype, the scale relationship of 24:1, prototype to model, was determined.

61. A sufficiently smooth surface was achieved by building it in concrete, the top surface being accurately finished to within $\frac{1}{32}$ th in. by using a mix made from cement and 'soft-washed' sand. This surface was then treated with two coats of varnish. Subsequent experiments verified that the channel was 'hydraulically smooth'.

62. Both Mr Montagu and Mr Smith commented on the wave angles observed in the model. An increase in roughness would of course reduce the velocity and Froude number. If the k value changed from 0.005 ft to 0.01 ft, the velocity at 4000 cusecs would be reduced by about 7% and the wave angle increased by $2\frac{1}{2}^\circ$. However it was thought that the original estimate of roughness was conservative; a k value of 0.01 ft corresponded to a typical brick-lined channel and it seemed unlikely that a well-constructed concrete channel would deteriorate to such a roughness.

63. It was true that, as Mr Smith pointed out in § 38 and as the Author stated in § 21, the value of β deduced in Table 1 was relatively insensitive to the method of calculation. If the substitution $d=A/B$ was made in equation (4), then the more general expression for x in equation (6) was

$$x = \pi A/B^2 \cos \beta$$

and in the comments of § 5 on the accuracy of equation (5), the term d/B could be replaced by A/B^2 . The value of this term was 0.176 at maximum discharge confirming that the shape had only a small effect on the wave angle.

64. Mr Smith had also commented on the choice of a trapezoidal shape for the channel section. This shape was the first choice for the Alphin Brook on the grounds of economy. Before the start of the investigation there were doubts as to whether the trapezoidal shape would be satisfactory. If that design had failed, the investigation would have been continued to study a section of rectangular shape. In the event however, the trapezoidal shape was satisfactory and it was found that the wave amplitudes recorded were acceptable. As mentioned in § 23, even with perfect suppression of the wave pattern—impossible to achieve in practice even with a rectangular section channel—a reduction in maximum water levels of only 9 in. would have been achieved in a total depth of over 7 ft. Substitution of a rectangular section would have reduced the superelevation at the bends by about 3 in.

65. Turning to the design of contractions, the Author wished to emphasize that the Froude number of a channel running under normal flow conditions did not vary very greatly over a large range of flow, a fact that was not very widely appreciated. He had attempted to clarify this point on two previous occasions^{12,13} in connexion with flow in part-full pipes and in natural channels. For a trapezoidal section channel the Froude number could be determined from equation (31) and for rough turbulent flow was:

$$F = v/\sqrt{gAB} = \sqrt{32i} \sqrt{RB/A} \log (14.8R/k)$$

Hence

$$F/\sqrt{32i} = \sqrt{B/p} \log (14.8R/k) \quad \dots \dots \dots (32)$$

where p = the wetted perimeter.

Since

$$B = b + 2sd \quad \dots \dots \dots (33)$$

$$p = b + 2d\sqrt{1+s^2} \quad \dots \dots \dots (34)$$

$$R = d(b+sd)/(b+2d\sqrt{1+s^2}) \quad \dots \dots \dots (35)$$

it followed that

$$\frac{F}{\sqrt{32i}} = \sqrt{\left\{ \frac{1+2s(d/b)}{1+2(d/b)\sqrt{1+s^2}} \right\} \log \left\{ 14.8(d/b)(b/k) \frac{1+s(d/b)}{1+2(d/b)\sqrt{1+s^2}} \right\}} \quad (36)$$

66. Equation (36) had been plotted in Fig. 16 for four values of side slope s and four values of width relative roughness b/k . It would be seen that the Froude number varied only gradually with depth: for example, the Froude number of a rectangular section channel with a b/k value of 100 was within 10% of $1.8\sqrt{32i}$ in the range $0.034 < d/b < 0.85$.

FIG. 16: FROUDE NUMBER/DEPTH RELATION FOR CHANNELS OF TRAPEZOIDAL CROSS SECTION

67. The Author was unable to understand why Mr Smith thought that the contraction shown in Fig. 12 would be more satisfactory than the straight wall contraction over a wide range of discharge. In the first place the Froude number varied very little with discharge if there was normal flow upstream of the contraction. In the second place Fig. 7 showed that the variation in wave angle β_1 with Froude number was no less at large values of θ than at small values. Thirdly, Ippen and Dawson³ had shown experimentally that straight wall contractions were superior to circular arc contractions in supercritical flow.

68. The spillway transition described by Mr Kabell was of course a different problem since it had to deal with an accelerating flow. The velocity upstream would be nearly constant but the depth would increase and the Froude number reduce as the flow increased. It would be exceedingly difficult to set up the theoretical equations describing the flow at a transition of trapezoidal cross section. However, the assumption made by Mr Kabell that at any flow the surface width of flow downstream was taken as the downstream width of an equivalent rectangular section contraction, appeared to have produced a successful design. Unfortunately pressure of other work prevented the Author from extending his research on this aspect of supercritical flow at present.

69. Finally in reply to Mr Montagu's remarks, it was necessary to say a few words about the nomenclature used in the Paper. Generally this followed current British and American usage. A recent British Standard¹⁴ gave the definition of 'hydraulic jump' as 'the abrupt transition from supercritical to subcritical flow'. Other terms used were not defined in the Standard but an American manual¹⁵ gave definitions of two of the terms:

"Standing wave" —A sudden rise in the water surface, generally fixed in position, such as a hydraulic jump; a standing wave may exist, however, where the principle of the hydraulic jump is not involved.

"Stationary wave" —Same as standing wave."

70. It would be clear, therefore, that the term 'stationary wave' (or 'standing

wave') was used as a general term to describe the wave phenomena discussed in the Paper. The term 'cross wave' referred to the wave formed initially by a *gradual* change in alignment of the wall of the channel and followed Knapp's² usage of the word. The term 'shock wave' or 'oblique hydraulic jump' referred to the wave formed by a *sudden* change in alignment.

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