The flow of water through smooth open channels of narrow rectangular and T-shaped cross sections

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Mr H. O. Phelps (Lecturer in Civil Engineering, University of the West Indies; on Fellowship Leave at University of Manchester Institute of Science and Technology) wrote that the Authors had provided additional data for open channel laminar flow which would be useful in hydraulic model design.

94. Some of the results appeared to need clarification. Measurements of velocity distribution indicated that the maximum filament occurred at some depth below the water surface in the narrow rectangular channel, and that the derivative of velocity at the surface was not zero. No explanation for this somewhat unexpected result had been given.

95. Depression of the filament of maximum velocity might result from any of the following:

- (a) drag of the air/water interface;
- (b) surface irregularities;
- (c) secondary currents;
- (d) inaccuracies in velocity measurement due to the effect of the measuring probe.

96. In the absence of other factors the momentum flux at the surface must be continuous, which was to say that the shear stress τ_w due to the velocity gradient at the water surface must be equal to the shear stress τ_a due to the velocity gradient in the air phase boundary layer formed on the water surface. The relative motion between air and water was equivalent to flow of a fluid over a flat plate at zero incidence, for which the boundary layer remained laminar up to a Reynolds number ($R_x = Ux/\nu$) of 5.5×10^5 . In this case U was the surface velocity, and X the distance of the point of measurement from the upstream end of the flume. For the first profile shown in Fig. 5, the value of R_x had been calculated to be approximately 8×10^5 , so that it was reasonable to assume that the boundary layer was laminar.

97. The corresponding shear stress τ_a was given by the equation

$$\tau_a = 0.332 \mu_a U \sqrt{\frac{U}{\nu x}} \quad \dots \dots \dots (22)$$

which yields a value of 0.14×10^{-6} lb f.ft⁻²

$$\tau_w = \mu_w \frac{dv}{dy} \quad \dots \dots \dots (23)$$

Using a value of 0.33 obtained from Fig. 5 for dv/dy , τ_w was calculated to be 6.9×10^{-4} lb f.ft⁻².

98. The discrepancy between τ_a and τ_w was obviously very large which meant that the velocity gradient at the surface could not possibly be accounted for by surface drag.

99. When surface irregularities were present a velocity gradient might be supported. However with the very low velocities experienced it was assumed that the surfaces were smooth.

100. Weak secondary currents were known to exist in turbulent flow in straight open channels, being especially associated with variations in shear stress over the wetted perimeter at sharp changes of configuration. They were responsible for distortion of the velocity profile and depression of the maximum filament of velocity below the surface. It would be interesting to hear the Authors' comments on the possibility of a similar phenomenon arising in their narrow rectangular channel.

101. Failing the presence of one or more of the foregoing factors, the Authors' results could be accounted for only by an inaccuracy in velocity measurement.

102. In § 63, the value of $K=2.47$ obtained experimentally by Straub *et al.* for a 12:1 width-depth ratio (Table 4) should be compared with the theoretical value of 2.7 and not the 'wide channel' value of 3.

103. Values of f calculated from equation (3) and those determined from velocity measurements using equations (18) and (19) were a little confusing. Since the ratio of the length of wetted perimeter due to the walls to that due to the bed, was 20:1, the average shear stress should be more nearly equal to the wall shear stress than to the bed shear stress. The opposite appeared to be the case; values of $f/2$ (shown in Fig. 11) for the bed were approximately equal to the average (Fig. 10) whereas the corresponding values for the walls (Fig. 12) were appreciably smaller.

104. It was implicit in the statements made in §§ 68-77 that the Authors had equated Reynolds numbers, expressed in terms of mean velocities and hydraulic radii, for geometrically dissimilar cross sections. This procedure was inconsistent with the laws of similarity since the hydraulic radius was merely a convenient length term which was used to express the ratio of inertia forces to viscous forces within a particular flow system.

105. Calculation of the D'arcy coefficient f for uniform flow requires that

$$\sigma P = \rho g i A$$

therefore

$$\frac{\sigma}{\rho} = g m i.$$

106. Thus hydraulic radius was correctly seen to be a significant factor which, however, did not provide a measure of the effect of shear stress distribution. Both f and R_N included the factor m and their values for a particular cross section and discharge could not justifiably be equated to those for a section having different boundary geometry.

107. Table 6 should be amplified to include the results of three investigations relevant to the discussion on critical Reynolds numbers. All show values of R_{NC} appreciably less than 500.

108. Although considerable success had been achieved in the theoretical analysis of laminar flow instability in terms of the occurrence of laminar boundary layer waves, prediction of the actual onset of transition remained obscure. Experimental evidence pointed to the great importance of local effects in promoting the change. None of the flow parameters provided a measure of the velocity distribution effect brought about by boundary geometry. The fact that similar values of $f/2$ were obtained at critical Reynolds numbers for different cross-sectional shapes must therefore be regarded as fortuitous and the suggested relationship between $f/2$ and R_{NC} purely empirical.

109. The Authors' explanation of the physical significance of the term $\sigma/\rho \bar{v}^2$ made it synonymous with the Reynolds number and this was precisely why, in the absence of other factors, the friction term f was inversely proportional to R_N in the laminar range.

110. Hydraulic models were often constructed with hydrodynamically rough surfaces and the commendable work of the Authors on narrow channels might, with advantage, be extended to include such surfaces. The Writer had investigated two-

dimensional laminar flow over rough boundaries and found that, in certain cases, the critical Reynolds number was substantially lowered as the relative roughness was increased.

TABLE 9

Nature of cross-section	Investigator	Dimensions Width Depth	R	m	R_{NC}	Min value of $f/2$
Circular pipes	Laird 1	Various diameters			413	0.005
Rectangular channels	Hopf 2				300-325	
" "	Jeffreys 3	4 Max 0.2	Min 20.0	Max 0.182	310	
" "	Woo 4	6 0.065	0.065	0.065	400	0.0114

Mr T. M. G. Cloughley (Cambridge University Engineering Laboratory) wrote that he was very interested to see the experimental results of the Authors' Paper for an open water-channel having a high depth to width ratio. The Authors' channel was 1 in. wide and velocity measurements in a non-uniform velocity field were made with a miniature screw-type current meter of 0.4 in. dia. He would like to ask whether an adjustment was made to the readings obtained from the meter to take into account the velocity distribution across it.

112. If no adjustment has been made, then all velocity readings indicated, especially those in the laminar flow range, would be less than the true values. For example, referring to Fig. 6, the distribution of velocity across the meter when placed on the channel centre-line was parabolic and the meter would give a measure of velocity which was somewhere close to the mean value across the meter. This value was less than the true velocity on the channel centre-line. A similar argument might be applied to the readings taken in the same horizontal plane at 0.205 in. off the centre-line.

113. In Fig. 10 the Authors had plotted curves of friction factor against channel Reynolds number for the four different stream depths examined. In the laminar flow regime, the curves moved progressively to the right as the channel depth increased. It would be expected that as the depth increased further the corresponding curves would move ever closer to the limiting curve for a channel having infinite depth to width ratio, i.e. the theoretical curve for laminar flow between two infinitely wide parallel plates, which was also plotted in Fig. 10. In fact the Authors' curve for the 10 in. depth was seen to lie to the right of this limiting curve which was incompatible with the above argument. This might be explained by the velocity readings taken from the current meter being less than the true values. Since the friction factor was inversely proportional to the square of velocity and the Reynolds number proportional to the velocity, an increment in velocity would cause the friction factor to decrease more rapidly than the Reynolds number increased. Thus if all velocities were higher than stated by the Authors all their curves in Fig. 10 would be moved to the left. It was then probable that the curve for the 10 in. depth would lie to the left of the theoretical limiting curve, as was to be expected.

114. This argument would explain the apparent anomaly in Table 3. The theoretical values of K for increasing depth approached the asymptotic value 3.00. The experimental value was 3.20 at the 10 in. depth, and this might be reduced to below 3.0 if the velocities were higher than measured.

115. The Authors arrived at the conclusion that for deep flow the filament of maximum velocity in the central vertical plane lies below the surface. This might be partially explained by the drag of the air above the stream retarding the flow near

the free surface. In the investigations of Iwagaki, Delleur and Straub *et al.*, quoted by the Authors, the maximum depths of flows were always less than 2.2 in. At these depths, the flow may be wholly laminar boundary layer on the channel bottom, and the drag of the air above the free surface might be insufficient to change the velocity profile giving the maximum velocity in the free surface.

116. The values of critical Reynolds number deduced from the dye traces were lower than those obtained from curves of f against R_N . This was to be expected. The glass tube through which the dye was ejected itself produced a disturbance in the stream, and at values of R_N approaching the beginning of transition, this disturbance could be sufficient to trigger transition.

117. The velocities near the bed measured by means of the deflexion of a sphere of plasticene should be treated with some suspicion because of the complex nature of flow around a sphere in a shear layer.

118. The investigation into the flow in a channel of T-section had provided some very interesting results.

The Authors, in reply, wished to express their gratitude to Mr H. O. Phelps and Mr T. M. G. Cloughley for the interest they had taken in the Paper.

120. Regarding Mr Phelps's remarks they disagreed that the relative motion between the water and air was analogous to that of fluid passing over a stationary flat plate at zero incidence. In the latter case there was the sudden change of conditions experienced by the fluid as it reached the leading edge of the plate, followed by a boundary layer of varying thickness tending to turbulent flow as the downstream distance increased. In the narrow open channels used by the Authors there was little if any increase in the tendency to turbulence as one travelled downstream, and the interaction between the two fluids would appear to be sensibly independent of the location along the length of the channel, except possibly at the inlet and outlet. Also the air, unlike the stationary plate, would have its own boundary layer.

121. It would therefore appear incorrect to employ the Reynolds number based on the distance from the entry as a criterion of laminar or turbulent flow, and to calculate the shearing stresses on that basis.

122. In addition to the four possible causes listed by Mr Phelps for the depression of the filament of maximum velocity, the Authors would mention surface tension which was probably quite important in such narrow passages.

123. Also there was very often a thin film of grease on the surface of slow moving liquids which tended to retard the flow. The Authors recalled that in the models designed by the late Professor A. H. Gibson he had always insisted on providing a small open-topped box at the downstream end of the model which served to skim any film of grease from the underside of the surface, thus eliminating distortion of the scale between the surface velocities of the model and those of the prototype. With the much higher velocities and greater surface area of the prototype the formation of a film of grease would be most unlikely as it would naturally get swept away downstream by the main body of the flow.

124. The value 2.47 of K obtained by Straub *et al.* was quoted simply because it was the value corresponding to the widest channel appearing in Table 4. It was then compared with the theoretical value, i.e. 3, for a channel of infinite width to depth ratio in order to demonstrate that even in the case of the widest experimental channel in Table 4 the results were still appreciably different from those which one would expect if the drag exerted by the side-walls were negligible.

125. The Authors had used the Reynolds number expressed in terms of the mean velocity and the hydraulic radius merely to ascertain how far the hydraulic radius was a criterion of the effect of shape as regards the critical Reynolds number and the friction coefficient.

126. It was not claimed in the Paper that there was conclusive evidence that the demarcation between laminar and turbulent flow occurred at the same value of the

friction coefficient f , irrespective of the shape of the cross section. Nevertheless there was certainly some evidence pointing in that direction. Since the Paper had been published, the Authors had discovered a Paper by Metzner and Reed¹⁹ in which they proposed a criterion for the onset of turbulence in pipes, based on the friction factor in the even more complex case of non-Newtonian fluids.

127. As Mr Phelps suggested there was a great deal of research still necessary into the principal factors influencing the type of motion occurring in channels with hydro-dynamically rough boundaries, but of course any investigations into transitional flow were of necessity very slow in yielding information.

128. The Authors agreed with Mr Cloughley that the current-meter was rather large compared with the width of the channel, but at the time when the research was performed it was the smallest instrument available. In order, however, to satisfy themselves that the readings of velocity were sufficiently accurate the Authors had several times calculated the total rate of discharge by the summation of the product of velocity and elementary area over the whole cross section. This was always found to agree so closely with the measured rate of discharge using the weighing machine and stop watch that it was considered unnecessary to apply any correction to the readings of the current-meter.

129. Perhaps the curve for the depth of 10 in. was slightly too far to the right in Fig. 10, but it was sufficiently near to the theoretical curve corresponding to infinitely wide parallel plates for the discrepancy to have arisen from quite a number of possible causes other than inaccuracy of the meter: for example the resistance imposed by surface tension or the drag of the atmosphere.

130. As Mr Cloughley remarked, the presence of the glass tube for the injection of the dye was itself a disturbing influence on the flow, but this method was nevertheless useful in providing an additional and more pictorial demonstration of the pattern of flow.

131. The measurements with the deflected sphere had been found to be in good agreement with those of the current meter when the two had been used at similar depths, and it was believed that although certainly not affording an ideal means of measuring the velocity, the sphere provided a satisfactory method in a region where so many of the other known techniques would have been useless.

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