Deformation of continuous prestressed concrete beams and its effect on ultimate load

E. W. BENNETT, N. COOKE & L. P. NAUGHTON

Dr N. F. Somes, Woolwich Polytechnic, Department of Civil Engineering

The Authors have presented an impressive amount of experimental data relating to the strength and deformability of continuous prestressed concrete beams. This information was obtained from tests on beams of small cross section, on model beams in fact. What prototype structures do the models truly represent? Taking a scale factor of say four and referring to Fig. 1 the models might represent a prestressed beam 24 in. $\times$ 16 in. with a cable duct 6 in. in diameter, through which are stressed and grouted three bars of 1.10 in. dia. The mild steel reinforcement, to the same scale factor would consist of four 1 in. dia. bars having a cover to their centres of something in the order of 3–4½ in. The provision of stirrups was not reported.

37. Do we construct full-scale beams like this in practice? The answer is that we might, but only rarely. If the model beams were not typical of the majority of sections in practice, what aspects of their behaviour might cause the results to be unrepresentative in terms of strength and deformability? I suggest that the most important aspect is bond, which governs the spacing of cracks and the crack widths.

38. A programme of experimental work on the same subject as the Paper has been pursued by the Cement and Concrete Association since 1962. It was found by the Association,¹³ and elsewhere,¹⁴ that the bond between steel and concrete had a most significant effect on curvature distribution and rotation values. It is already widely known that bond has a significant effect on the strength of sections. The following data from reference 13 will demonstrate the influence of bond. Two very similar beams of section 11 $\times$ 6 in. were simply supported on a span of 110 in., and subjected to equal point loads at the ¼ span points. Each beam was post-tensioned with seven 0.276 in. dia. wires within a metal sheath of 1½ in. internal diameter. Neither beam had secondary reinforcement or stirrups. One beam was grouted and the other was left ungrouted. The beams therefore represented extreme cases of bond. Table 4 shows a comparison between the conditions in the beams at their maximum load.

Table 4

	Grouted beam	Ungrouted beam
Number of visible cracks	14	5
Total rotation along the span in radians . .	0.0458	0.0321
Moment of resistance (kip-in.)	492	358

It is seen that the two extreme conditions of bond produce greatly differing values for strength, cracking, and deformability. A similar result was obtained from a comparison of beams having a single point load at mid-span.

DISCUSSION

39. Where did the Authors' beams lie between the extremes of good bond and no bond? I would welcome some information on the numbers of cracks which developed at critical zones and the crack spacing factor which Guyon defined as $\Delta L/d$

where ΔL is the average crack spacing
 d is the overall depth of the section.

40. Of the 69 beams discussed by various workers it has already been noted¹³ that those of references 3 and 6 possessed poor bond, with $\Delta L/d$ frequently exceeding 1.0. A crack spacing factor of this magnitude is unlikely to be met with in full-scale structures.

41. Several points of construction were not covered in the Paper. Were the mild steel bars located in the section by means of stirrups, and if there were stirrups, where were they placed along the beam? The effect on the moment-rotation relation of just one stirrup placed strategically in a critical zone is very significant. How were the various cable profiles of Fig. 1 achieved in practice, and what was the means of forming the ducts?

42. The values for α in Table 1 suggest high shears at supports B and C. In the apparent absence of stirrups, what effect did shear have on the failure mechanism?

43. Were the effects of friction in the cable ducts considered in arriving at the value of the prestressing force? It would be interesting to know the reasons for the change in the diameter of the cable ducts from $\frac{3}{4}$ in. in series E to $1\frac{1}{2}$ in. in series F, particularly since the latter represents $\frac{1}{2}$ of the beam depth.

44. In § 13 the Authors state: 'the strains are measured over a flexural crack'. Surely, what is meant is 'displacements' rather than 'strains'. Furthermore, is not the 'third method' of measuring curvature, namely that of measuring strains in the compression zone, the only realistic means of doing so? Curvature should not be based on the measurement across air gaps, i.e. crack widths, which is what methods 1 and 2 would achieve.

45. It would be helpful to know what the phrase 'ultimate load' means in the context of these beam tests. Is it the instantaneous value of load including rate-of-straining effects, or the value at a strain held reasonably constant? The Paper does not make clear just how load was controlled.

46. In §§ 18 and 22 the Authors have fallen into a trap which has previously caught me. According to Professor A. L. L. Baker, the experimental θ or θ_p , as it is known in the 1962 ICE Research Committee Report, is not given by the hatched area on the experimental curvature distribution diagrams in Fig. 8. Professor Baker states¹⁵ that the measurement of θ_p should commence when the concrete in the critical zone reaches a strain of 0.002 namely, state L_1 . The net result of this is that the experimental rotation angles listed in Table 2 are not derived on the basis that the ICE Research Committee Report intended.

47. I applaud the Authors' decision (§ 29), to use experimental values for maximum moments of resistance rather than values calculated arbitrarily. The accuracy of the calculation of these moments under these concentrated loads is considerably improved, however, if the enhancement of the flexural compressive strength in these conditions is taken into account, as described in reference 13.

48. Fig. 11 shows the value of taking a beam test as far as one reasonably can. The falling branch to the $M-\Delta$ curve for section C is quite steep. It is now known, however,^{13, 16} that, with the addition of suitable stirrups, $dM/d\Delta$ after crushing can be held zero for considerable increases in deflection or even given small positive values.

49. To date, the simplest and most attractive means of assessing the collapse load for simpler hyperstatic structures is the application of simple plastic theory developed by Professor J. F. Baker for steel. Four two-span prestressed concrete beams of section 10×5 in. tested at the laboratories of the Cement and Concrete Association were all shown¹⁶ to exceed the collapse load derived using simple plastic theory and flexural capacities based on BS CP 115. On average they exceeded the calculated

Fig. 12 $(M/M_u)/(\theta/\theta_u)$ relation (average values)

collapse load by 19%, the range being 13–23%. If this excess, ranging 13–23%, is regarded as a margin to accommodate lower standards of construction prevailing outside research laboratories what better method can the designer want to assess the collapse load of such simple hyperstatic structures?

Drs S. K. Mallick and M. K. L. N. Sastry, Indian Institute of Technology, Kharagpur

The results of the investigations on the behaviour of continuous prestressed concrete beams which we have conducted are already available in published form,^{17–20} and in order to avoid unnecessary repetition, reference will be made to the relevant tables or figures directly.

51. The main problem in the formulation of any method for the computation of redistribution in continuous beams is to find a suitable procedure for the assessment of the plastic rotations that may take place at the critical hinge sections at failure of the structure.

52. The examination of the results of 26 three-span beams and 32 two-span beams by the application of Guyon's method indicates that the degree of redistribution is underestimated (Ref. 19, Tables 9 and 10). The possible reasons for the deviation have been discussed elsewhere.^{17,19,20} They may be summarized as follows.

53. The states of deformations at the two critical sections are different: the first section exhibits larger compressive strain and curvature (Ref. 17, Table 5; Ref. 19, Table 5).

54. Whereas the ultimate moment is not significantly affected by the variations in the ultimate compressive strain in concrete, the curvature is directly proportional to it. As such, the moment–curvature relation will vary from one section to another in contrast to the standard form assumed by Guyon. This is shown very clearly in Ref. 17, Figs 10–12.

55. It is interesting to note that the test values in Fig. 9 of the Paper do not show large scatter about the proposed standard curve for *all sections*. Our experiences have been altogether of a different nature and we have not been able to derive any reasonable standard curve either for the first or the second section.

56. The properties of the moment–curvature relations for 26 beams, summarized in Ref. 19, Table 5, and their average values plotted in Fig. 12, clearly support our contentions. The observations made by Ghosh¹⁸ are similar.

DISCUSSION

57. We have attempted to assess the rotations actually taking place at the hinge sections experimentally. The preliminary results were presented by Sastry (Ref. 19, Fig. 3) and the detailed results for continuous and simply supported beams are available elsewhere.^{19,20} We are pleased to note that the Authors' approach is similar, that is to plot the distribution of curvature in the tests all over the beams.

58. Reference to the three curvature distribution diagrams (Ref. 18, Fig. 3; Ref. 19, Figs 7 and 8) illustrates the physical behaviour of the beams quite clearly: at the first hinges the rotations are larger than those obtained theoretically on the basis of a standard curve and at the second hinges, the rotations are smaller.

59. The curvature along the length of the beam is invariably related to the nature of cracking²⁰ and at best, the hinge rotations can be assessed approximately.

60. The tests we conducted cover several hard cases^{17,19}: in two-span beams, loads have been placed as close as $0.225 L$ (where L = the span) on both sides of the support, and the ratio of the side to central span for the three-span beams loaded at the centre of the mid span has been as high as 1.50. The results show that complete redistribution is a possibility in all but the exceptional cases, and we note that the Authors have also drawn similar conclusions.

61. We have not been able to understand clearly the method proposed by the Authors in § 25. The term θ is defined as rotation, i.e. radians, and then one has to refer to the curvature diagram to compute the ultimate moment. Is it implied that the moment-curvature diagram for a hinge section in dimensionless form is identical with moment-rotation diagram for the section?

62. The proposed method gives close agreement with the results of our tests and it would be useful to know the basis on which the expressions are established.

Dr Bennett, Dr Cooke, Mr Naughton

The Authors are grateful to Drs Somes, Mallick and Sastry for raising a number of interesting points from their own experience of similar tests.

64. Further details of the experimental work are contained in two theses.^{21,22} All the beams were provided with $\frac{1}{4}$ in. mild steel stirrups, generally at 6 in. centres but at 3 in. centres in the regions of high shear to obviate the risk of premature shear failure noted by Dr Somes. The ducts were formed by an inflated Ductube held in position by steel rings secured to the sides of the mould by threaded rods and bolts and the increase of the duct diameter to $1\frac{1}{2}$ in. in series F was made in order to use commercially available polystyrene spacers instead of specially made spacers. The prestress in the cable was measured at the time of testing by dynamometers consisting of calibrated steel tubes at each end, visible in Figs 2 and 3. The friction loss was found to be only a few hundred pounds when tensioning from one end; nevertheless some of the beams in series E and all in series F were tensioned from both ends.

65. The tests were performed by applying predetermined values of load, but to keep the deformation approximately constant while measurements were made, the deflexion rather than the load was maintained constant. In view of the large number of measurements the overall rate of loading was rather slow. In the latter stages of some tests the load tended to decrease as the deflexion continued to increase as in Fig. 11, and the maximum value of load was taken as the ultimate load.

66. The spacing of the cracks in the critical lengths of the beams in series F varied between 2.5 in. and 3.9 in. and was usually about 3 in.; similarly, in series E the ratio of crack spacing to depth was between 0.4 and 0.6. The Authors cannot understand the conclusion drawn by Dr Somes¹³ from the earlier Leeds curvature tests,⁶ as in that series the only beam at all comparable in section to the continuous beams was C5 which displayed a spacing/depth ratio of 0.64 although only 5 in. deep. To write, in the present context, of the ratio as 'frequently exceeding 1.0' is therefore misleading. The bond in all the beams with grouted cables was good, although the spacing/depth ratio in a larger beam with well-designed reinforcement would have been somewhat less.

67. The decision to base the measured curvature on the strain in the compressive zone was made on grounds of expediency rather than principle. Methods 1 and 2 are regarded as more correct in that they take account of the full depth of the member, part of which should not be ignored in assessing the deformation of the whole member simply because cracks are present. Provided one does not require the average curvature over lengths less than the crack spacing, the results obtained by the three methods are about the same, but errors can arise when widely spaced cracks occur in conjunction with a sharp peak in the moment diagram so that the critical zone only contains one or two cracks.

68. The Authors consider this last point to be the major factor in the problem of scale, to which Dr Somes draws attention. It has been shown elsewhere⁶ that the effect is to increase the randomness of the amount of redistribution, and tests on small specimens are therefore as likely to overestimate as to underestimate the amount of redistribution for larger beams.

69. Dr Somes is correct in pointing out that the shaded areas in Fig. 8 do not correspond exactly to the rotation angles θ and ψ in the ICE Report. A more accurate assessment could be made by drawing the elastic part of the curvature diagram with the average EI values referred to in § 19, instead of the values for the uncracked member (EI).

70. The work of Drs Mallick and Sastry is of interest in clearly demonstrating that greater curvature can occur under the restrained conditions present in a continuous beam. This would account for the different shape of the experimental specific moment-curvature diagram in the Author's Fig. 9(c) as compared with the theoretical curves of Guyon and Macchi. In using a similar relationship for specific moment-hinge rotation the Authors have assumed that the shape of the moment diagram at all the hinge points is approximately the same, and here the method may need some adjustment, as in the example of F15 in § 27.

71. In design practice it would certainly be simple and attractive to assume complete moment redistribution as for steel structures, but the Authors hesitate to endorse this suggestion of Dr Somes in view of the considerable evidence, not all obtained from tests of small beams, that this does not always occur. The margin of strength in the BS CP 115 formula for ultimate moment is provided to allow for fluctuations in the quality of materials and workmanship and if this is correct for statically determinate members it is obviously insufficient also to cover incomplete moment redistribution in hyperstatic structures. The Authors have, therefore, advanced their proposal as a simpler alternative to the method of Guyon and the modifications developed by Drs Mallick and Sastry.

References

13. SOMES N. F. *Moment-rotation characteristics of prestressed concrete members. Stage I—Rectangular sections.* Cement and Concrete Association, London, 1966, pp. 80, TRA/398.
14. WARWARUK J. *Strength and behaviour in flexure of prestressed concrete beams.* University of Illinois, Urbana, September, 1960. Structural Research Series No. 205.
15. Private communication, Professor A. L. L. Baker to Cement and Concrete Association, 27 February, 1967.
16. SOMES N. F. *Moment-rotation characteristics of continuous prestressed concrete members.* PhD thesis. University of London, 1966.
17. MALLICK S. K. Redistribution of moments in two-span prestressed concrete beams. *Mag. Concr. Res.*, 1962, **14** (Nov.) 171–183.
18. GHOSH S. K. and SASTRY M. K. L. N. Discussion, ref. 17. *Mag. Concr. Res.*, 1964, **16** (Sept.) 173–176.

DISCUSSION

19. MALICK S. K. and SASTRY M. K. L. N. Redistribution of moments in continuous prestressed concrete beams. *Mag. Concr. Res.*, 1966, **18** (Dec.) 207-220.
20. MALICK S. K. and SASTRY M. K. L. N. Plastic rotations in the computation of redistribution in continuous prestressed concrete beams. *Ind. Concr. J.*, 1967, **41** (Feb.) 83-88.
21. NAUGHTON L. P. *A study of some statically indeterminate structures in prestressed concrete.* MSc thesis, University of Leeds, 1963.
22. COOKE N. *The redistribution of bending moments in continuous prestressed concrete beams.* PhD thesis, University of Leeds, 1965.