Investigation of multi-folded plate structures

by

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Mr M. Sas-Skowronski (Senior Engineer, Greater London Council) wrote that although not primarily aimed at presenting a new idea, the Paper was nevertheless useful in overcoming the inadequate recognition of the special advantages offered by this type of computation and he wished to congratulate the Authors on providing a better general understanding of the behaviour of folded plate structures. However there were a number of points on which a comment should be made.

47. In recent years folded plates had become increasingly popular, their use ranging through a variety of structures to which the engineer's imagination alone set limits. It was therefore understandable that the design theories of folded plates had during the past few decades made considerable strides towards both simplicity and accuracy. Out of a number of approaches so far developed, few had been acknowledged by engineers, and the writer considered the 'degenerate theory' which required additional knowledge of the shell design, had very little chance to be accepted as a popular theory.

48. The folded plate shape of roof structure had come into wide use because of its low construction cost for long span, high load-carrying capacity, rigidity, and because of its aesthetic appearance. The profile of the folded plate roof had an important bearing on the economy of the structure. The configuration could, however, have many variations to fit the requirements of the construction.

49. Any analysis would indicate that the corrugated configuration was unsuitable for conventionally reinforced concrete because of the very small area of concrete available at the ridge, which caused high stresses and thus unnecessarily increased the amount of reinforcement. This could be attributed primarily to the shallow ratio of depth from ridge to valley for the long span, and to the absence of sufficient concrete area in the ridge. Several remedies for this condition were apparent, one of them being to prestress the structure in order to use the whole of the concrete area as a homogeneous section. Tensile stresses, especially at the supports, could easily be prevented by arranging longitudinal prestressing wires parabolically to locate the resultant force between the kern points.

50. Since a considerable number of plate structures constructed today were prestressed, the method given by the Authors would be considerably enhanced if it were to be extended to include the determination of stress induced by prestress force. Some designers had suggested using the folded plate analysis as an approximate solution for barrel shell structures. The writer did not agree either with them or with the Authors in their use of the shell analysis as an approximation in the analysis of folded plates. Each structure had its own characteristics, and therefore a proper individual, simple and logical approach should be made. Applied theory should be simple (if possible), giving an undistorted picture of the statical behaviour of the structure.

51. The writer had the impression that mathematical analysis was frequently regarded as the essence of theory, but it was often forgotten that the assumptions underlying any theory were the most basic and important elements of that theory. For the convenience of readers, the assumption on which the theory was based should have been indicated. The writer assumed that the Authors' assumptions were those usually made in folded plate analysis but these were not given in the Paper. The most important were that the structural material was homogeneous and isotropic (uncracked), and that it had a linear elastic stress/strain relationship. Folded plates constructed of reinforced concrete certainly did not satisfy these assumptions. A theory should be based on assumptions concerning the properties of materials.

52. From the study of the Paper, the writer could not understand why the Authors had made long mathematical analyses which did not appear to be accurate. Most laboratory tests on the structural models ignored shrinkage, creep, and thermal expansion, and it was thus difficult to know exactly how much total expansion and contraction could be expected on a job. Folded plates, like shells, used thin sections with concrete distributed over a large surface. The load-carrying capacity of these sections, because of their thinness, depended upon the local as well as the general stability of the section.

53. Folded plate design varied from that specified in the typical Code of Practice in the method of analysis, precautions against buckling instability, camber and deflexion controls, and permissible combined flexural, shear and axial stresses. Somewhat higher factors of safety were recommended than those normally required for other types of structure, and on most concrete folded plates a factor of safety of 3.5 to 5 was provided. Changes in stresses caused by geometrical displacements should be included in the design. In cases where the plate width was narrow compared to the span, and the break in the surface at the connecting edges was small, strains in the plane of the plate might cause large local displacements, a smaller edge support, and consequent buckling of the member. While the cracking in the deep corrugated sections might be less than in slender conventional beams, the light reinforcement was rather more difficult to hold to fine tolerances, particularly on the sloped sections. At points of restraint, such as stiffener ribs, frames, and end-beams, special consideration should be given to the plate design. Although materials expanded uniformly as the temperature rose, structures did not. A structure would tend to deform or buckle until restraint was overcome. Normally, folded plates were able to accommodate volumetric changes by small sideways and flexural adjustments. However these thin plates were subject to quick and intensive volume changes and sections would invariably crack where these motions were restrained by thicker, slower-acting frames and rigid beams. At this point basic assumptions were no longer valid, unless at these restraining points the slab was reinforced at the juncture to resist and distribute cracking.

54. It had been common practice to disregard the additional transverse and longitudinal stresses resulting from the resistance of the slabs by transverse bending to the relative displacements of the joints, and the writer believed that the Authors had disregarded them also. If two plates met at an angle considerably different from 180°, the displacement of the common joint could not occur without appreciable deflexion of one or both plates in their own planes. Since the deflexion was associated with plate bending, which involved spanning of the plates as deep beams between end diaphragms, the relative displacements of the joints of a cross-section would usually be small. It was safe to neglect the stresses resulting from the plate rotation only in structures of certain proportions: in many structures those stresses might equal or exceed the stresses calculated from other causes. While they probably had only a minor effect on the ultimate strength of a properly reinforced structure, they might nevertheless result in offensive cracking and deflexions. The geometry of the structure was very important, and having chosen an almost ideal model to prove the correctness of their theory, the Authors' theories were of doubtful value.

55. In engineering, evidence was gathered from various sources, namely imagination, analysis, experiments, and experience. Each of these sources was important. Research in structural engineering, as in other fields, was welcome and was receiving increasing attention from industry. An important aspect of analytical, experimental, and design research was to develop and apply new methods suitable for obtaining solutions to problems in civil and structural engineering. Attention should be given to the development of simplified analytical procedures and, by means of analytical methods, models, or analogues, to determining quick, logical, and easy solutions. In any case, these studies should and must be correlated with the results of observations made on actual structures.

56. What should be kept in mind was that most folded plate structures could be designed quite easily without the use of differential equations or an awkward number of simultaneous equations. Considerable additions had recently been made to analytical knowledge and experience, but a review of all known theories was beyond the scope of the discussion.

The Authors thanked Mr Sas-Skowronski for his contribution and in reply offered the following comments.

58. They had been mainly concerned with demonstrating the accuracy of the 'degenerate theory' when applying it to folded plate structures of the types commonly used in practice. If a design office or a computer service had a multi-cylindrical shell programme available, then the analysis and hence design of any type of folded plate structure was extremely simple if the 'degenerate theory' was used. In fact all that was necessary was that in the data tape the half angles of all shells should be altered to 1° and the radii were then adjusted to $R = C/2 \sin 1^\circ$ where C was the width of plate. From the designer's view point knowledge of shell theory was not necessary. The use of such a computer programme thus allowed a whole range of possible types of folded plate structures to be examined and the most suitable aesthetically and economically to be selected very quickly.

59. The Authors would agree that if uniform plate thickness was preserved throughout this was insufficient at the intersections for economical use of reinforcement. It was however sometimes standard practice to introduce a local thickening of the plates at these points to overcome this. The alternative of post tensioning at suitable points had of course been employed in practice.

60. As to whether or not prestressing should be incorporated in the programme, the Authors would refer Mr Sas-Skowronski to an earlier paper² in which use was made of a general multi-shell programme with edge beams, the latter being prestressed if desired. In the present Paper, folded plates had been analysed using this particular programme together with the 'degenerate theory', and prestressing of edge beams might thus be incorporated if required. Perhaps the following example of a treble bay monitor roof, each bay consisting of three plate elements, the outer two plates being inclined at 45° to the vertical whilst the central connecting plate is horizontal, would prove of interest. Slabs are 4 in. thick and have equal widths of 14 ft as shown in Fig. 12, the extreme inclined slabs being supported by edge beams 3 ft by 1 ft, and the junctions of all other slabs supported by edge beams of 1 ft square cross-section. The overall length of the structure is 60 ft and the total applied load is 80 lb/sq. ft. The structure is symmetrical about the central horizontal slab and to conserve space only the results for slab 1 will be given for two cases:

- (i) no prestressing force in any edge beams (see Table 1);
- (ii) outer edge beams prestressing force 120 000 lb, eccentricity 1.0 ft, lower inner edge beams prestressing force 50 000 lb, no eccentricity (see Table 2).

61. It would be seen that the edge beam moments M_1 and M_2 and the direct force F have been considerably reduced by post tensioning. The transverse bending moment M_s in the plate has also been considerably reduced and the normal deflexion

FIG. 12: TREBLE BAY MONITOR ROOF

TABLE 1: PLATE RESULTANTS, NO PRESTRESSING

Edge beam A

$$M_1 = 136\,879 \text{ lb/ft/ft.} \quad M_2 = 15\,551 \text{ lb/ft/ft.} \quad F = 121\,857 \text{ lb.} \quad (N_x)_A = -6510 \text{ lb/ft.}$$

Plate (1)

ϕ	M_ϕ lb ft/ft	N_ϕ lb/ft	N_x lb/ft	$10^6 w/E$	$N_{x\phi}$ lb/ft
1°	1027.2	-304.7	-6510.4	-30.6	-6380.4
-1°	-3389.6	-2330.1	-5162.0	-0.0	-2107.6
0.5°	1114.0	-1108.1	-6306.5	-25.8	-5203.9
-0.5°	-1027.4	-2106.0	-5382.7	-6.2	-3065.8
0°	441.5	-1703.7	-5151.9	-16.7	-4090.6

TABLE 2: PLATE RESULTANTS, PRESTRESSED EDGE BEAMS

Edge beam A

$$M_1 = 57\,816 \text{ lb/ft.} \quad M_2 = 6862.5 \text{ lb/ft} \quad F = -27\,057 \text{ lb.} \quad (N_x)_A = -11\,279.5 \text{ lb.}$$

Plate (1)

ϕ	M_ϕ lb ft/ft	N_ϕ lb/ft	N_x lb/ft	$10^6 w/E$	$N_{x\phi}$ lb/ft
1°	256.7	-481.8	-11\,279.5	-13.5	-6583.3
-1°	-2617.9	-2131.9	-2\,833.6	0.8	-1711.2
0.5°	742.7	-1256.6	-8\,726.8	-11.9	-4760.1
-0.5°	-650.5	-2011.1	-4\,500.7	-2.0	-2376.7
0°	444.9	-1741.3	-6\,473.8	-7.7	-3375.2

of the plate is likewise considerably diminished at the edge $\phi = 1^\circ$. In fact the post-tensioning force of 120 000 lb at 1 ft eccentricity in the outer edge beam A is just sufficient under working load as it gives a tension stress in the bottom fibre of the beam of some 290 lb/sq. in., which is probably just allowable in the case of pre-stressing.

62. The Authors would agree that applying plate analysis to barrel vault structures was a considerable waste of computer time compared with using a shell programme for the same case.¹

63. The 'degenerate theory' was not approximate, its accuracy being at least equal to that of the classical plate theory of Goldberg and Leve⁵ as developed by Scordelis,⁶ and hence considerably more accurate than the so-called 'ordinary methods' of Winter and Pei,⁷ Gafaar,⁸ Simpson,⁹ and Yitzhaki.¹⁰ In other words it was as accurate as the standard elastic theories. Even though Mr Sas-Skowronski did not agree with using the degenerate theory he must agree that its assumption, for one plate, of small angle and large radius was close to a flat plate and that experimental and theoretical values agreed remarkably well.

64. The assumptions involved were those of elastic shell theory and did assume that the material was homogeneous, isotropic, and linear elastic. The same assumptions were made however for the elastic analysis of any type of statically indeterminate concrete structure; what was surely important was the load range over which the analysis was applicable. In a recently conducted investigation by the senior Author at the University of Manchester, tests on large scale multi-folded plate structures of micro-concrete reinforced with mild steel had indicated that such structures behaved much more elastically up to working load conditions than previously suspected, despite obvious cracking.

65. Mr Sas-Skowronski had mistaken the Authors' intentions, since the mathematics and hence the programme was for a previously derived cylindrical shell analysis.² It was discovered by the senior Author that this general multi-cylindrical shell programme normally used for multi-shell investigation could in fact be used also for the analysis of folded plate systems. Moreover, its accuracy was equal to that of the elastic plate theory when compared with the theoretical and experimental work of Scordelis⁴ in the United States.

66. Of the models tested, the cement mortar model was certainly, during its construction and testing, subjected to both shrinkage and creep and also to any temperature rises, small though they might be in the laboratory. Later tests on much larger folded plate structural models had, as far as possible, been subjected to the conditions which might be expected in practice, the aim being to keep the tests as realistic as possible.

67. The majority of folded plates in practice did not have edge beams at plate junctions, and moments were relatively small at the traverses or edge beams so that cracking was unlikely to be due to these. However, diagonal tension due to shear at the end frames or traverses must be guarded against just as in the case of shells.

68. The degenerate theory considered compatibility of displacements, rotations, and equilibrium at each edge of every plate. Thus the displacements and rotations of one plate, at a joint, were considered to affect the forces and displacements in the next plate. The accuracy of the 'degenerate theory' had already been partly established by comparing it with test results by Scordelis⁴ on a much more complicated model and with previous tests by the Authors.¹

69. In the Paper the models had been selected to resemble those more commonly used in practice and thus to investigate the applicability of the degenerate theory for practical design. The geometry of the models was chosen with a length/width ratio of 3 as this was approximately the smallest size used in practice; moreover the overall geometry was typical.

70. The Authors completely agreed with Mr Sas-Skowronski that structural research should receive not only attention but encouragement from industry. In

point of fact quite a few consulting firms had used this particular programme to design folded plate structures, as it was extremely quick and reliable in producing design results.

71. As to the correlation of theory with stresses and deflexions observed on actual structures, this was obviously the best model test of all. Unfortunately, such full scale tests on actual structures were seldom conducted because of the cost involved.

72. It was doubtful whether any method of analysis save that of the more approximate type (the value of which was questionable) was as quick or reliable as that given by computer analysis.

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